



W. Macquorn Rankine

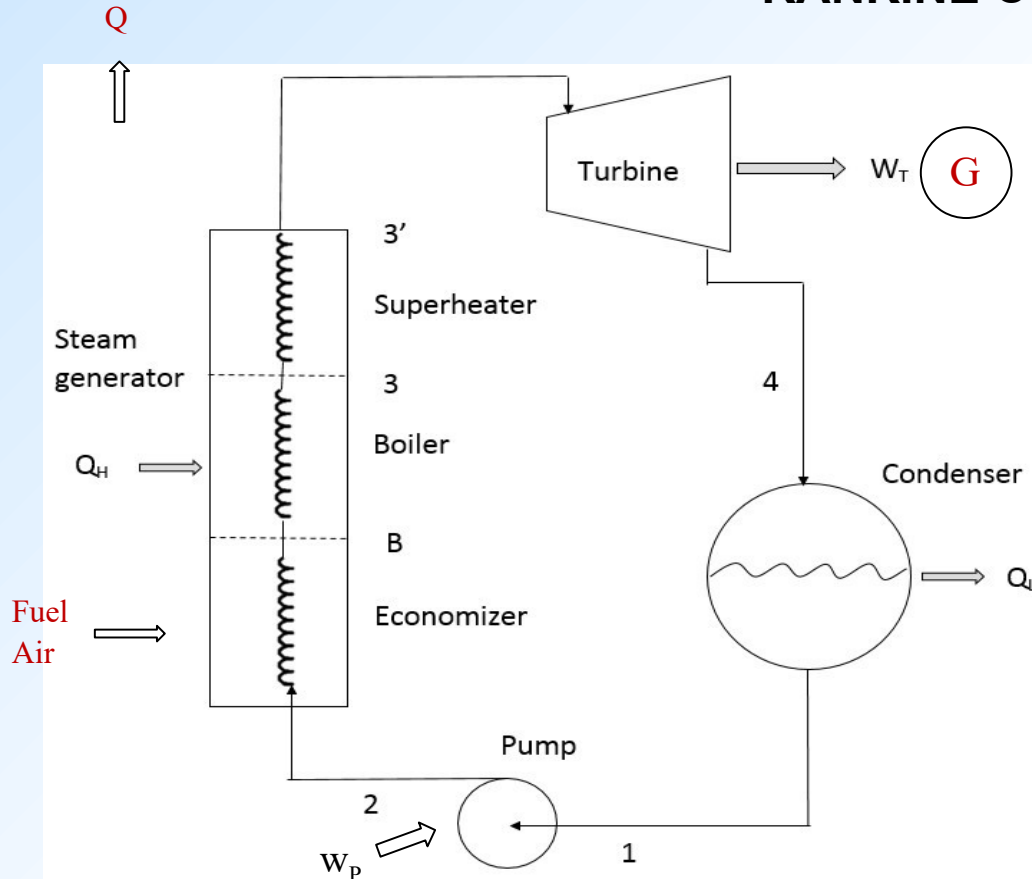
William John Macquorn Rankine

Scottish Engineer

1820 - 1872



RANKINE CYCLE



Heat exchangers are used for heat addition and rejection.

External heating / combustion

- Generally less polluting than internal heating
- Can use lower grade fuels
- Can use solar, nuclear, geothermal sources

Common working fluids are: water, potassium, sodium, rubidium, ammonia, hydrocarbons, etc.

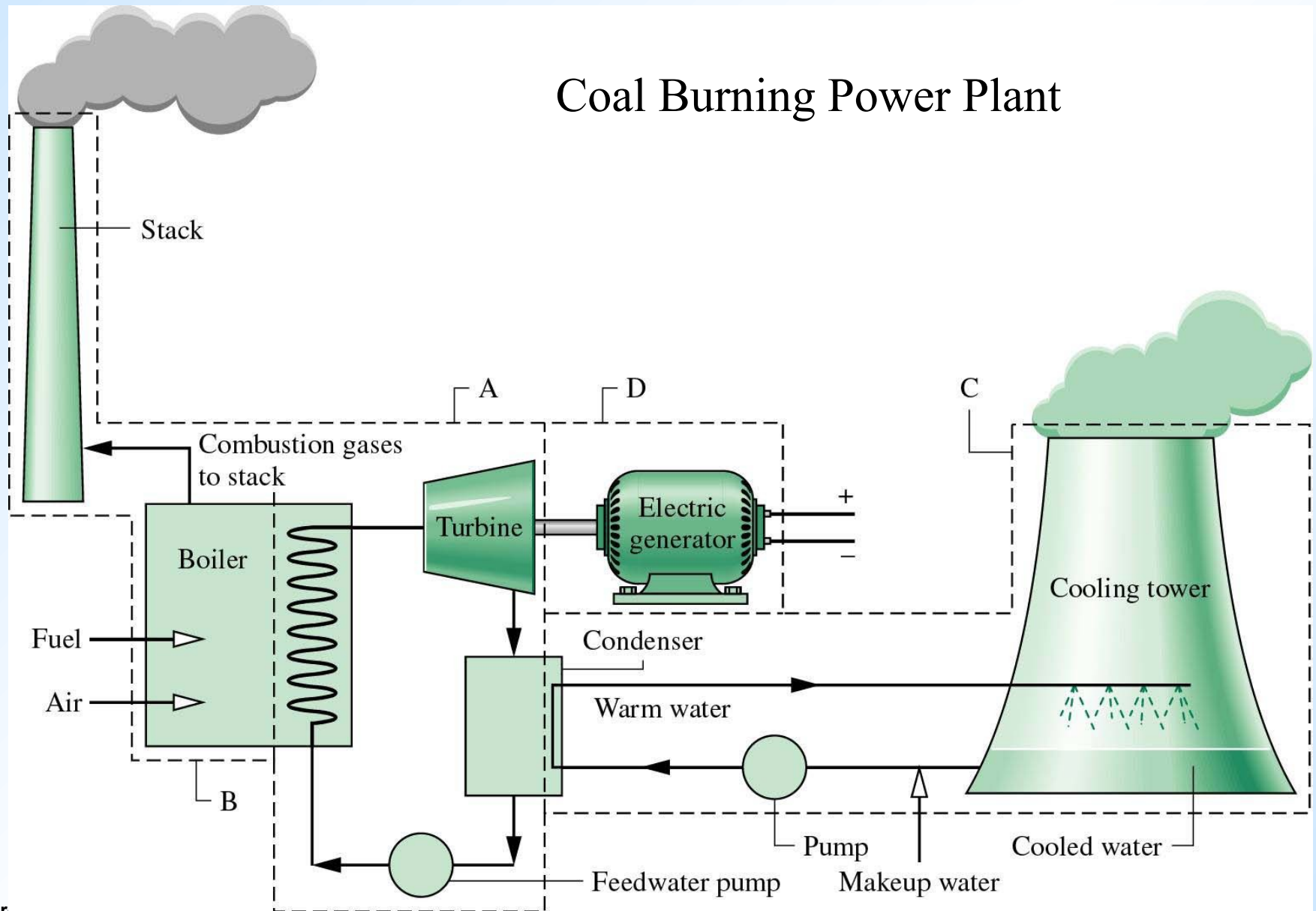


Components of steam thermal plant

- | | |
|---------------------------|---------------------------|
| 1. Boiler | 9. Super heater |
| 2. Steam turbine | 10. Re heater |
| 3. Generator | 11. Air heater |
| 4. Condenser | 12. Boiler chimney |
| 5. Cooling towers | 13. Water treatment plant |
| 6. Circulating water pump | 14. Coal mill |
| 7. Boiler feed pump | 15. Control room |
| 8. Economiser | |

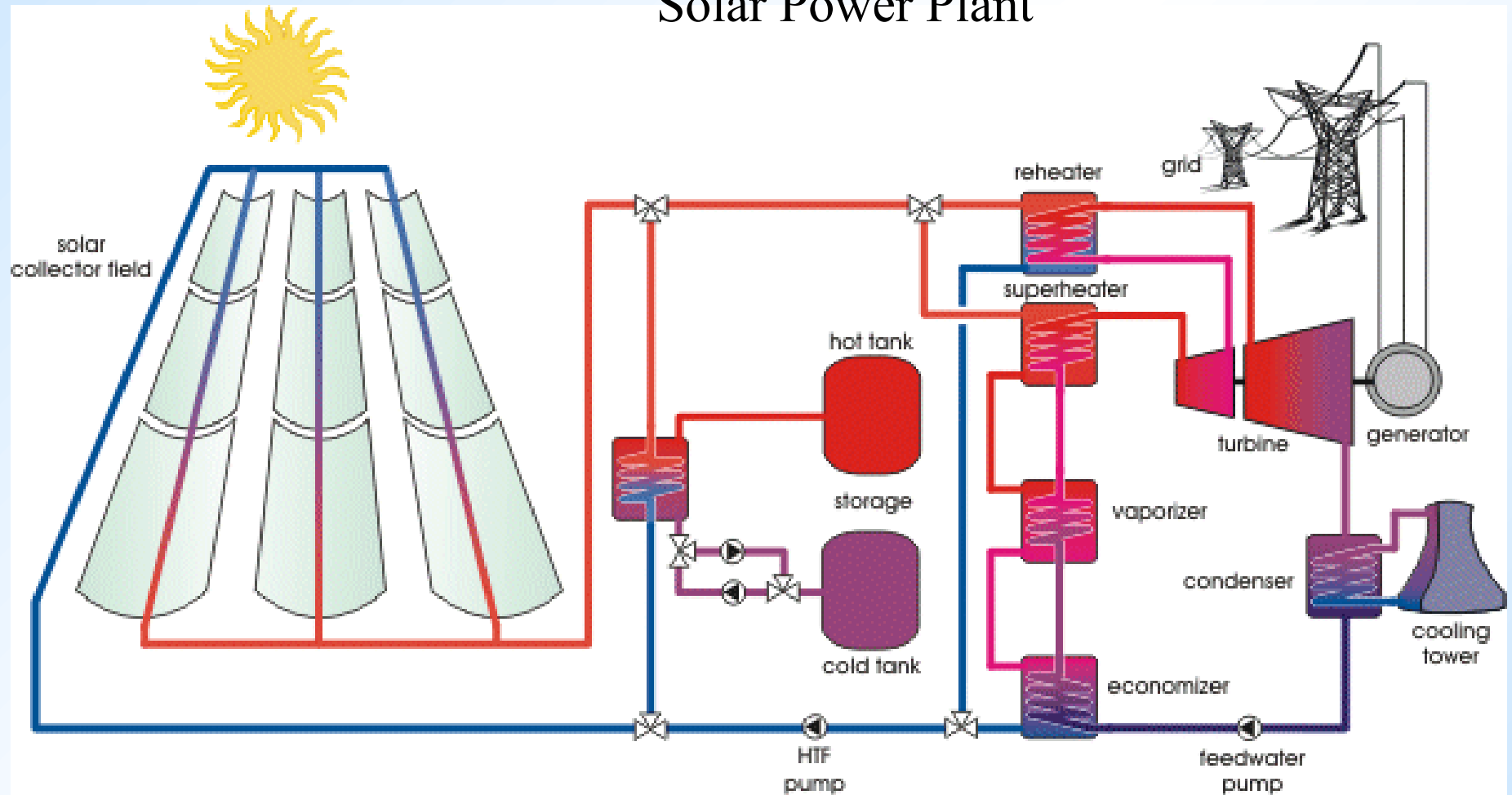


Coal Burning Power Plant



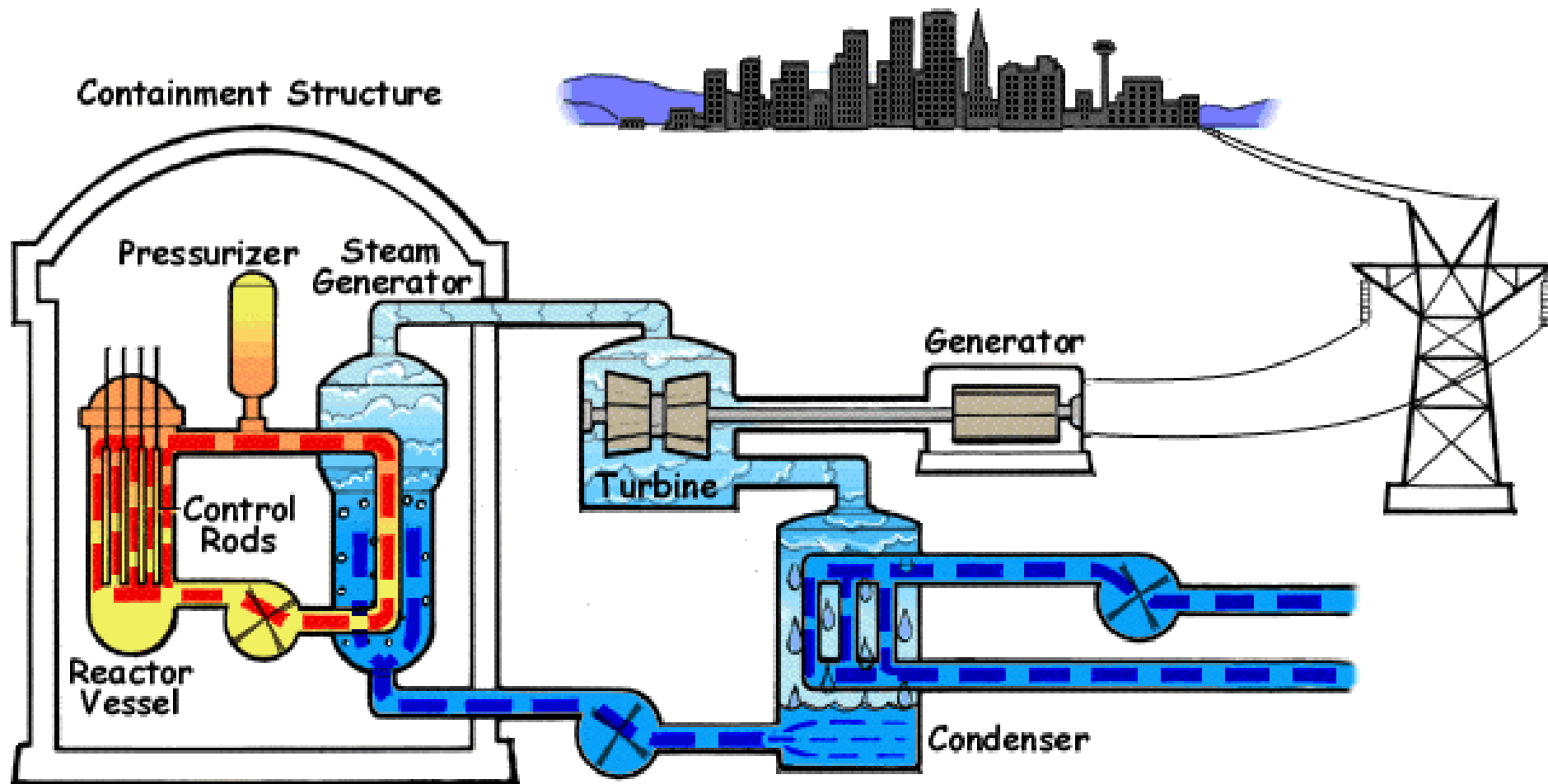


Solar Power Plant





Nuclear Power Plant





Ideal Rankine Cycle

1 - 2: Isentropic Compression

(Pump)

2 – 3: Isobaric Heat Addition

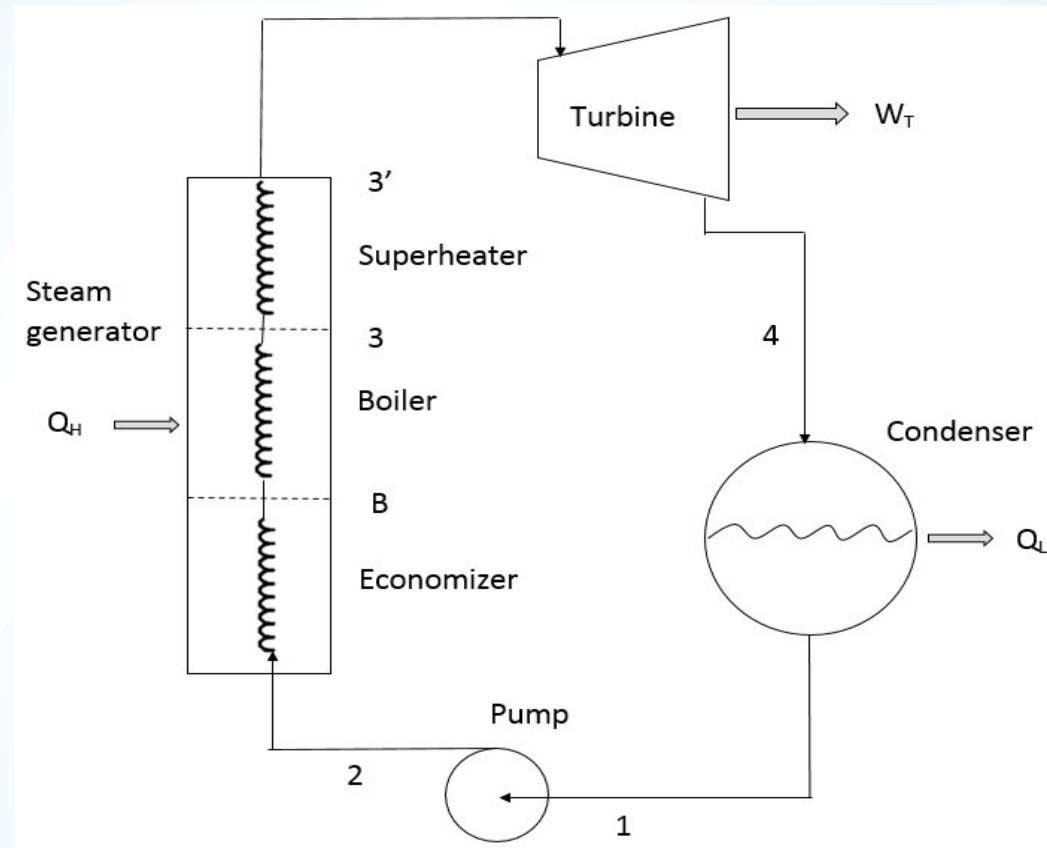
(Steam Generator or Boiler)

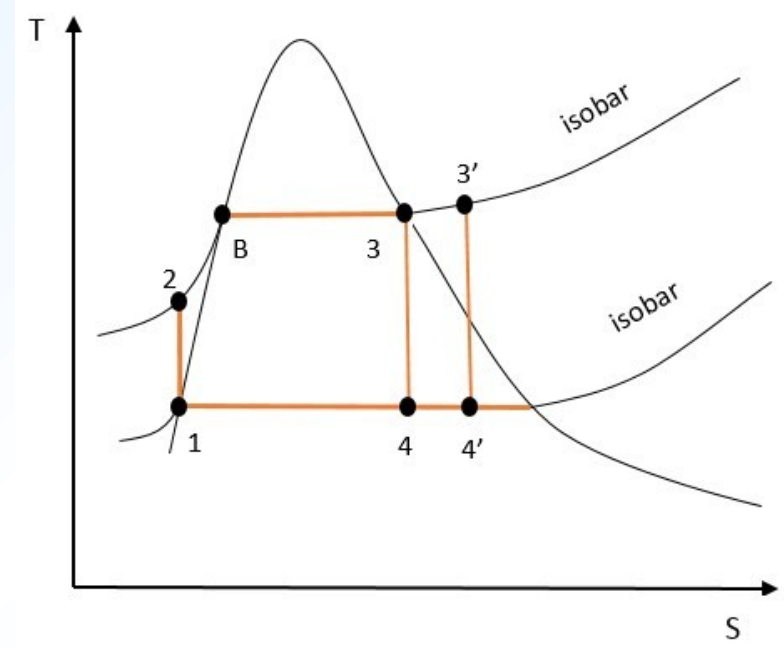
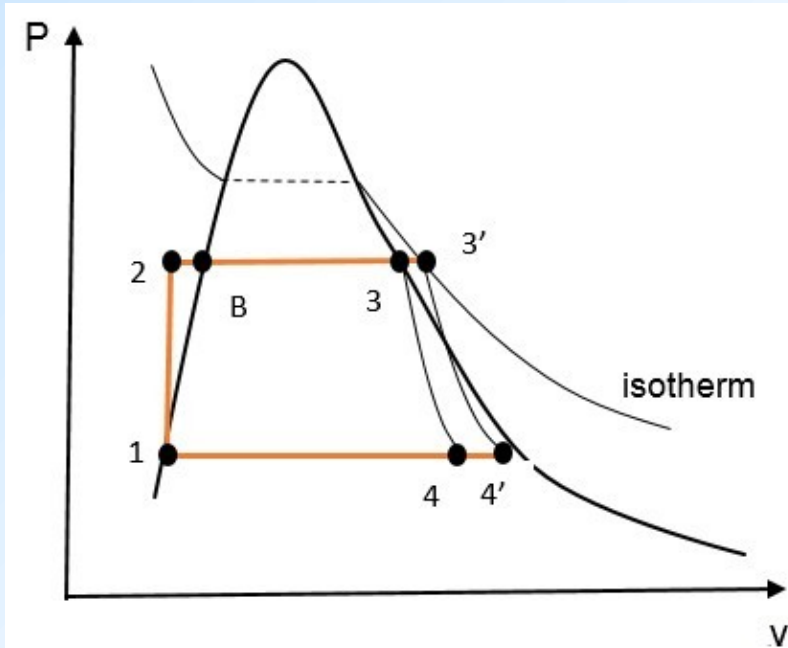
3 – 4: Isentropic Expansion

(Turbine)

4 – 1: Isobaric Heat Rejection

(Condenser)





Saturated Rankine Cycle: 1 – 2 – B – 3 – 4 – 1

Superheated Rankine Cycle: 1 – 2 – B – 3 – 3' – 4 -1

2 – B: Economizer - Brings subcooled liquid up to saturation temperature

B – 3: Boiler / Evaporator – Saturated liquid to saturated vapor

3 – 3': Superheater – Saturated vapor to superheated vapor



Analyzing Rankine Cycle requires determination of **enthalpies**.

1st Law: $q - w = \Delta e = \left[\underbrace{(u + P v)}_{h_{in}} + \underbrace{\frac{1}{2} v^2 + g z}_{\text{kinetic \& potential energy}} \right]_{out} - \left[\underbrace{(u + P v)}_{h_{out}} + \underbrace{\frac{1}{2} v^2 + g z}_{\text{kinetic \& potential energy}} \right]_{in}$

Generally can neglect kinetic & potential energy changes

$q - w = \Delta h$

Processes: 1 – 2 isentropic compression (pump) - $w_P = h_2 - h_1 = v_1 (P_2 - P_1)$

2 – 3 , 3' isobaric heat addition (steam generator) $q_H = h_{3,3'} - h_2$

3 , 3' – 4,4' isentropic expansion (turbine) - $w_T = h_{4,4'} - h_{3,3'}$

4 , 4' – 1 isobaric heat rejection (condenser) $q_L = h_1 - h_{4,4'}$



Net Work: $w_{\text{net}} = w_T - |w_P| = (h_{3,3'} - h_{4,4'}) - (h_2 - h_1)$

Thermal efficiency: $\eta_{th} = \frac{w_{\text{net}}}{q_H} = \frac{(h_{3,3'} - h_{4,4'}) - (h_2 - h_1)}{(h_{3,3'} - h_2)}$

Work ratio = $\frac{\text{net work}}{\text{gross work}} = \frac{w_{\text{net}}}{w_T}$

Pump work (1 – 2)

For small units where P_2 is not too large compared to P_1 , $h_1 \approx h_3$. So, pump work is negligible compared to turbine work.



Pump work (1 – 2)

For modern power plants,

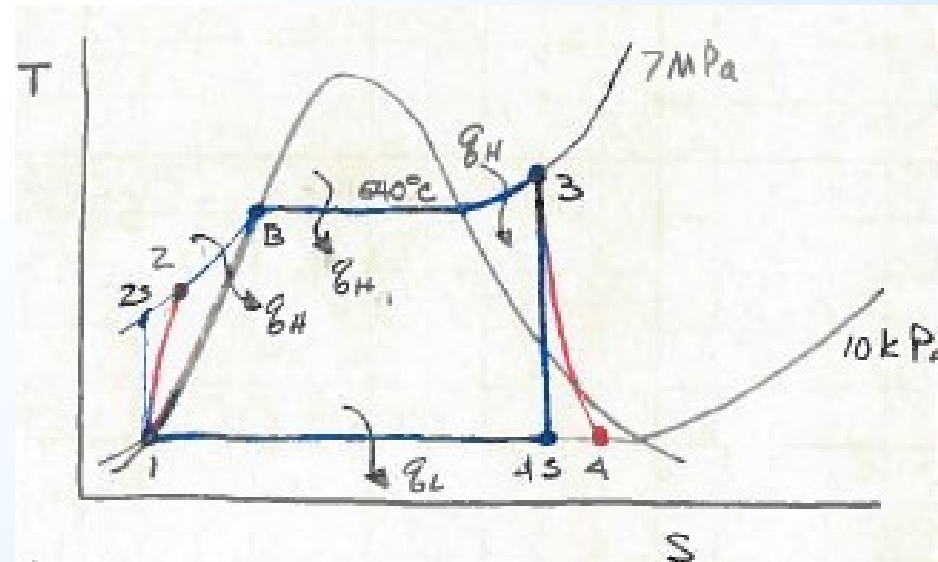
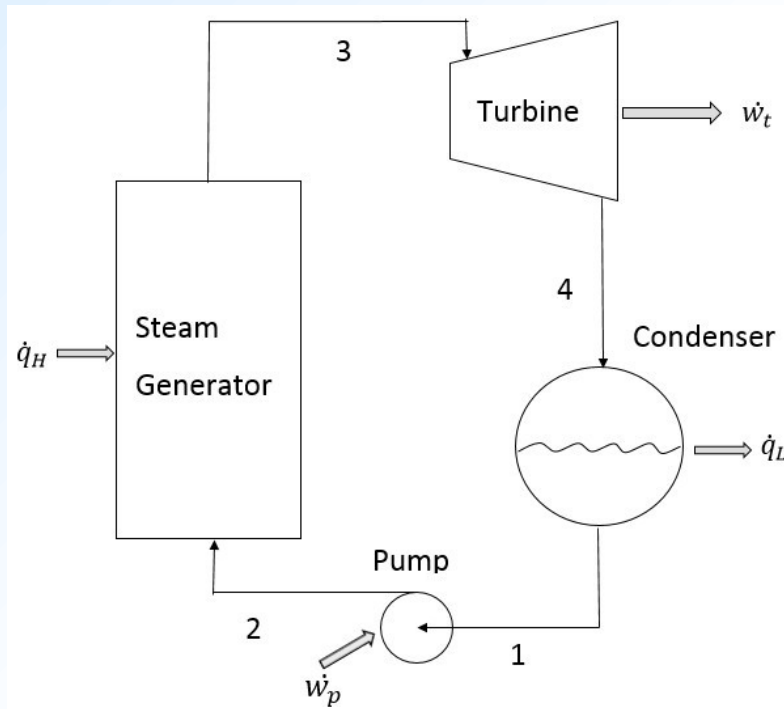
- $P_2 > 70$ bar and $P_1 \approx 0.07$ bar
- h_1 is the saturated liquid enthalpy at P_1 (h_{1f})
- h_2 is found from subcooled liquid tables at T_2 and P_2
- Generally, $T_2 \approx T_1$ and the latter is used in lieu of T_2 which is difficult to obtain
- A good approximation for the pump work can be obtained from the change in “flow work”

$$|w_P| = h_2 - h_1 = \underbrace{(u_2 - u_1)}_{\approx 0 \text{ since } T_2 \approx T_1} + (P_2 v_2 - P_1 v_1) \approx v_1 (P_2 - P_1)$$



Example 1

Find the thermal efficiency and the specific work of a simple Rankine cycle if the maximum temperature and pressure are 540 °C and 7 MPa and the minimum pressure is 10 kPa. The turbine and pump efficiencies are both 85 %.





State 1: Saturated liquid: $P_1 = 10 \text{ kPa}$

Use steam table: $h_1 = h_f = 191.53 \text{ kJ/kg}$ and $v_1 = 0.0010102 \text{ m}^3/\text{kg}$

State 2: Subcooled (compressed) liquid: $P_2 = 7000 \text{ kPa}$ and $v_2 = v_1$

Pump work:
$$w_P = \frac{v_1 (P_1 - P_2)}{\eta_P} = \frac{(0.0010102) (10 - 7000)}{0.85} = - 8.31 \text{ kJ/kg}$$

$$h_2 = h_1 - w_P = 191.53 + 8.31 = 199.84 \text{ kJ/kg}$$

State 3: Superheated vapor: $P_3 = P_2 = 7000 \text{ kPa}$ and $T_3 = 540 \text{ }^\circ\text{C}$

Use steam table: $h_3 = 3506.9 \text{ kJ/kg}$ and $s_3 = 6.9193 \text{ kJ/kg.K}$

State 4: Saturated liquid & vapor ($0 < x_4 < 1$): $P_4 = P_1$ and $s_{4s} = s_3$

Use steam tables: $s_{4g} = 8.1502 \text{ kJ/kg.K}$ and $s_{4f} = 0.6493 \text{ kJ/kg.K}$

$$h_{4g} = 2584.7 \text{ kJ/kg} \quad \text{and} \quad h_{4f} = h_1 = 191.53 \text{ kJ/kg}$$



$$x_{4s} = \frac{s_{4s} - s_{4f}}{s_{4g} - s_{4f}} = \frac{6.9193 - 0.6493}{8.1502 - 0.6493} = 0.836$$

$$h_{4s} = h_{4f} + x_{4s} (h_{4g} - h_{4f}) = 191.53 + (0.836) (2584.7 - 191.53) = 2192.0 \text{ kJ/kg}$$

$$h_4 = h_3 + \eta_T (h_3 - h_{4s}) = 3506.9 - (0.85) (3506.9 - 2192.0) = 2389.2 \text{ kJ/kg}$$

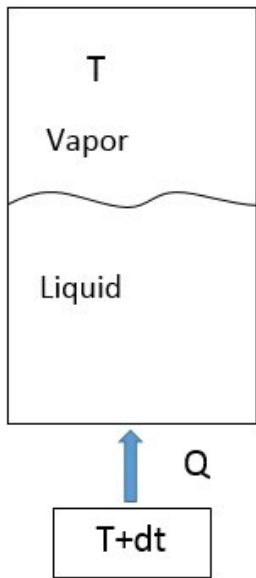
$$\text{Turbine work: } w_T = h_3 - h_4 = 3506.9 - 2389.2 = 1117.7 \text{ kJ/kg}$$

$$\text{Net work: } w_{\text{net}} = w_T + w_P = 1117.7 - 8.31 = 1109.7 \text{ kJ/kg}$$

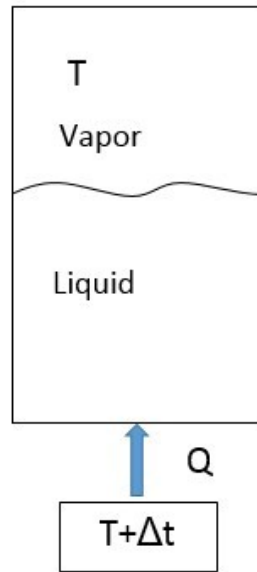
$$\text{Thermal efficiency: } \eta_{\text{th}} = 1 - \frac{q_L}{q_H} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{2389.2 - 191.8}{3506.9 - 199.84} = 0.3355$$



Rankine Cycle – External Irreversibilities



Reversible
heat transfer
external and
internal



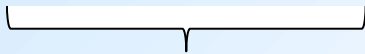
Internally
reversible
heat transfer
but not externally

Evaporation and condensation
are reversible processes
(temperature remains constant)

External irreversibilities are
primarily due to finite
temperature differences, ΔT



For steam generator, external irreversibilities are primarily due to ΔT between primary heat sources and working fluid.

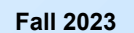


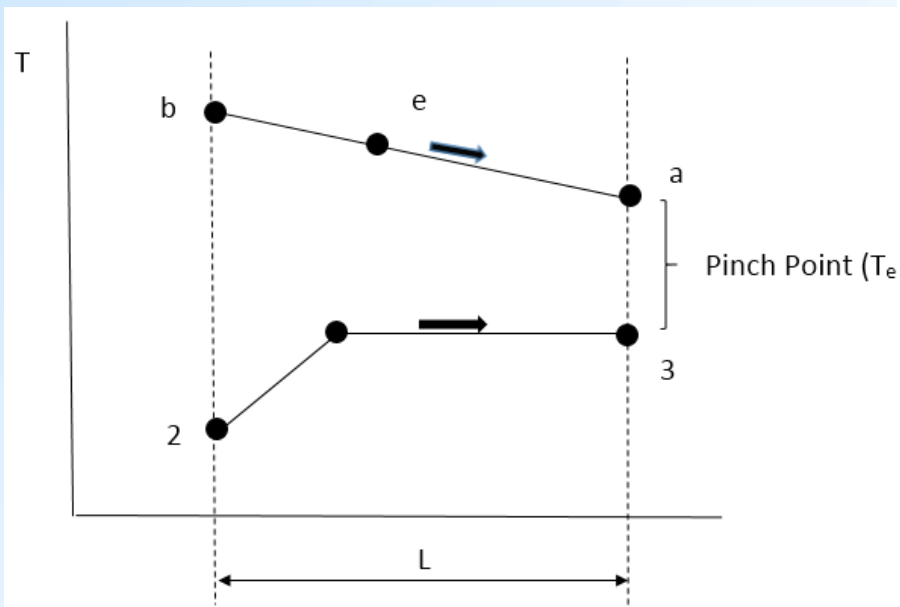
Primary heat sources are:

- Combustion gases
- Primary coolant from nuclear reactor
- Solar concentrator

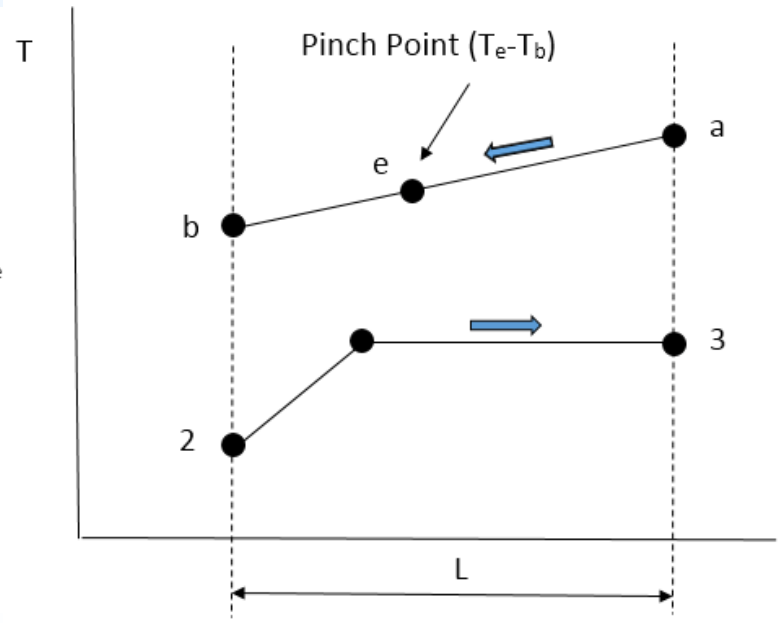
Working fluid can be:

- Steam
- Ammonia
- Refrigerants





Parallel flow steam generator



Counter flow steam generator

- The net ΔT is greater for parallel flow than for counter flow
- Heat transfer process is also more efficient with counter flow
- So, use counter flow



Pinch-point temperature difference, $T_e - T_B$

Small Pinch-point means

- Lower temperature difference between steam lines & primary heat source lines
- Requires large, costly steam generators

Large Pinch-point means

- Small, inexpensive steam generators
- Higher overall temperature difference, hence higher external irreversibilities and reduction in plant efficiency

Most economical pinch-point ΔT is obtained by considering both capital costs (price of steam generator) and recurring costs (operating and fuel costs).



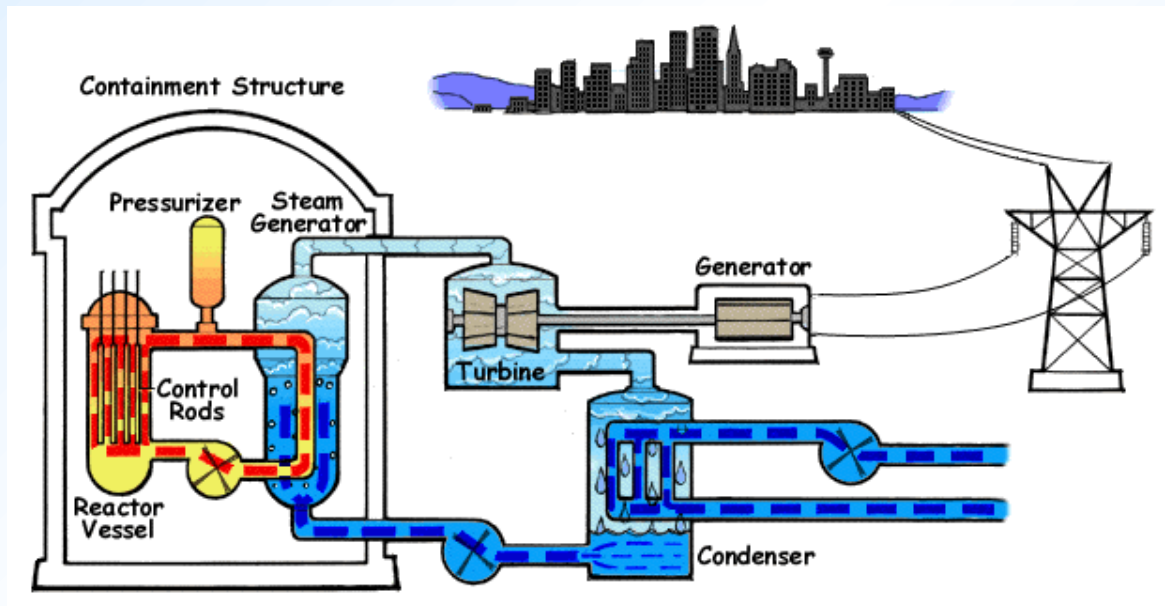
Types of Heat Source Fluids for Steam Generator:

Gas: Combustion gases

Primary coolant from a gas-cooled reactor, CO₂, He

Liquid: Water; pressurized-water reactor (PWR)

Liquid metal; Liquid-metal fast breeder reactor (LMFBR)



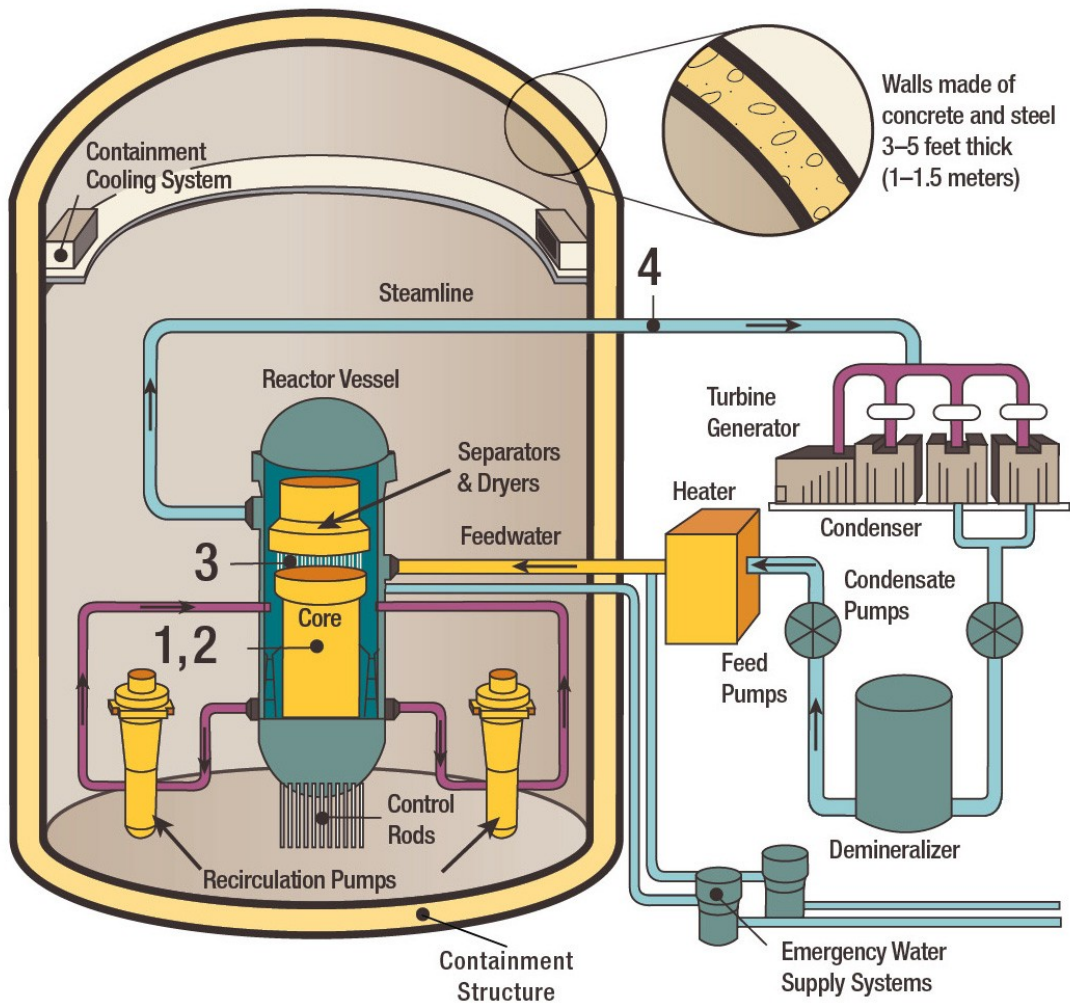
Heat Transfer:

$$\dot{Q} - \dot{W} = \dot{m} c_p \Delta T$$

Heat source fluids
have a variety of c_p &
 \dot{m} required to achieve
 \dot{Q}



Typical Boiling-Water Reactor



Fuel: Pellets of enriched UO_2

Cladding: Zircaloy

Moderator: Coolant

Coolant: Light water

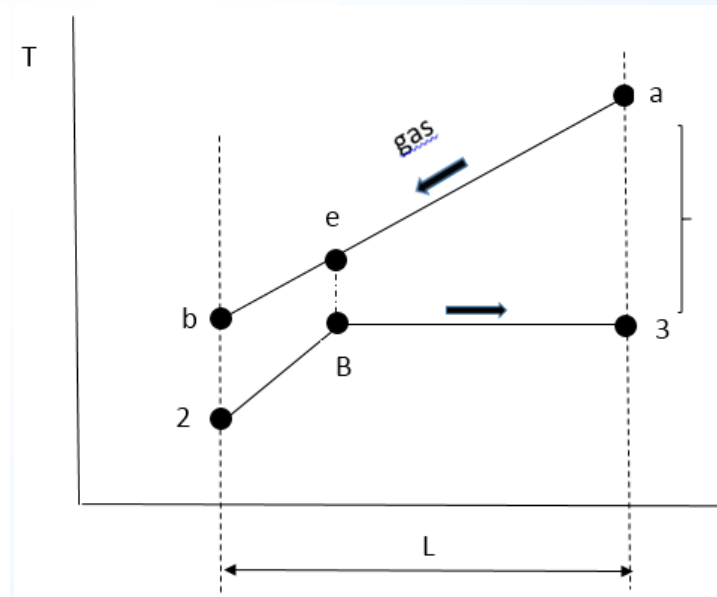
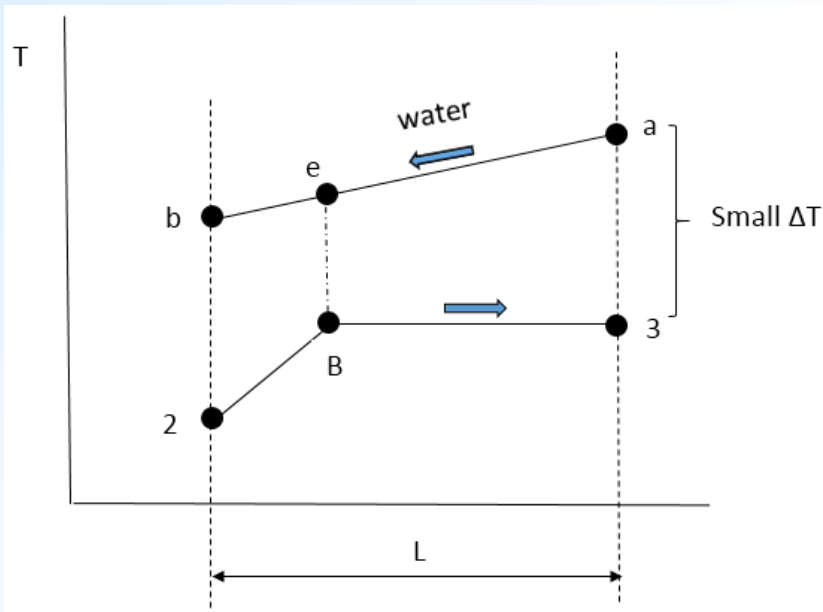
Control: Boric acid in the
first loop and control rods

Control rods: Silver-Indium-
Cadmium alloy

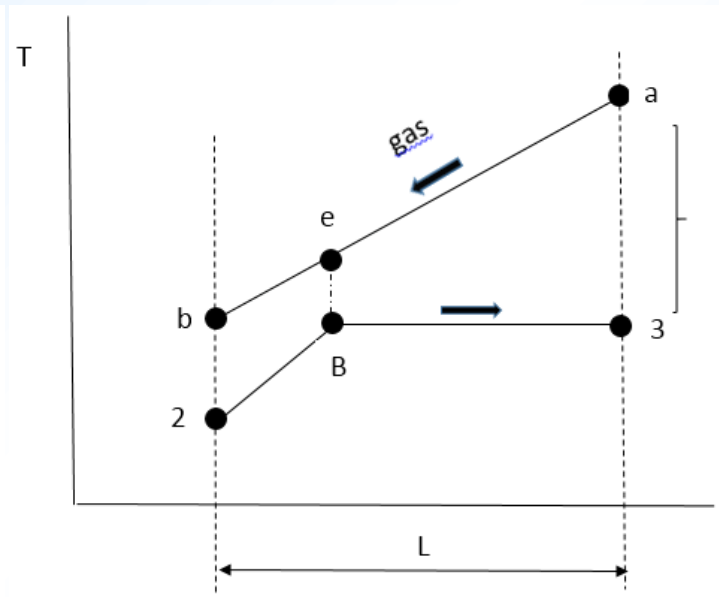
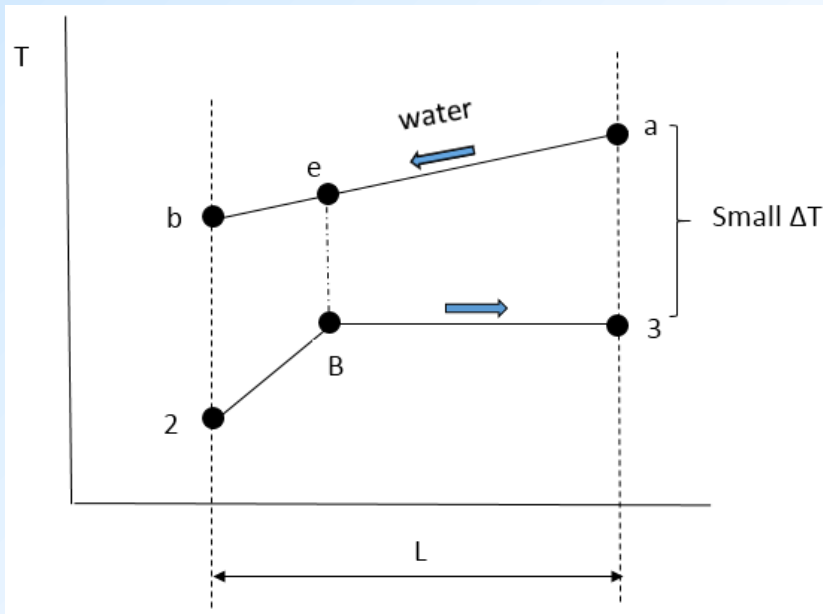


PWR (Pressurized Water Reactor):

Water has a higher c_p than gas, but also has a much higher mass flow rate in order to limit temperature rise in water flowing through the reactor – need to maintain uniform moderation (?) of neutrons – Therefore, much greater ($m c_p$) for water than gases.



Large ΔT
So, large
external
irreversibilities



$$\frac{dQ}{dL} = \dot{m} c_p \frac{dT}{dL}$$

Slope of line a-b is proportional to $\frac{1}{\dot{m} c_p}$

Or $\underbrace{\frac{dT}{dL} \propto \frac{1}{\dot{m} c_p}}_{\text{For a fixed } \dot{Q}}$

For primary heat source fluid

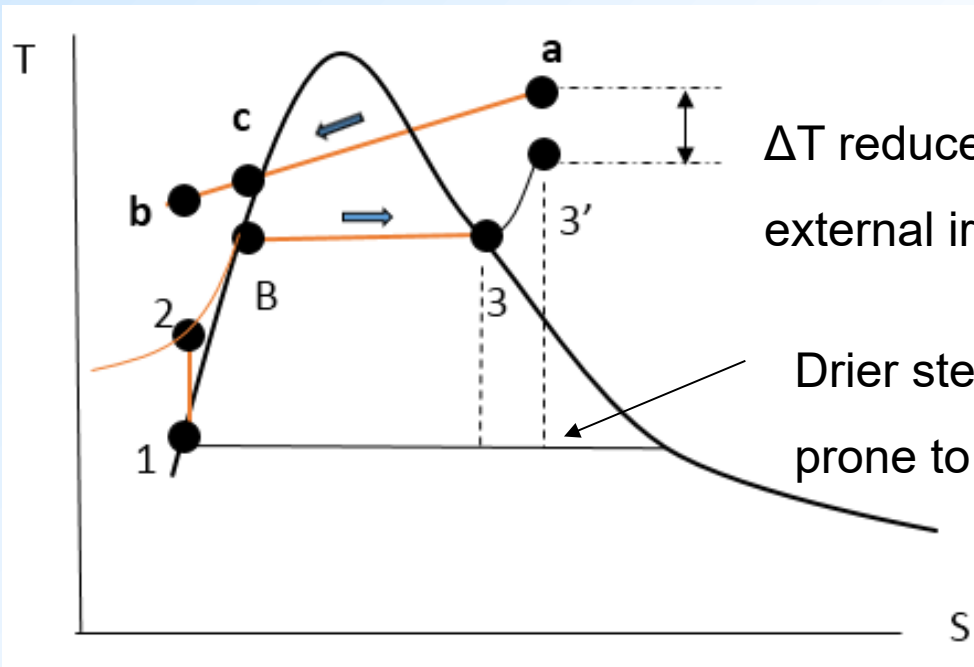


Small external irreversibilities

PWR and BWR



To Superheat or Not To Superheat?



ΔT reduced by using superheat. Therefore, external irreversibilities are also reduced.

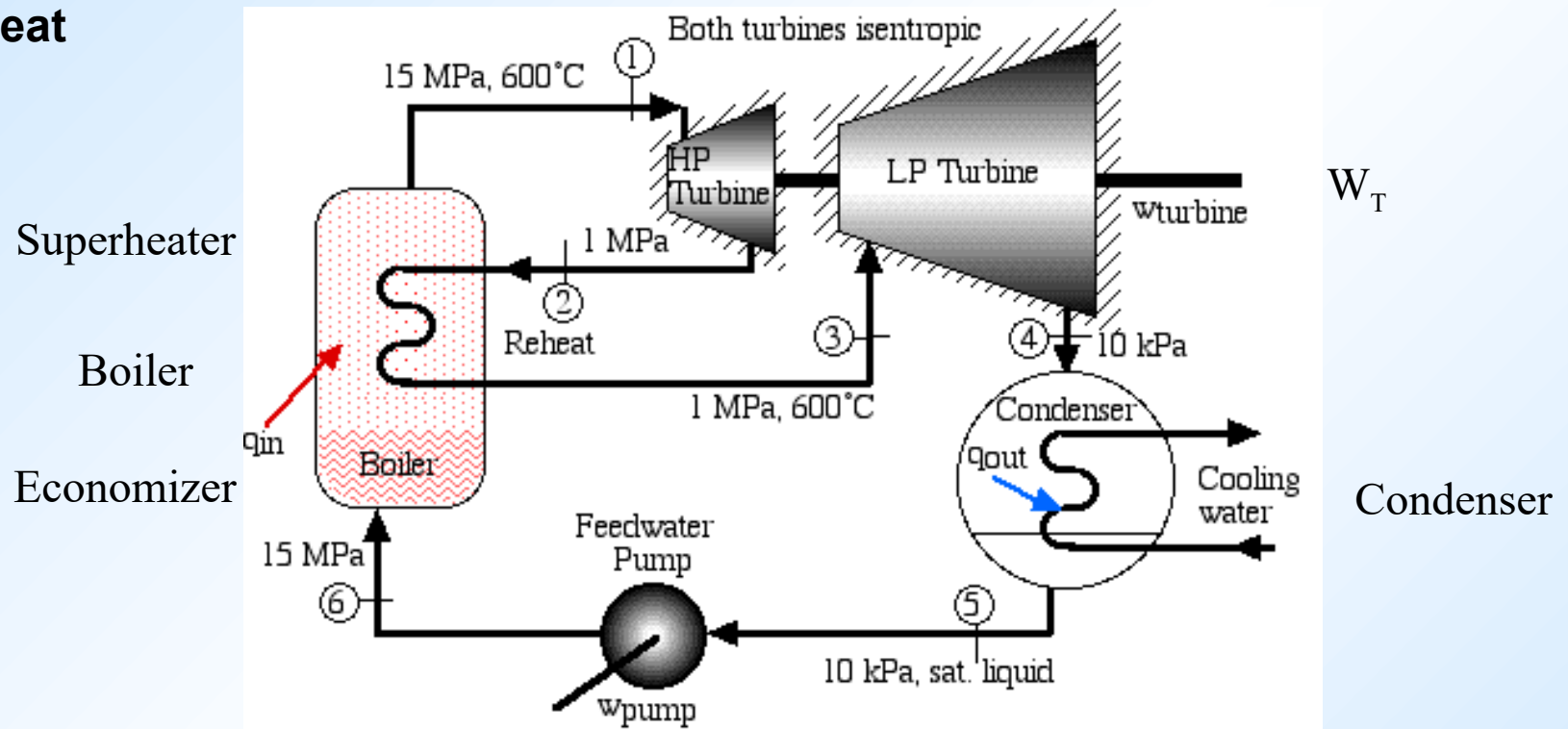
Drier steam \Rightarrow more efficient and less prone to turbine blade damage

Gas or liquid metal as primary heat source fluid

Fossil fuel, gas-cooled and liquid-metal-cooled reactors



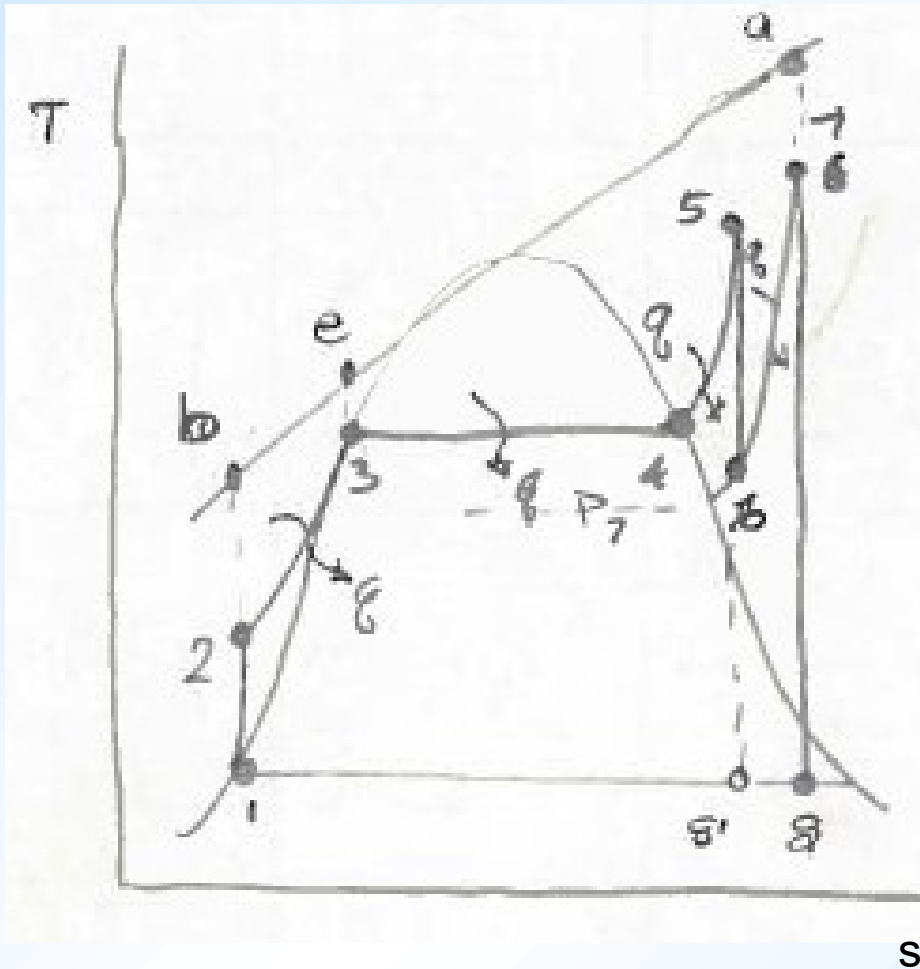
Reheat



Improved efficiency

Drier steam at exhaust

All modern power plants use at least 1 stage of reheat



$$-w_T = (h_6 - h_5) + (h_8 - h_7)$$

$$-w_P = (h_2 - h_1)$$

$$w_{\text{net}} = (h_5 - h_6 + h_8 - h_7) - (h_2 - h_1)$$

$$q_h = (h_5 - h_2) + (h_7 - h_6)$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_h} = \frac{(h_5 - h_6 + h_8 - h_7 - h_2 + h_1)}{(h_5 - h_2) + h_7 - h_6}$$

Superheat Reheat power plant is often designated as $P_5/T_5/T_6$ (turbine inlet)

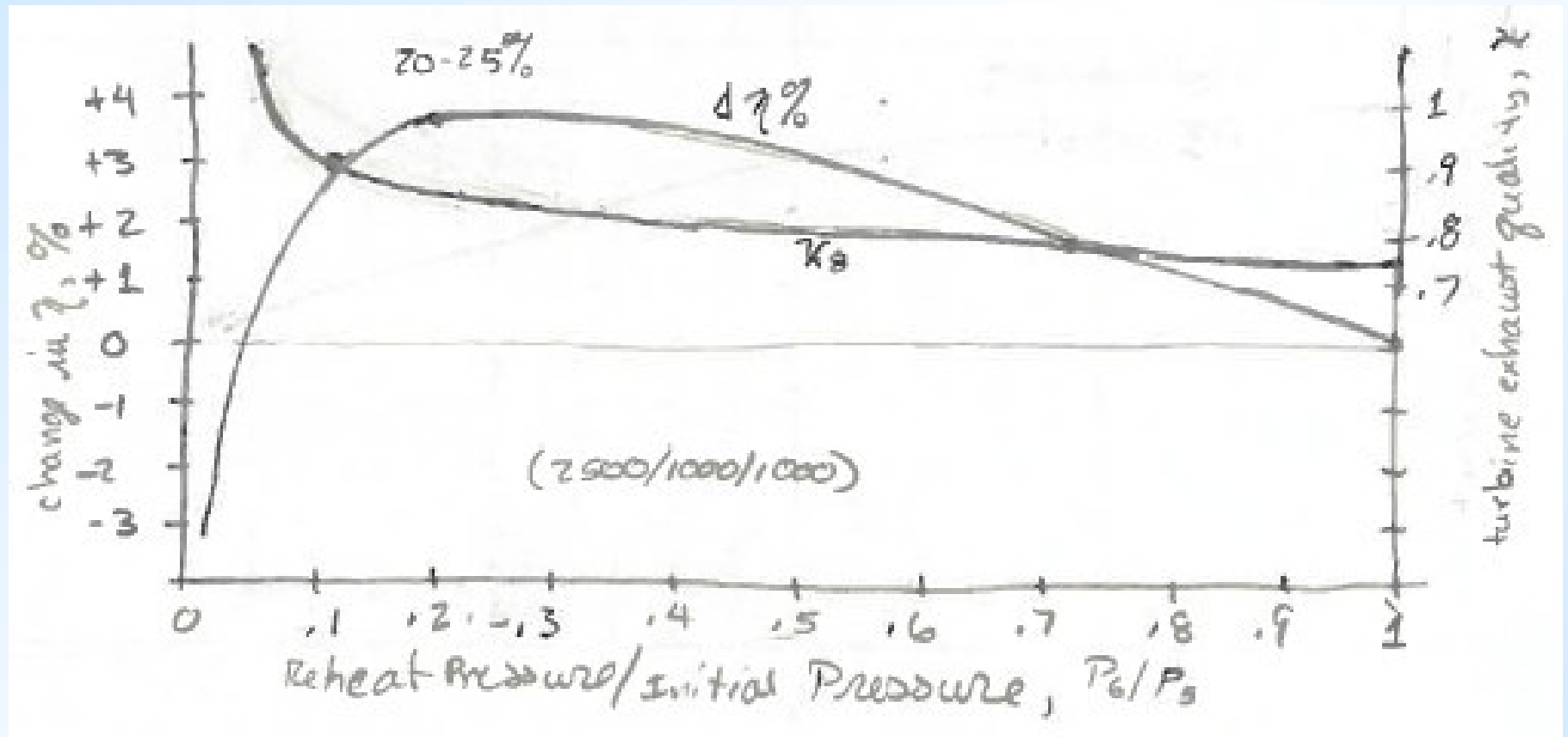


The pressure at which the steam is reheated is critical.

A reheat pressure ($P_6 = P_7$) too close to the initial pressure ($P_2 = P_3 = P_4 = P_5$) results in little increase to the cycle efficiency

The efficiency increase due to reheat improves as the reheat pressure ($P_6 = P_7$) is lowered, reaching a peak at a pressure ratio P_6/P_5 of 0.2 to 0.25.

Too low of a pressure ratio could result in superheated exhaust steam, unfavorable for condenser.





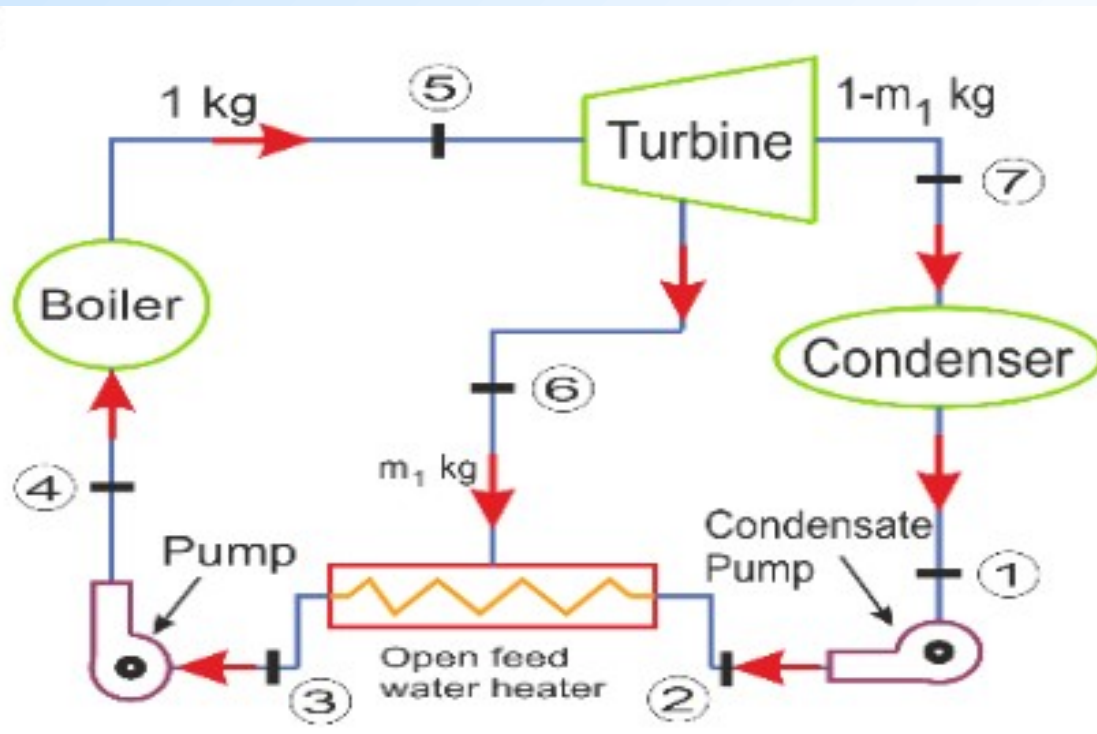
Regeneration – Feed Water Heating

Superheat addresses external irreversibilities in the boiler.

Regeneration would address external irreversibilities in the economizer;

- Transfer heat from exhaust side of cycle to compressed liquid prior to heat addition process
- For the Rankine cycle, heat rejection at the condenser is isothermal and below the temperature of the compressed liquid; cannot transfer thermal energy from a cold fluid to a warm fluid.

Regeneration is therefore accomplished by “bleeding” a small amount of steam from the turbine and heating the “feed water” (compressed liquid) prior to the feed water entering the steam generator.

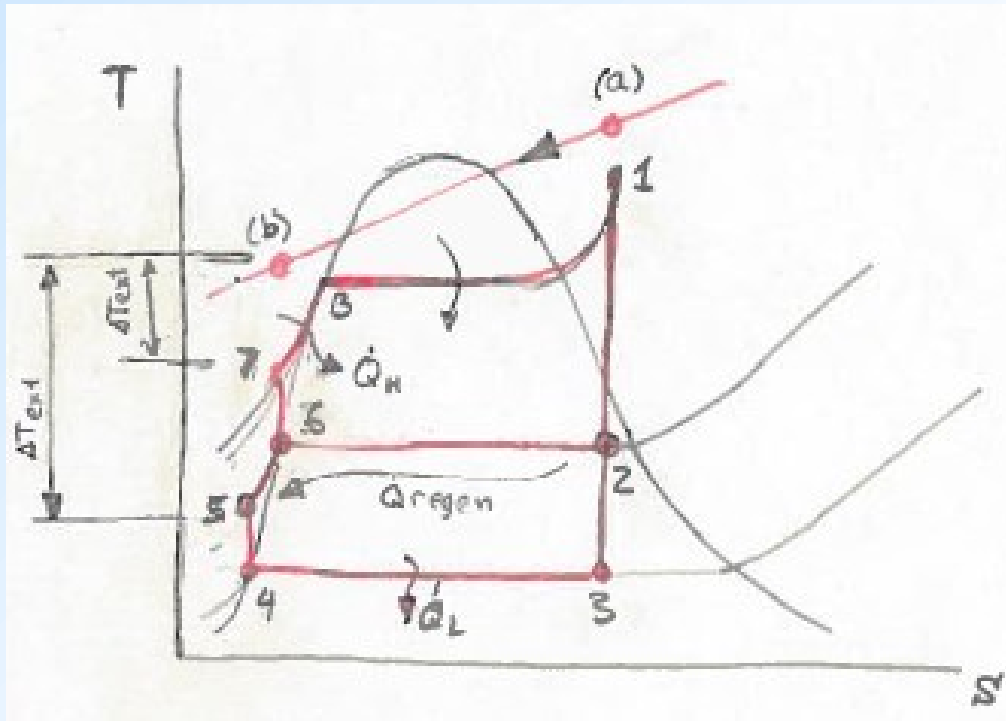


Feedwater is the compressed liquid flowing towards the steam generator.

Compressed liquid at (2) is heated with small amount of steam (6) to bring the feedwater back to saturated liquid state (3).

Heat transfer takes place in a heat exchanger called “feedwater heater”.

Dates back to 1920's, same time period that steam temperatures reached 725 °C



Modern power plants use 5 and 8 feedwater heaters

Still need an economizer (7 – B), but a much smaller one than without feedwater heating. Compare the temperature difference from (b) to (7) and from (b) to (5)

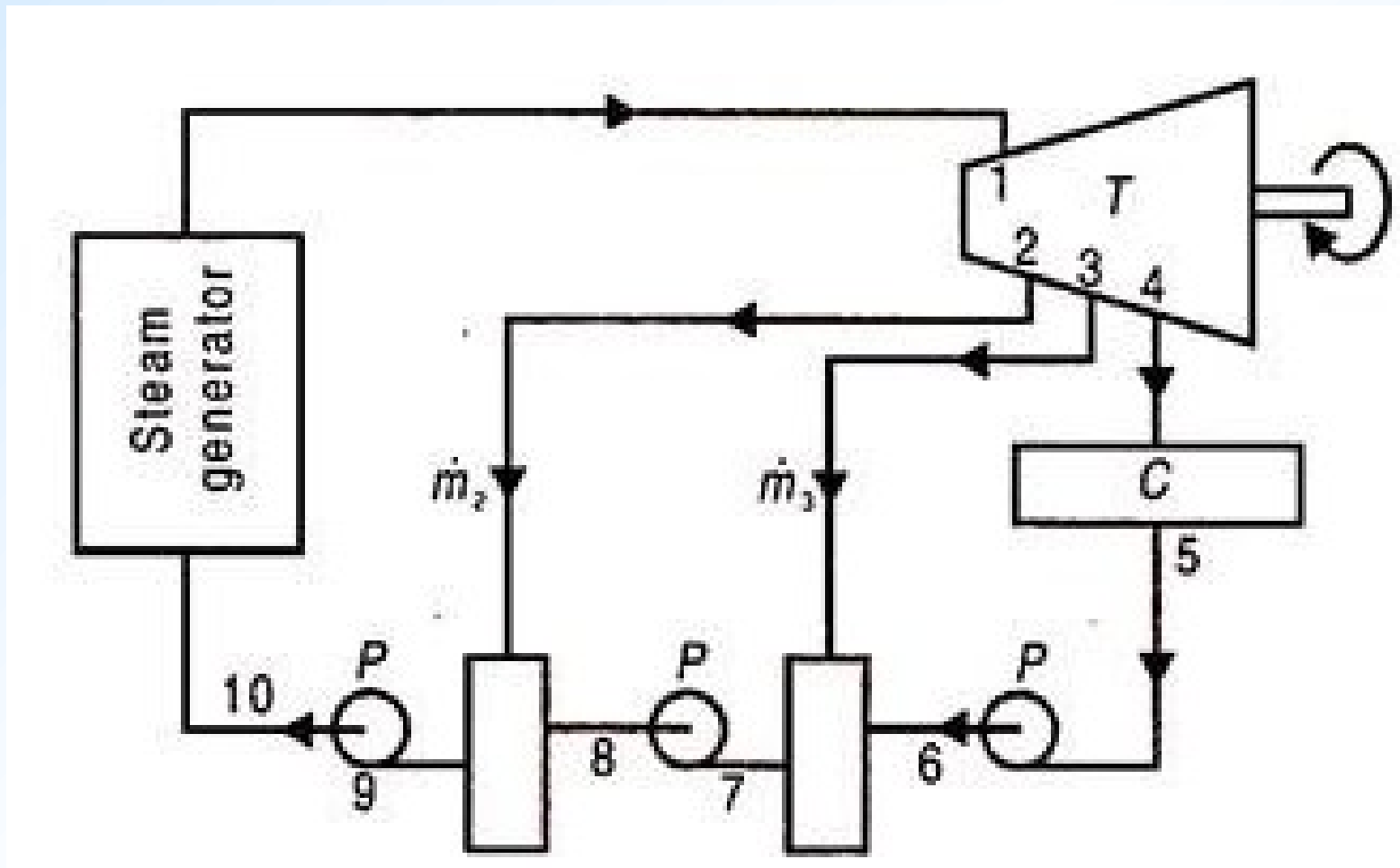
Three types of feedwater heaters:

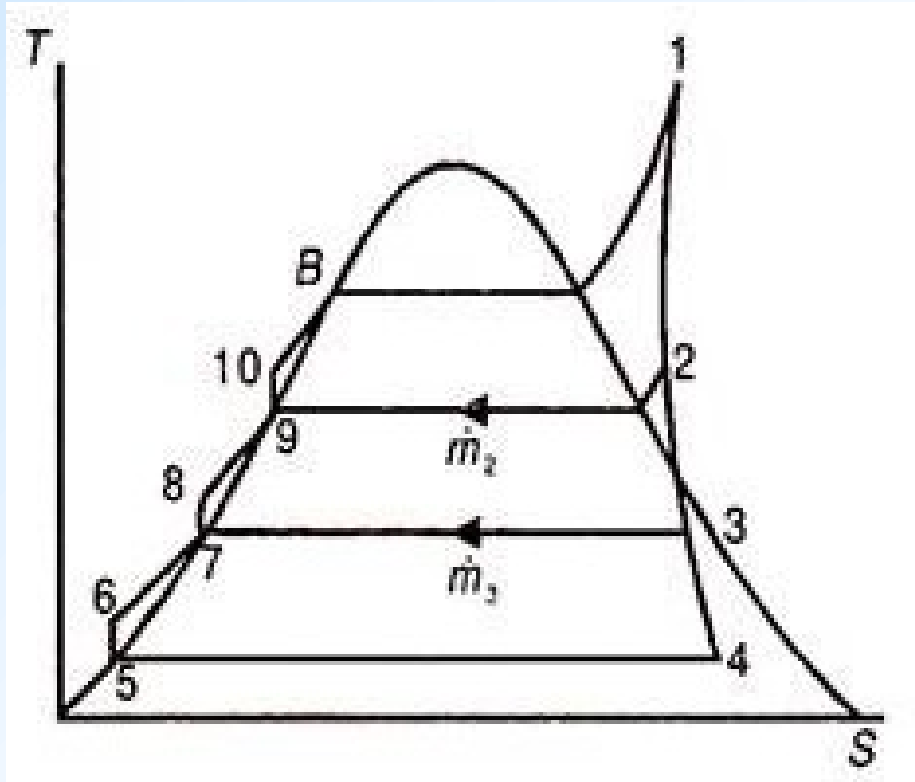
1. Open (direct contact) → steam is mixed with feedwater
2. Closed with drains cascading backwards
3. Closed with drains cascading forwards

Steam is run through a tube-and-shell heat exchanger



Open Feedwater Heater

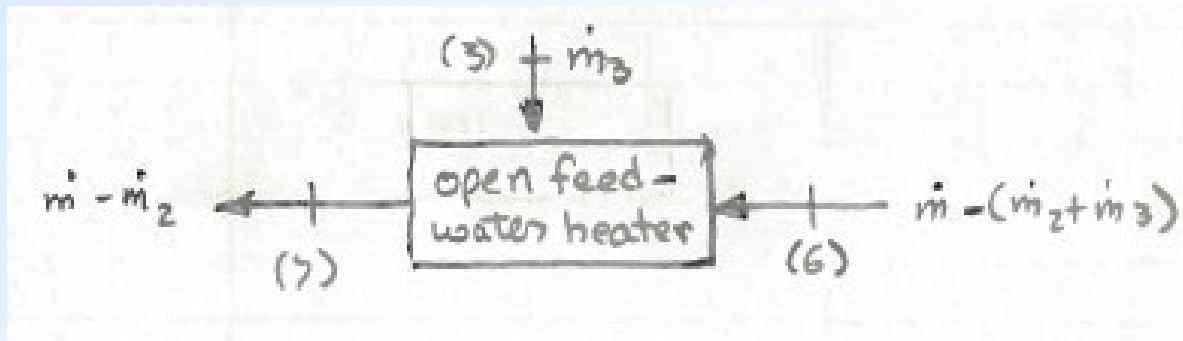




Large amount of energy released from extracted steam as it condenses; small amount energy required to bring feedwater back to saturated liquid.

Therefore, m_2 and m_3 are small fractions of the total m through the turbine.

Steam is “bled off” at (2) and (3) and mixed with subcooled water at (6) and (8) to produce saturated water at the temperature of the bled steam.



State (3) and state (6) mix to become state (7)

The amount of steam bled is what saturates the subcooled liquid $\dot{m}_3 = \dot{m} - (\dot{m}_2 + \dot{m}_3)$

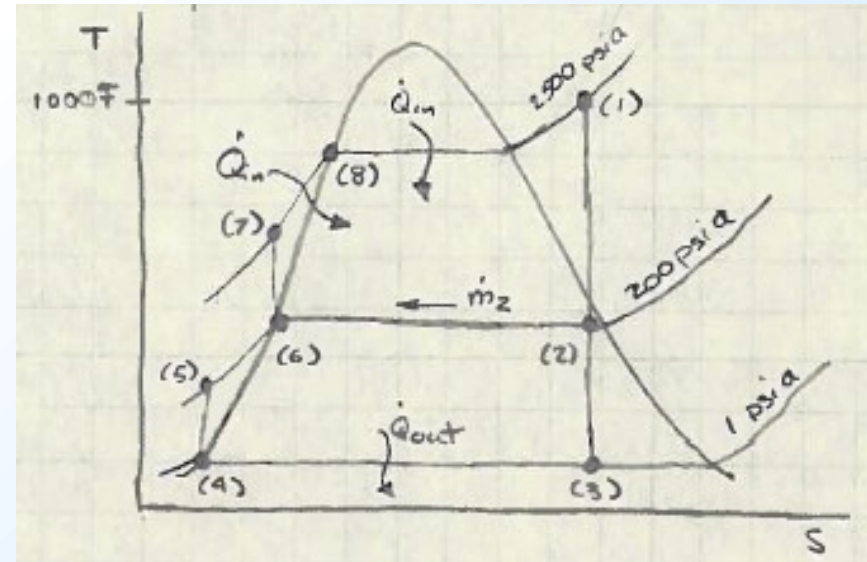
$P_6 = P_7 \leq P_3$ Reverse flow into turbine would occur if $P_7 > P_3$

Requires a pump after the feedwater heater and each pump has to handle nearly all of the mass flow in the cycle

Open feedwater heaters also serve as deaerators to remove non-condensable gases such as O_2 , N_2 , and CO_2 .



An ideal Rankine cycle operates between 2500 psia and 1000 °F at throttle and 1 psia in the condenser. One open feedwater heater is placed at 200 psia. Calculate the net work and heat input in Btu/lbm.



**Solution**

$$\left. \begin{array}{l} T_1 = 1000 \text{ F} \\ P_1 = 2500 \text{ psia} \end{array} \right\} \text{Superheated steam} \left\{ \begin{array}{l} h_1 = 1457.5 \text{ Btu/lbm} \\ s_1 = 1.15269 \text{ Btu/lbm.R} \end{array} \right.$$

$$\left. \begin{array}{l} P_2 = 200 \text{ psia} \\ s_2 = s_1 = 1.15269 \text{ Btu/lbm.R} < s_g (500 \text{ psia}) \end{array} \right\} \text{Saturated steam}$$

$$\left. \begin{array}{l} s_{2f} = 0.5438 \text{ Btu/lbm.R} \\ s_{2g} = 1.5454 \text{ Btu/lbm.R} \end{array} \right\} x_2 = 0.9815$$

$$\left. \begin{array}{l} h_{2f} = 355.5 \text{ Btu/lbm} \\ h_{2g} = 1198.3 \text{ Btu/lbm} \end{array} \right\} h_2 = 1182.7 \text{ Btu/lbm}$$

$$\left. \begin{array}{l} x_2 = 0.9815 \\ h_2 = 1182.7 \text{ Btu/lbm} \end{array} \right\} \dot{m}_2 \text{ is drawn off at this point}$$

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

$$\left. \begin{array}{l} P_3 = 1 \text{ psia} \\ s_3 = s_1 = 1.15269 \text{ Btu/lbm.R} < s_g (1 \text{ psia}) \end{array} \right\} \left. \begin{array}{l} s_{3f} = 0.1326 \text{ Btu/lbm.R} \\ s_{3g} = 1.9781 \text{ Btu/lbm.R} \end{array} \right\} x_3 = 0.7555$$

$$\left. \begin{array}{l} h_{2f} = 355.5 \text{ Btu/lbm} \\ h_{2g} = 1198.3 \text{ Btu/lbm} \end{array} \right\} h_3 = 852.2 \text{ Btu/lbm}$$



Saturated liquid }
$$\begin{aligned} h_4 &= h_{3f} = 69.73 \text{ Btu/lbm} & s_4 &= s_{3f} = 0.1326 \text{ Btu/lbm.R} \\ v_4 &= 0.016136 \text{ ft}^3/\text{lbm} & P_4 &= 1 \text{ psia} \end{aligned}$$

Subcooled liquid }
$$\begin{aligned} P_5 &= 200 \text{ psia} \\ v_5 &\approx v_4 = 0.016136 \text{ ft}^3/\text{lbm} \\ s_5 &= s_4 \\ w_{P_1} &= | -_4 w_5 | = h_5 - h_4 = v_4 (P_5 - P_4) = 0.59 \text{ Btu/lbm} \\ h_5 &= h_4 + w_{P_1} = 70.32 \text{ Btu/lbm} \end{aligned}$$

Saturated liquid }
$$\begin{aligned} P_6 &= 200 \text{ psia} & v_6 &= 0.01839 \text{ ft}^3/\text{lbm} \\ h_6 &= h_{2f} = 355.5 \text{ Btu/lbm} \end{aligned}$$

Subcooled liquid }
$$\begin{aligned} P_7 &= 2500 \text{ psia} \\ w_{P_2} &= | -_6 w_7 | = h_7 - h_6 = v_6 (P_7 - P_6) = 7.83 \text{ Btu/lbm} \\ h_7 &= h_6 + w_{P_2} = 363.33 \text{ Btu/lbm} \end{aligned}$$



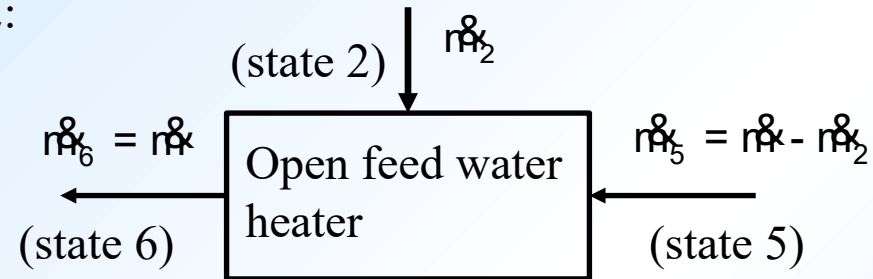
Saturated liquid } $P_8 = 2500 \text{ psia}$
 $h_8 = 731.7 \text{ Btu/lbm}$

Summary of states:

State		h (Btu/lbm)
1	Superheated vapor	1457.5
2	$x = 0.9815$	1182.7
3	$x = 0.7555$	852.2
4	Saturated liquid	69.73
5	Saturated liquid	70.32
6	Saturated liquid	355.5
7	Saturated liquid	363.33
8	Saturated liquid	1699.92



Mass fraction at state 2:



Apply first law (conservation of energy): $\dot{Q} - \dot{W} = 0 = \dot{m}_6 h_6 - \dot{m}_5 h_5 - \dot{m}_2 h_2$

$$\dot{m}_6 h_6 = (\dot{m} - \dot{m}_2) h_5 + \dot{m}_2 h_2 = \dot{m} h_5 + \dot{m}_2 (h_2 - h_5)$$

$$\dot{m}_2 = \dot{m} \left(\frac{h_6 - h_5}{h_2 - h_5} \right) = 0.2564 \dot{m}$$

Turbine work: $\dot{W}_T = \dot{m} \left[(h_1 - h_2) + \left(1 - \frac{\dot{m}_2}{\dot{m}} \right) (h_2 - h_3) \right]$

$$w_T = 520.56 \text{ Btu/lbm}$$

Pump work: $\dot{W}_P = (\dot{m} - \dot{m}_2) w_5 + \dot{m}_6 w_7 = \dot{m} \left[\left(1 - \frac{\dot{m}_2}{\dot{m}} \right) w_5 + w_7 \right]$

$$w_P = 0.44 + 7.83 = 8.27 \text{ Btu/lbm}$$



Net work: $w_{\text{net}} = w_T - w_P = 512.29 \text{ Btu/lbm}$

Heat input: $q_H = h_1 - h_7 = 1094.23 \text{ Btu/lbm}$

Thermal efficiency: $\eta_{\text{th}} = 46.8 \%$



Closed Feedwater Heaters – reduce size and number of feedwater pumps

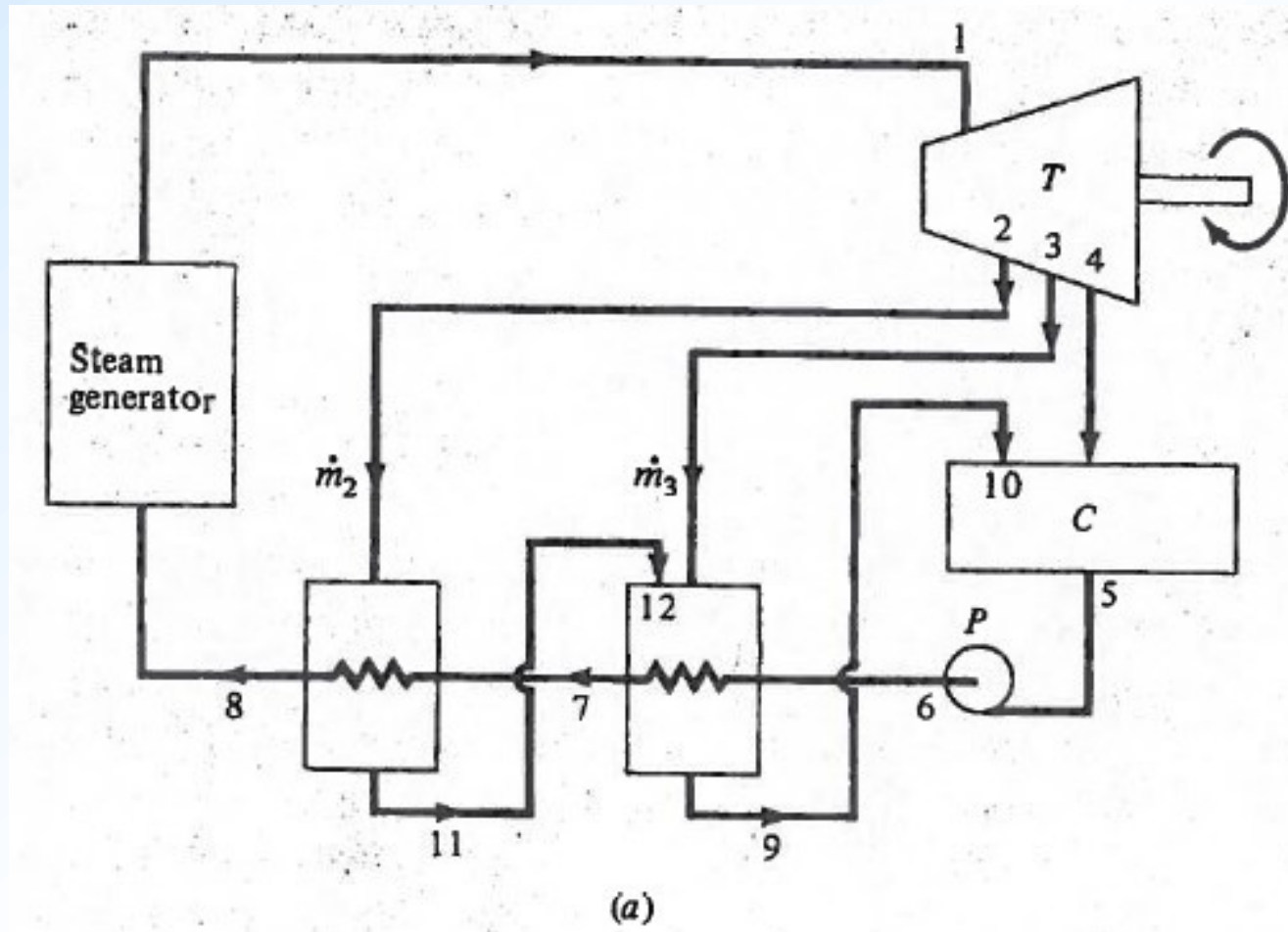
Two types - Drains Cascading Backward
- Pump forward

Drains Cascading Backwards:

- Counterflow heat exchanger (shell & tube type); no mixing as in open feedwater heaters
- Need to throttle (“trap”) high pressure bleed back to low pressure bleed

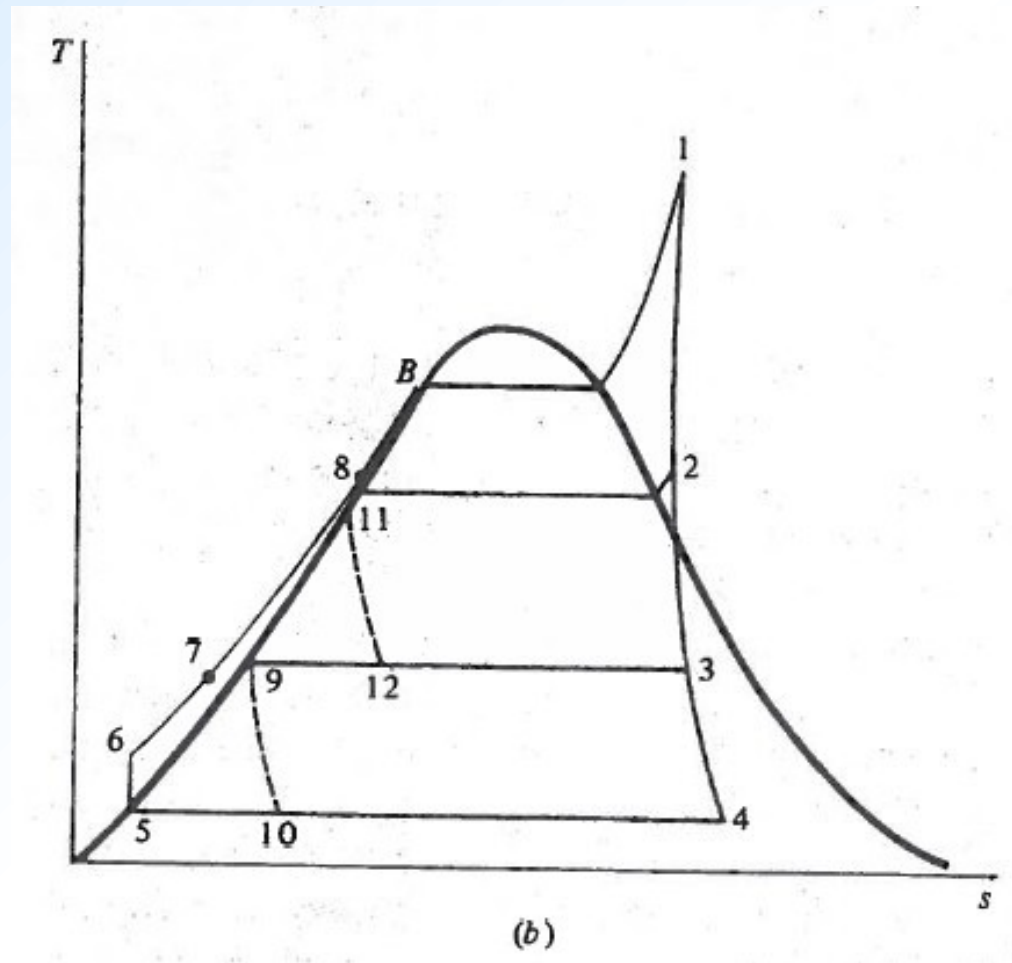


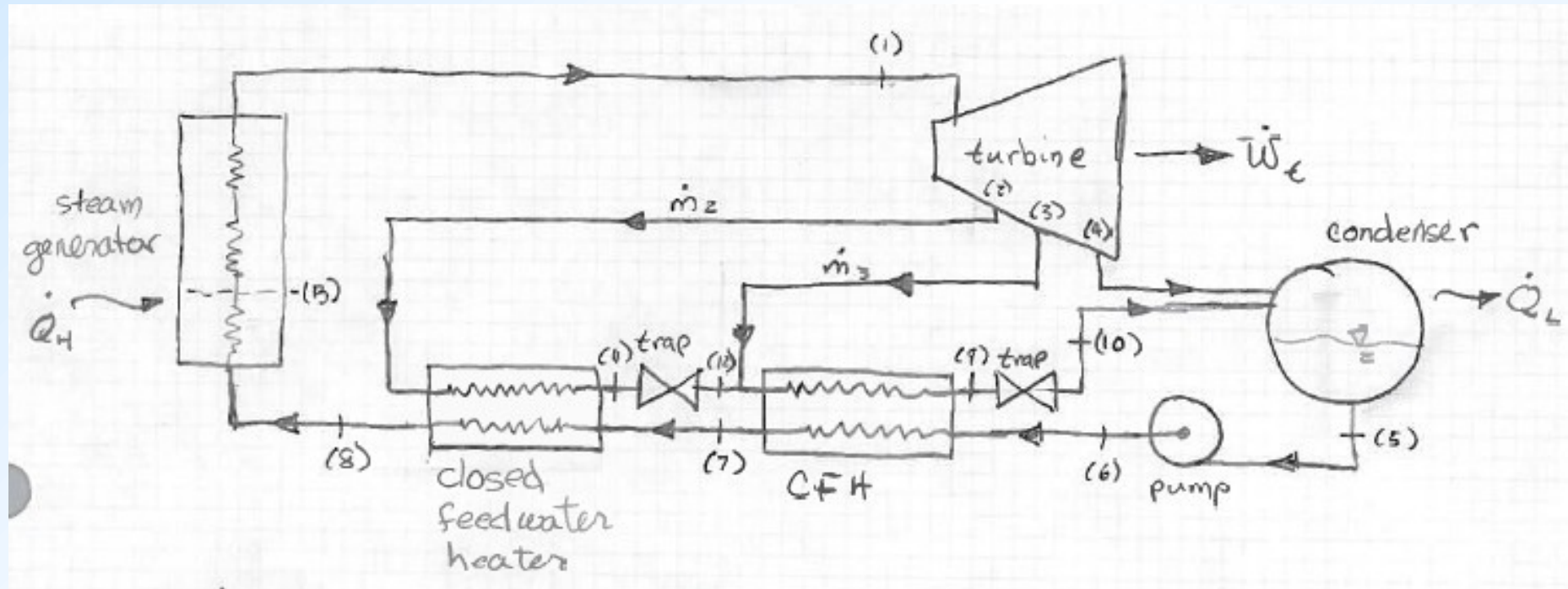
Closed feedwater Heater – Cascading Drain Backwards

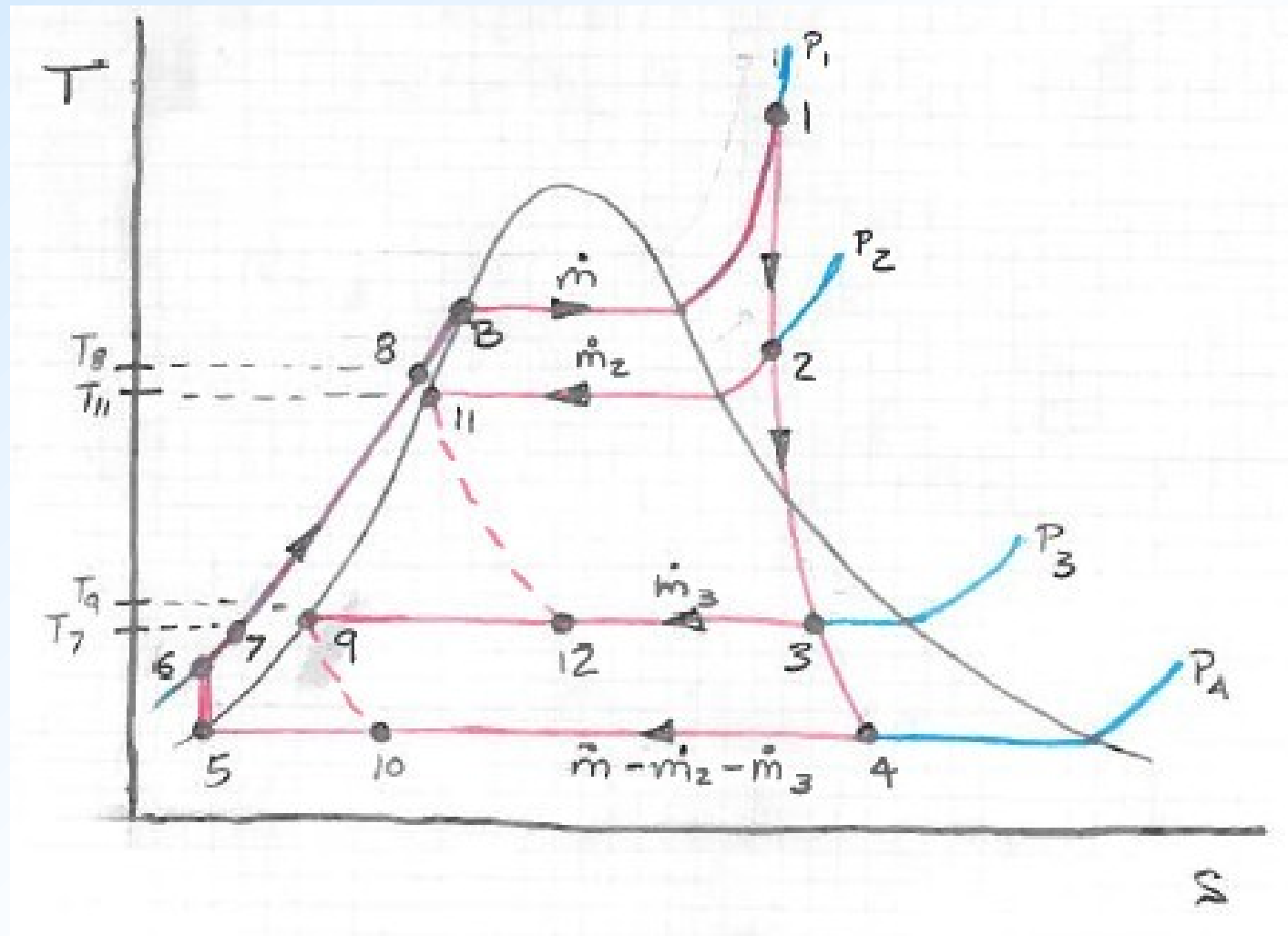




Closed feedwater Heater – Cascading Drain Backwards







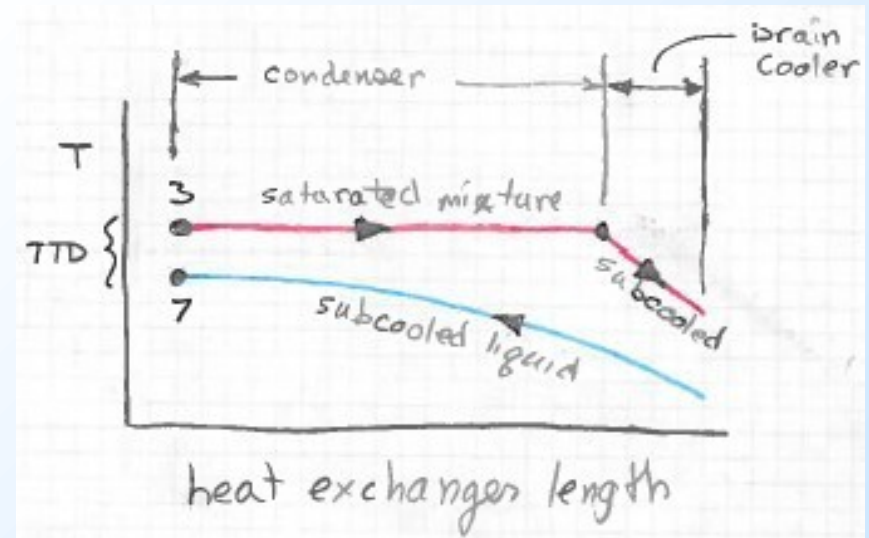
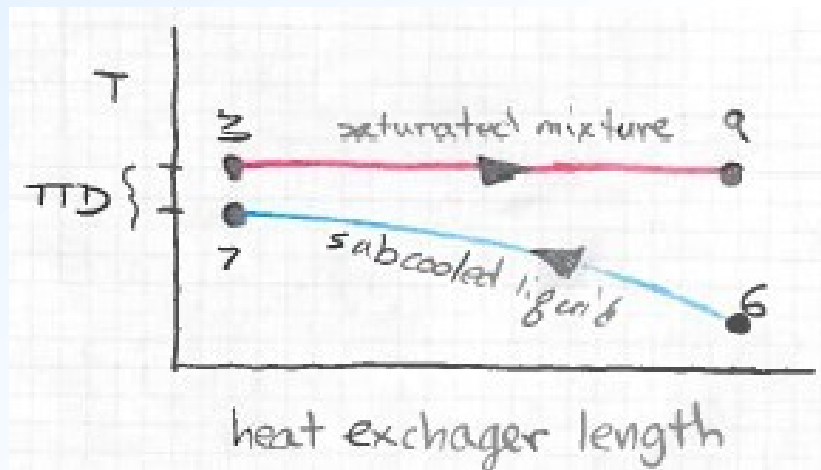


Terminal Temperature Difference (TTD or TD)

$$TTD = \left[\begin{array}{c} \text{Saturation temperature} \\ \text{of bled steam} \end{array} \right] - \left[\begin{array}{c} \text{Exit temperature} \\ \text{of feedwater} \end{array} \right]$$

TTD is a measure of feedwater effectiveness

For low-pressure feedwater heater:

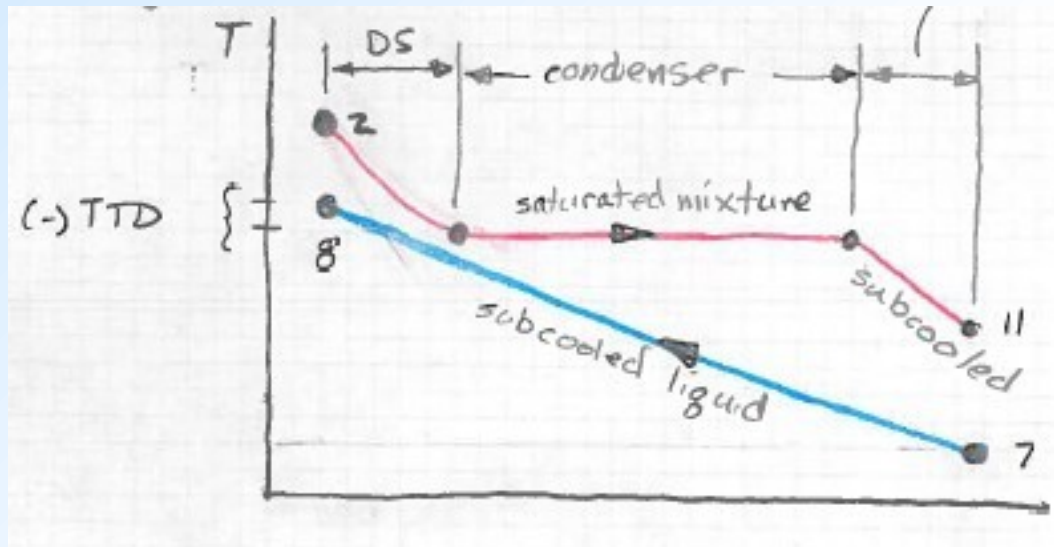




For low-pressure feedwater heater:

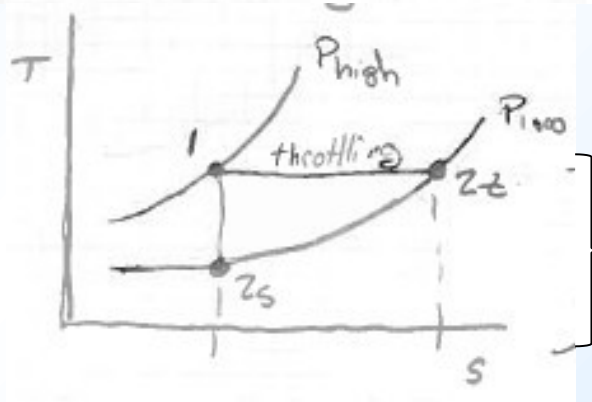
DS: Desuperheater

Drain Cooler



The drains must be throttled (“trapped”) to lower the pressure back to the pressure of the earlier stage. This throttling results in a loss of availability.

Recall for a gas

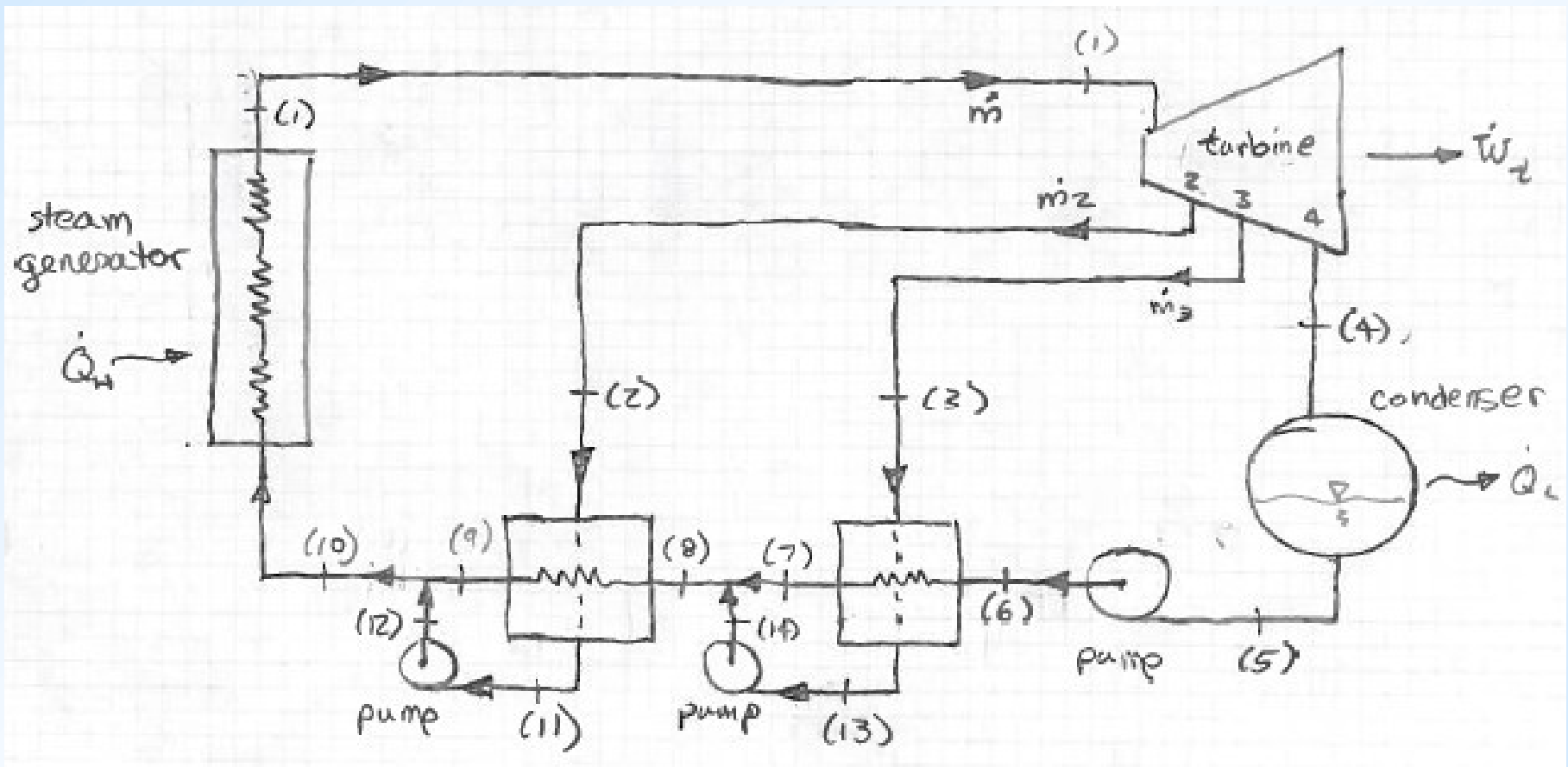


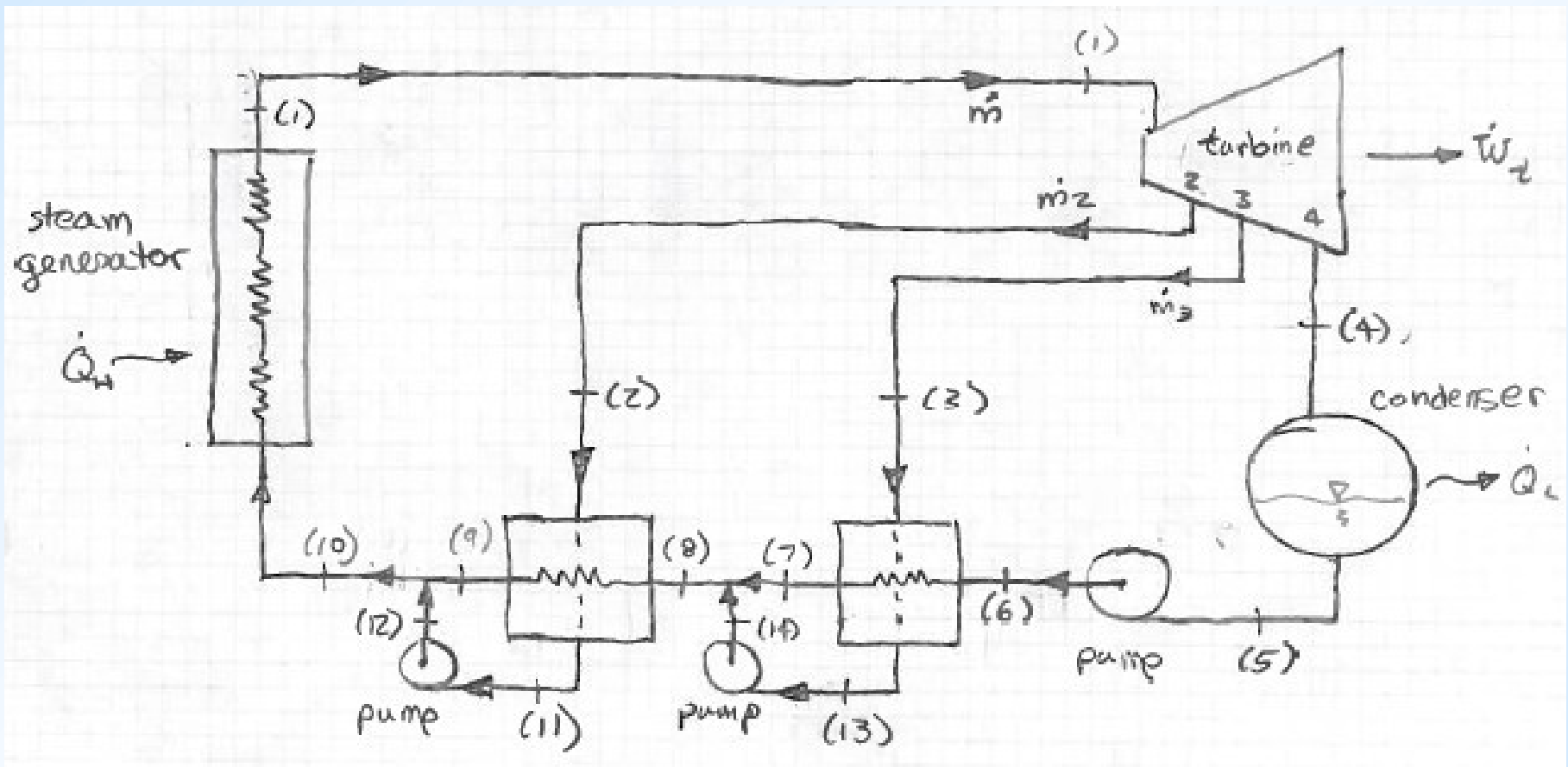
Loss of potential to extract work which is availability.
Also a loss of availability due to finite temperature difference during heat transfer – external irreversibility



Closed feedwater Heater – Drains Pumped Forward

- To reduce loss of availability
- Adds some complexity with small pumps
- Drains pumped forward into the main feedwater line
- Pumps only handle a fraction of the flow, not all of the flow as with the open feedwater heaters
- Efficiency is better
- Typically used at the lowest feedwater pressure point in order to prevent throttling of a large flow (combined cascaded drains) into the condenser

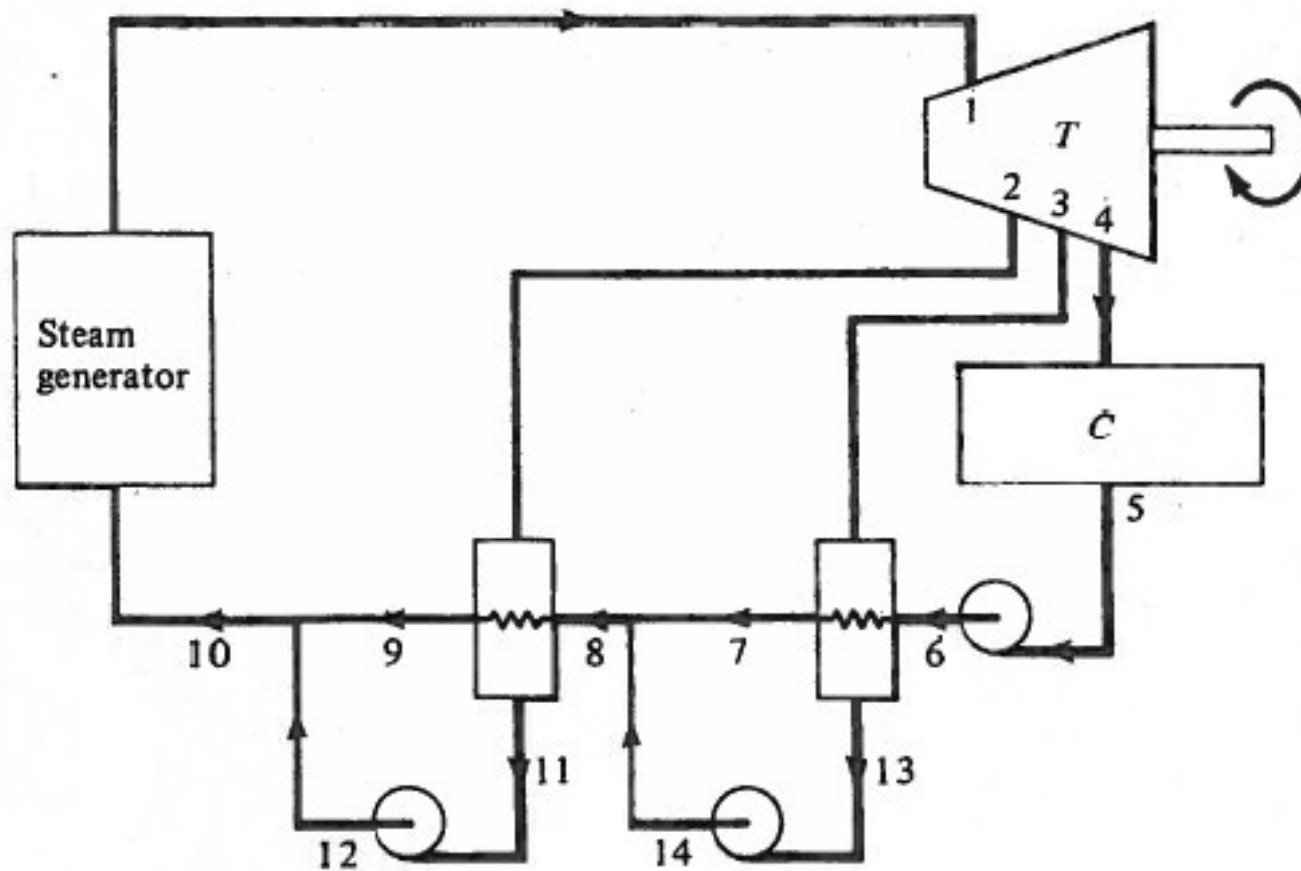


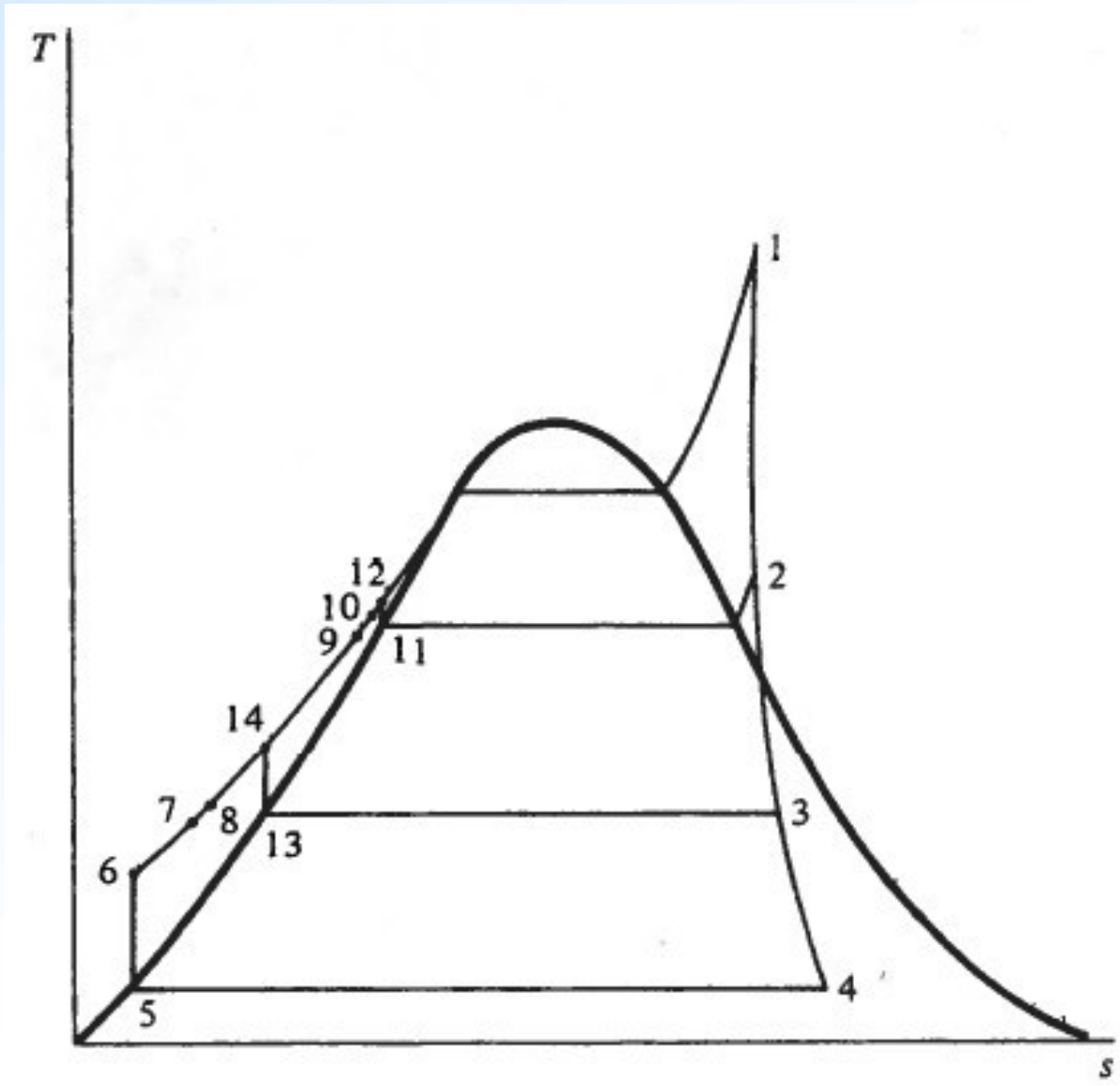






Closed Feedwater Heater - Pumped Forward



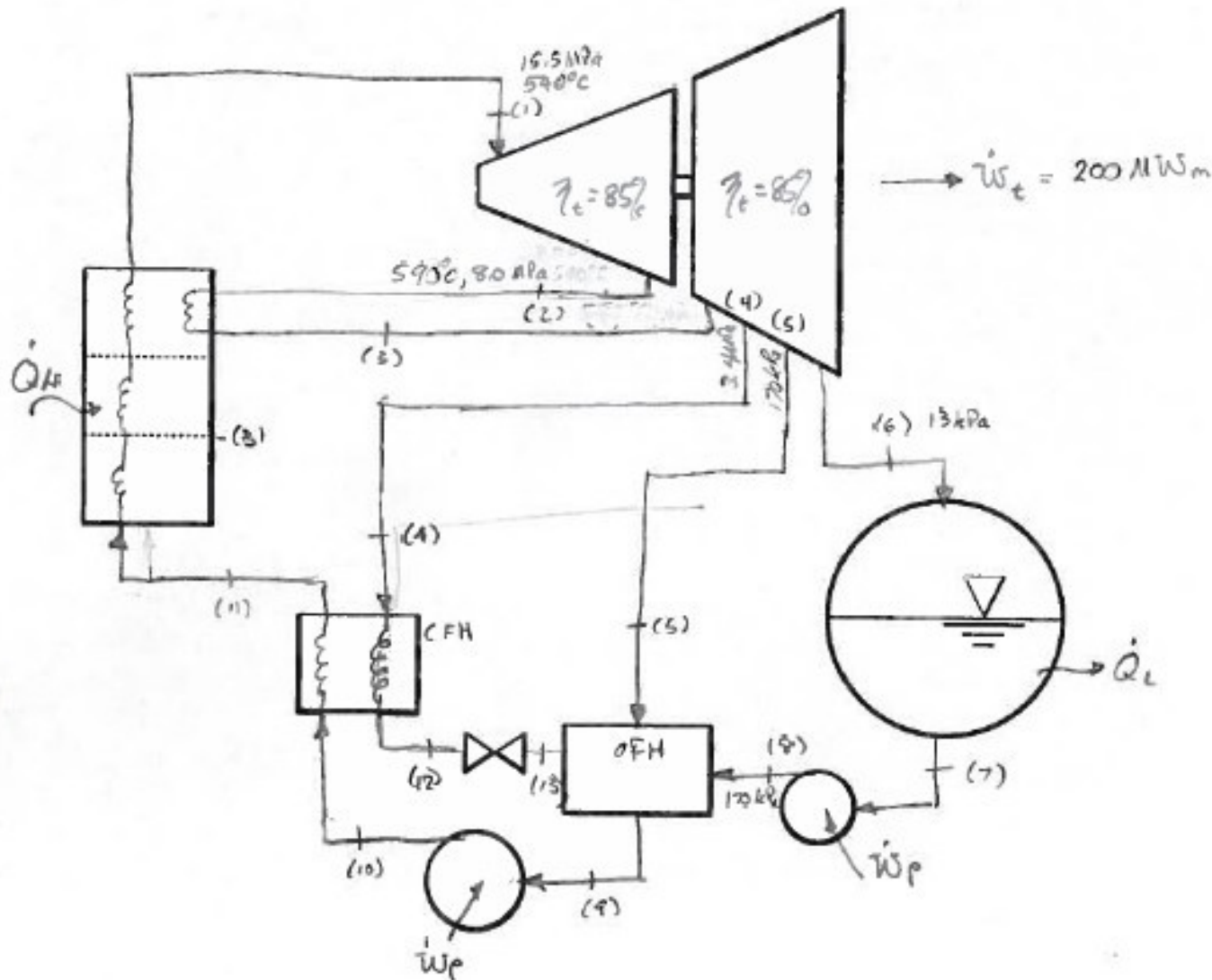




Example 3

Determine the thermal efficiency, the required steam flow rate, and the moisture at the turbine exhaust for a reheat/regenerative cycle which is to produce 200 MW at the turbine coupling if the turbine throttle conditions are 15.5 MPa and 540 °C; reheat is at 8 MPa and 590 °C; one closed feed water heater is at 3.4 MPa; an open feed water heater is at 170 kPa; and the condenser pressure is 13 kPa.

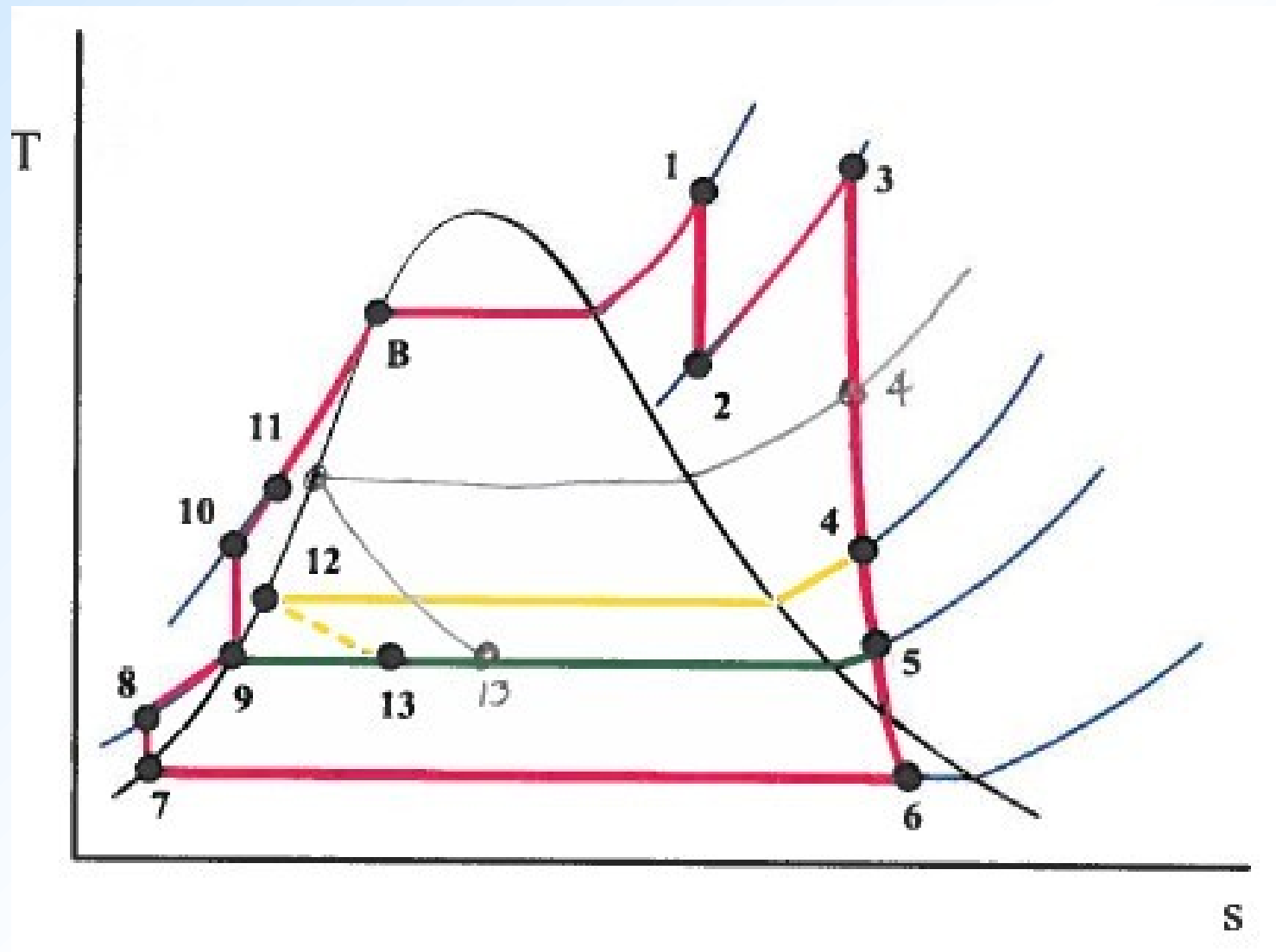
The adiabatic turbine and pump efficiencies are 84 %. The temperature difference in the closed feed water heater is 3 °C and the drain for the closed feed water heater is trapped to the open feed water heater. Sketch the cycle on a T-s diagram.



$$\eta_{th} = ?$$

$$\dot{m} = ?$$

$$1 - x = ?$$



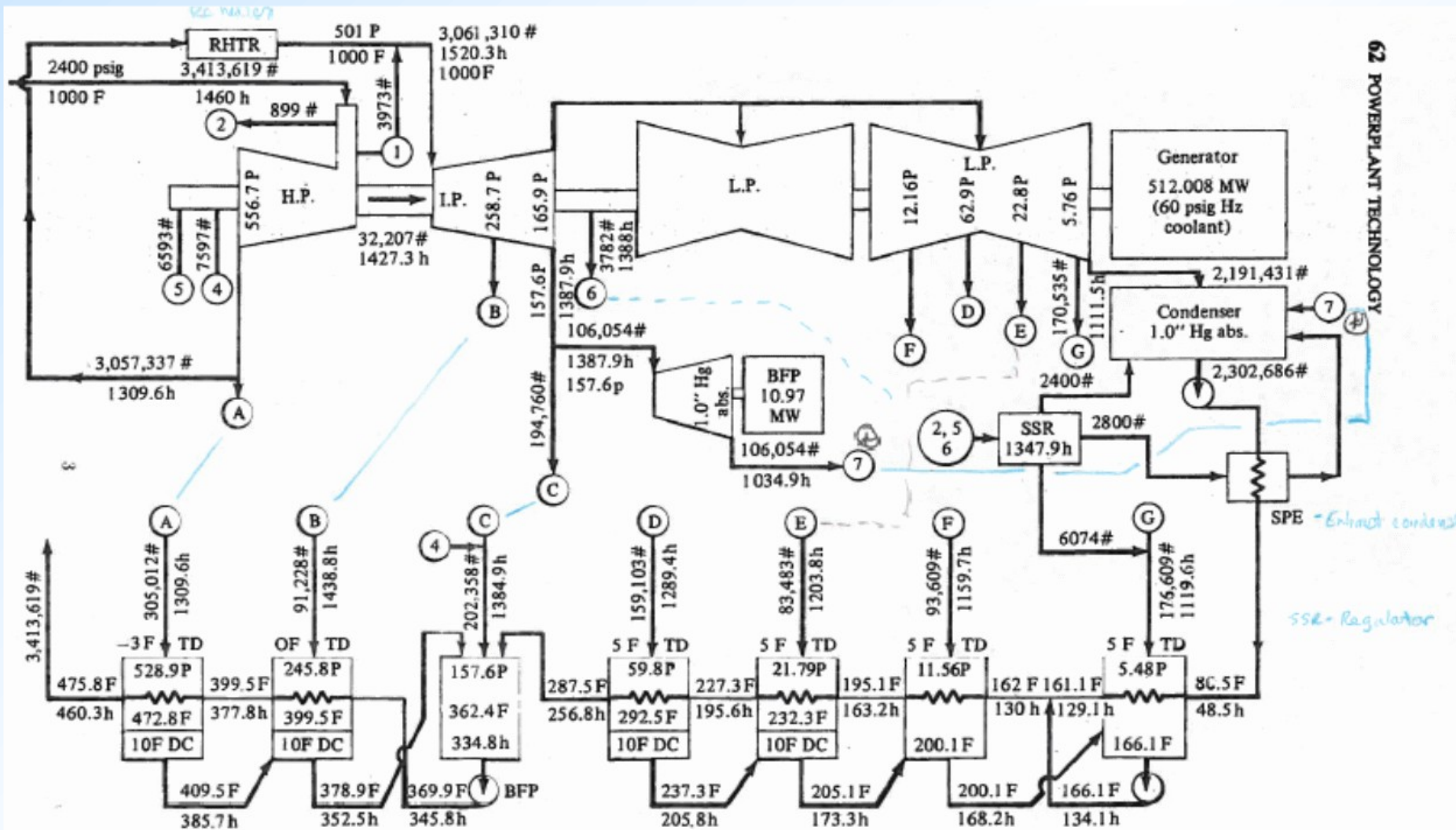
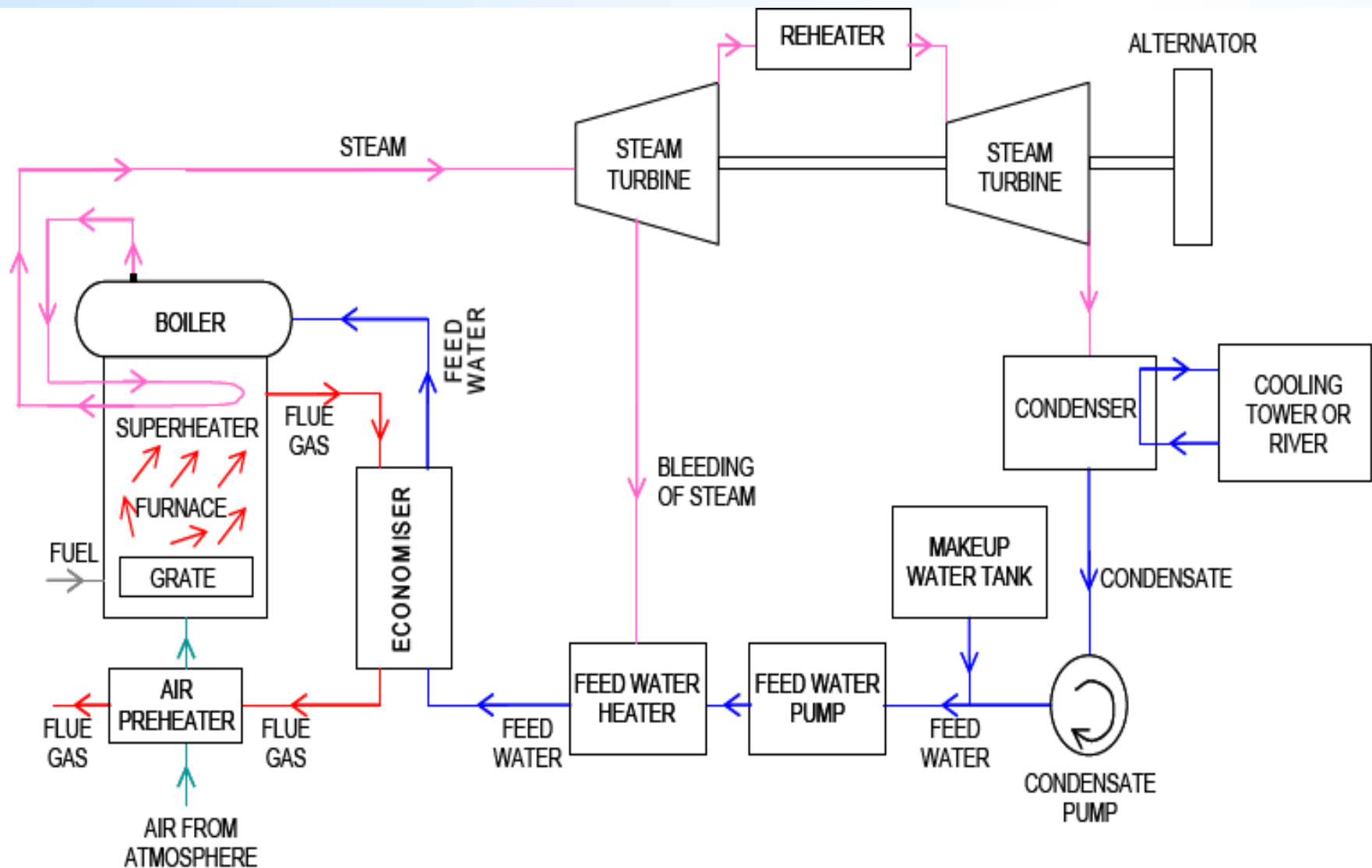
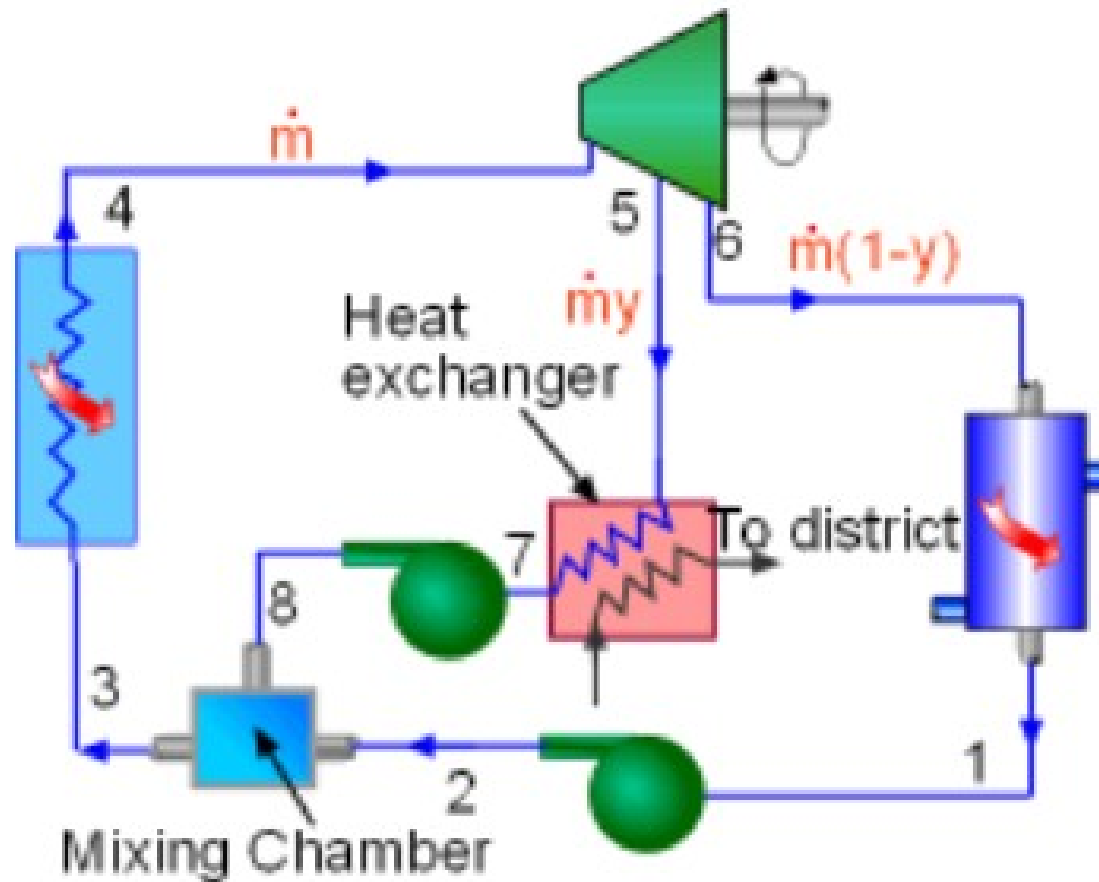


Figure 2-20 Flow diagram of an actual 512-MW 2400 psig/1000°F/1000°F reheat powerplant with seven feedwater heaters. (Courtesy Wisconsin Power & Light Co.)





Schematic of the
Cogeneration Power Plant



Typical Combined Cycle Plant

