Energy and Mass

$\Delta E = \Delta m\ c^2$

c: speed of light in vacuum = $2.9979\ 10^8$ m/s

m: mass in g, kg, or **amu** (atomic mass unit, au)

1 amu = $1.66\ 10^{-27}$ kg

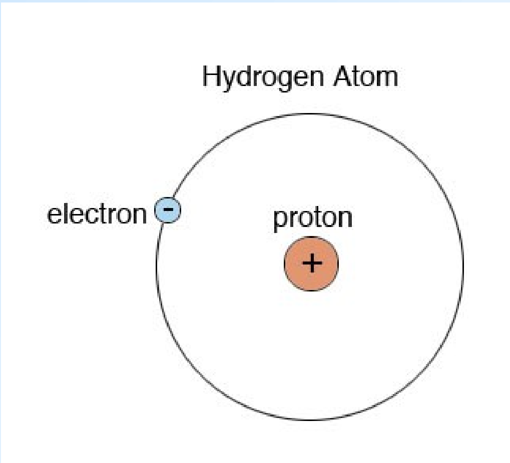
E: Energy in J or kJ or **eV** (electron volt) or MeV (Mega electron volt)

1 eV = $1.602\ 10^{-19}$ J

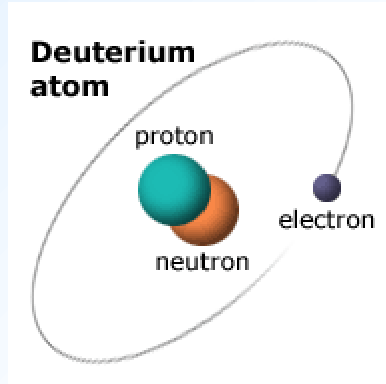
By definition, **eV** is the amount of energy gained (or lost) by the charge of a single electron moved across an electric potential difference of 1 Volt. Thus, it is 1 Volt (1 Joule per Coulomb, 1 J/C) multiplied by the elementary charge (*e*, or $1.602176565(35)\times10^{-19}$ C).

Mass-Energy Conversion Factors

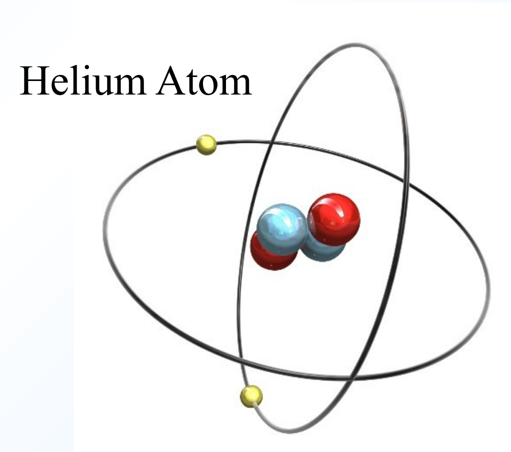
Mass	J	MeV
1 kg	$8.983\ 10^{16}$	$5.6094\ 10^{29}$
1 amu	$1.4924\ 10^{-10}$	**931.494**



$^{1}_{1}H$



$^{2}_{1}H$ Heavy hydrogen
Exists in 1:6660



$^{4}_{2}He$

Protons and neutrons in the nucleus are called **nucleons**

Electron mass: 0.0005486 amu

Proton mass: 1.007277 amu

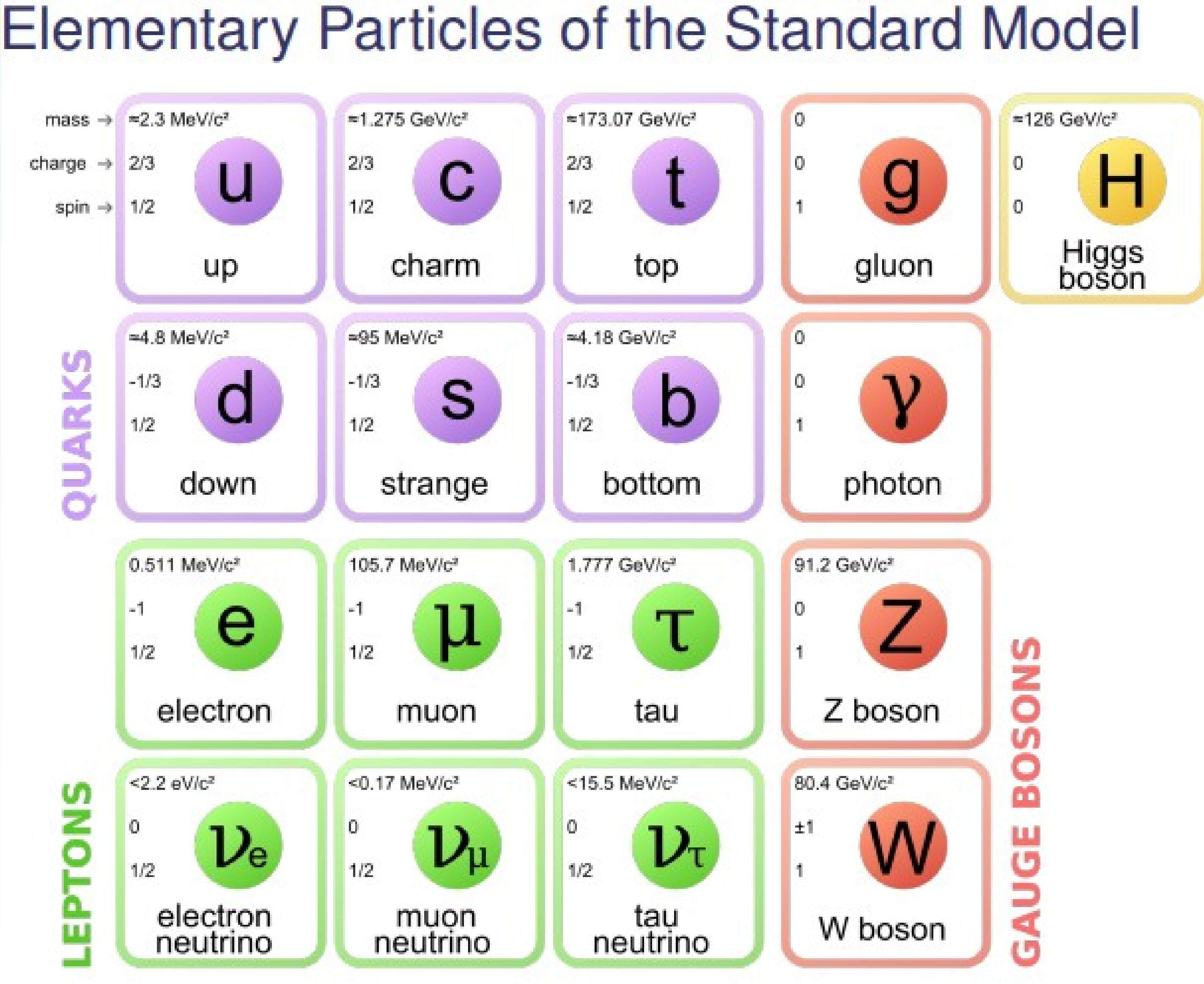
Neutron mass: 1.008665 amu

Nuclear symbol: $^{A}_{Z}X$

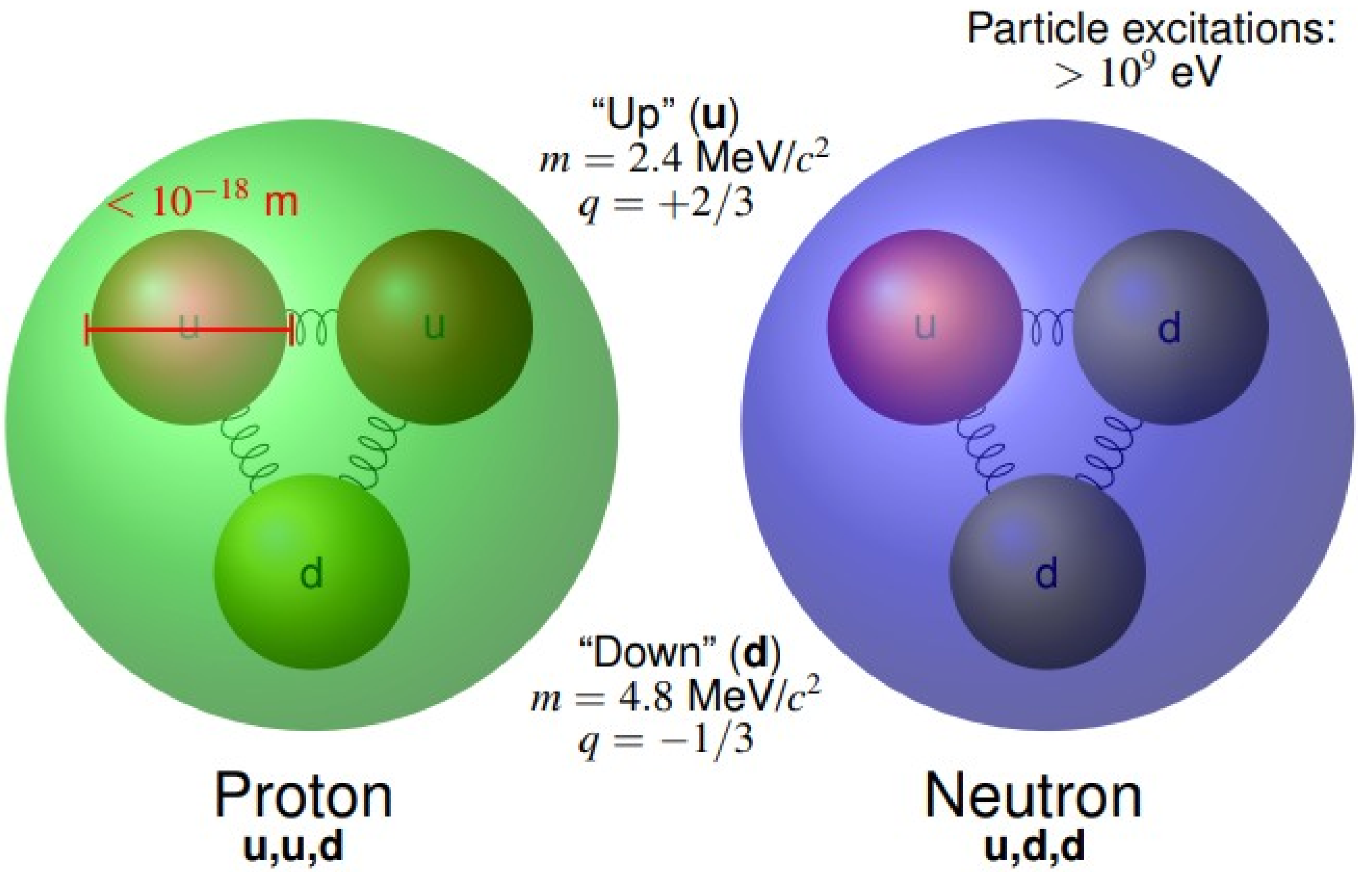
X: Chemical symbol

Z: Atomic number (number of protons)

A: Mass number (number of nucleons)


Elementary Particles of the Standard Model
mass →
charge →
spin →
≈2.3 MeV/c^2 2/3 1/2 u up
≈1.275 GeV/c^2 2/3 1/2 c charm
≈173.07 GeV/c^2 2/3 1/2 t top
0 0 1 g gluon
≈126 GeV/c^2 0 0 H Higgs boson
≈4.8 MeV/c^2 -1/3 1/2 d down
≈95 MeV/c^2 -1/3 1/2 s strange
≈4.18 GeV/c^2 -1/3 1/2 b bottom
0 0 1 γ photon
0.511 MeV/c^2 -1 1/2 e electron
105.7 MeV/c^2 -1 1/2 μ muon
1.777 GeV/c^2 -1 1/2 τ tau
91.2 GeV/c^2 0 1 Z Z boson
<2.2 eV/c^2 0 1/2 ν_e electron neutrino
<0.17 MeV/c^2 0 1/2 ν_μ muon neutrino
<15.5 MeV/c^2 0 1/2 ν_τ tau neutrino
80.4 GeV/c^2 ±1 1 W W boson
QUARKS
LEPTONS
GAUGE BOSONS

The Structure of Nucleons



Z in the nuclear symbol is often dropped since the chemical symbol, X, and the number of protons represent the same information. Helium-4 or He-4 instead of $^{4}_{2}He$

Chemical and physical properties are dictated by number of protons

Nuclear properties are dictated by number of nucleons ➔ **isotopes**

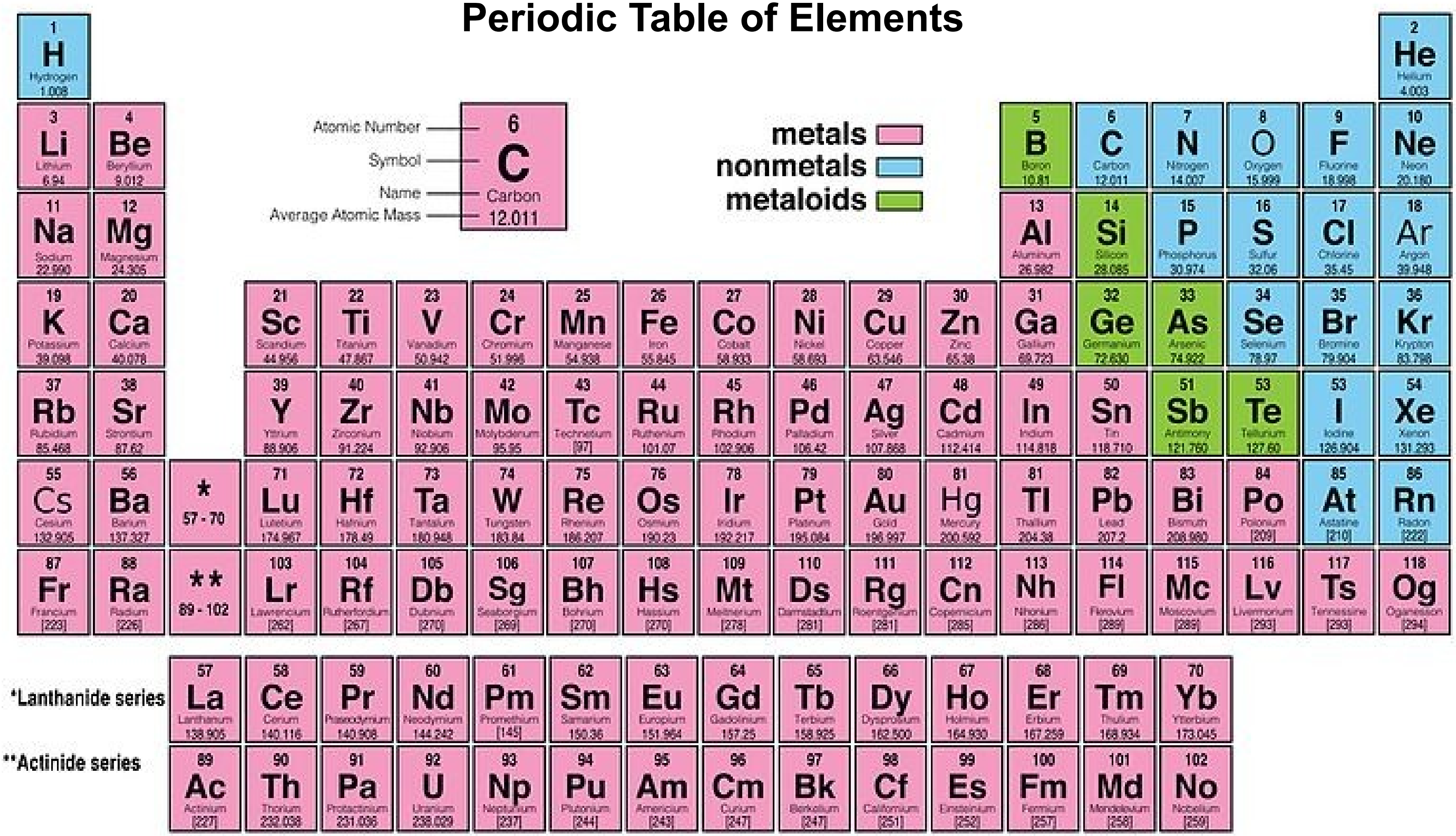
Natural Uranium: 99.282 mass % U-238 (atomic number 92)

0.712 mass % U-235

0.006 mass % U-234

Many isotopes that do not naturally occur in nature appear in the lab or during nuclear reactions. For example, Uranium has 14 known isotopes ranging from 227 to 240.

Periodic Table of Elements



See: http://en.wikipedia.org/wiki/Table_of_nuclides

Half-lives
(example: Gd)

^{145}Gd	< 1 day
^{146}Gd	1–10 days
^{149}Gd	10–100 days
^{153}Gd	100 days–10 a
^{148}Gd	10–10,000 a
^{150}Gd	10 ka–103 Ma
^{152}Gd	> 700 Ma
^{158}Gd	Stable

n ↓ / Z →	0 (n)	1 (H)	2 (He)	3 (Li)	4 (Be)	5 (B)	6 (C)	7 (N)	8 (O)	9 (F)	10 (Ne)	11 (Na)	12 (Mg)	13 (Al)	14 (Si)
0		^{1}H													
1	^{1}n	^{2}H	^{3}He	^{4}Li	^{5}Be										
2	^{2}n	^{3}H	^{4}He	^{5}Li	^{6}Be	^{7}B	^{8}C								
3		^{4}H	^{5}He	^{6}Li	^{7}Be	^{8}B	^{9}C	^{10}N							
4	^{4}n	^{5}H	^{6}He	^{7}Li	^{8}Be	^{9}B	^{10}C	^{11}N	^{12}O						
5		^{6}H	^{7}He	^{8}Li	^{9}Be	^{10}B	^{11}C	^{12}N	^{13}O	^{14}F					
6		^{7}H	^{8}He	^{9}Li	^{10}Be	^{11}B	^{12}C	^{13}N	^{14}O	^{15}F	^{16}Ne				
7			^{9}He	^{10}Li	^{11}Be	^{12}B	^{13}C	^{14}N	^{15}O	^{16}F	^{17}Ne	^{18}Na			
8			^{10}He	^{11}Li	^{12}Be	^{13}B	^{14}C	^{15}N	^{16}O	^{17}F	^{18}Ne	^{19}Na	^{20}Mg		
9				^{12}Li	^{13}Be	^{14}B	^{15}C	^{16}N	^{17}O	^{18}F	^{19}Ne	^{20}Na	^{21}Mg	^{22}Al	
10					^{14}Be	^{15}B	^{16}C	^{17}N	^{18}O	^{19}F	^{20}Ne	^{21}Na	^{22}Mg	^{23}Al	^{24}Si
11						^{16}B	^{17}C	^{18}N	^{19}O	^{20}F	^{21}Ne	^{22}Na	^{23}Mg	^{24}Al	^{25}Si
12							^{18}C	^{19}N	^{20}O	^{21}F	^{22}Ne	^{23}Na	^{24}Mg	^{25}Al	^{26}Si
13								^{20}N	^{21}O	^{22}F	^{23}Ne	^{24}Na	^{25}Mg	^{26}Al	^{27}Si
14									^{22}O	^{23}F	^{24}Ne	^{25}Na	^{26}Mg	^{27}Al	^{28}Si
15										^{24}F	^{25}Ne	^{26}Na	^{27}Mg	^{28}Al	^{29}Si
16											^{26}Ne	^{27}Na	^{28}Mg	^{29}Al	^{30}Si
17												^{28}Na	^{29}Mg	^{30}Al	^{31}Si
18													^{30}Mg	^{31}Al	^{32}Si
19														^{32}Al	^{33}Si
20															^{34}Si

Half-lives (example: Gd)

^{145}Gd	< 1 day
^{146}Gd	1–10 days
^{149}Gd	10–100 days
^{153}Gd	100 days–10 a
^{148}Gd	10–10,000 a
^{150}Gd	10 ka–103 Ma
^{152}Gd	> 700 Ma
^{158}Gd	Stable

Z ›	90														
n ↓	Th	91													
131	^{221}Th	Pa	92												
132	^{222}Th	^{223}Pa	U	93											
133	^{223}Th	^{224}Pa		Np	94										
134	^{224}Th	^{225}Pa	^{226}U		Pu	95									
135	^{225}Th	^{226}Pa	^{227}U	^{228}Np		Am									
136	^{226}Th	^{227}Pa	^{228}U	^{229}Np			96								
137	^{227}Th	^{228}Pa	^{229}U	^{230}Np		^{232}Am	Cm								
138	^{228}Th	^{229}Pa	^{230}U	^{231}Np	^{232}Pu										
139	^{229}Th	^{230}Pa	^{231}U	^{232}Np	^{233}Pu	^{234}Am		97	98	99	100				
140	^{230}Th	^{231}Pa	^{232}U	^{233}Np	^{234}Pu	^{235}Am	^{236}Cm	Bk	Cf	Es	Fm				
141	^{231}Th	^{232}Pa	^{233}U	^{234}Np	^{235}Pu	^{236}Am	^{237}Cm				^{241}Fm				
142	^{232}Th	^{233}Pa	^{234}U	^{235}Np	^{236}Pu	^{237}Am	^{238}Cm	^{239}Bk	^{240}Cf	^{241}Es	^{242}Fm				
143	^{233}Th	^{234}Pa	^{235}U	^{236}Np	^{237}Pu	^{238}Am	^{239}Cm	^{240}Bk	^{241}Cf	^{242}Es	^{243}Fm				
144	^{234}Th	^{235}Pa	^{236}U	^{237}Np	^{238}Pu	^{239}Am	^{240}Cm	^{241}Bk	^{242}Cf	^{243}Es	^{244}Fm	101			
145	^{235}Th	^{236}Pa	^{237}U	^{238}Np	^{239}Pu	^{240}Am	^{241}Cm	^{242}Bk	^{243}Cf	^{244}Es	^{245}Fm	Md	102		
146	^{236}Th	^{237}Pa	^{238}U	^{239}Np	^{240}Pu	^{241}Am	^{242}Cm	^{243}Bk	^{244}Cf	^{245}Es	^{246}Fm		No	103	104
147	^{237}Th	^{238}Pa	^{239}U	^{240}Np	^{241}Pu	^{242}Am	^{243}Cm	^{244}Bk	^{245}Cf	^{246}Es	^{247}Fm	^{248}Md		Lr	Rf
148	^{238}Th	^{239}Pa	^{240}U	^{241}Np	^{242}Pu	^{243}Am	^{244}Cm	^{245}Bk	^{246}Cf	^{247}Es	^{248}Fm	^{249}Md	^{250}No		
149		^{240}Pa	^{241}U	^{242}Np	^{243}Pu	^{244}Am	^{245}Cm	^{246}Bk	^{247}Cf	^{248}Es	^{249}Fm	^{250}Md	^{251}No	^{252}Lr	^{253}Rf
	150		^{242}U	^{243}Np	^{244}Pu	^{245}Am	^{246}Cm	^{247}Bk	^{248}Cf	^{249}Es	^{250}Fm	^{251}Md	^{252}No	^{253}Lr	^{254}Rf
		151		^{244}Np	^{245}Pu	^{246}Am	^{247}Cm	^{248}Bk	^{249}Cf	^{250}Es	^{251}Fm	^{252}Md	^{253}No	^{254}Lr	^{255}Rf
			152		^{246}Pu	^{247}Am	^{248}Cm	^{249}Bk	^{250}Cf	^{251}Es	^{252}Fm	^{253}Md	^{254}No	^{255}Lr	^{256}Rf
			153		^{247}Pu	^{248}Am	^{249}Cm	^{250}Bk	^{251}Cf	^{252}Es	^{253}Fm	^{254}Md	^{255}No	^{256}Lr	^{257}Rf

Nuclear particles without any protons:

Electron: e^-, β^- Beta particle

Neutron: n

Positron: e^+, β^+

Neutrino (little neutron): ν -> carries 5 % of total energy produced in fission

Isotope	Mass in amu
Electron	0.000549
Proton	1.007277
Neutron	1.008665
Hydrogen, H-1	1.007825
Deuterium, H-2	2.01410
Helium, He-4	4.00260

$m_e + m_p = 1.007826$ amu (compare Hydrogen, H-1)

$m_e + m_p + m_n = 2.016491$ amu (compare Deuterium, H-2)

Mass defect

The mass of an isotope is less than the sum of the masses of individual particles. This is called "**mass defect**" (MD). It is the negative mass "glue" that prevents the Coulomb forces associated with protons from tearing the nucleus apart.

In order to break the nucleus, a minimum energy equivalent to MD has to be added to the nucleus.

$$MD = Z\, m_p + (A - Z)\, m_n - \text{nucleus mass}$$

- Nucleus mass is difficult to measure.
- Atomic mass is easier to measure, but it includes the orbiting electrons.

$$MD = Z\, m_{H_1} + (A - Z)\, m_n - \text{atomic mass}$$

Energy equivalent of MD is the total **Binding Energy** (BE) – absolute minimum energy required to break a nucleus into Z protons and (A-Z) neutrons, or A nucleons.

Example 1

Calculate the binding energy per nucleon of the following isotopes:

(a) H-2 or D-2 (Z = 1, A = 2, atomic mass = 2.0141 amu)

(b) Fe-56 (Z = 26, A = 56, atomic mass = 55.934934 amu)

(c) Ni-59 (Z = 28, A = 59), atomic mass = 58.9342 amu)

(d) U-238 (Z = 92, A = 238, atomic mass = 238.0289 amu)

Mass of H-1 = 1.007825 amu

Mass of a neutron = 1.0086625 amu

1 amu (or au) = 931.5 MeV

(d)

U-238 (Z = 92, A = 238, atomic mass = 238.0289 amu)

Binding Energy in MeV = (Mass Defect in amu) (931.5 MeV7amu)

Mass Defect = (Z) (m_{H-1}) + (A – Z) (m_n) – Atomic Mass

= (92) (1.007825) + (238 – 92) (1.0086625) – 238.0289

= 1.955725 amu

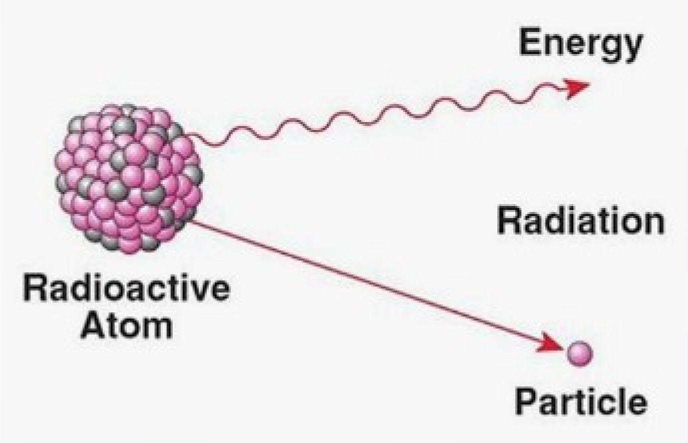
BE = (1.955725) (931.5) = 1821.758 MeV

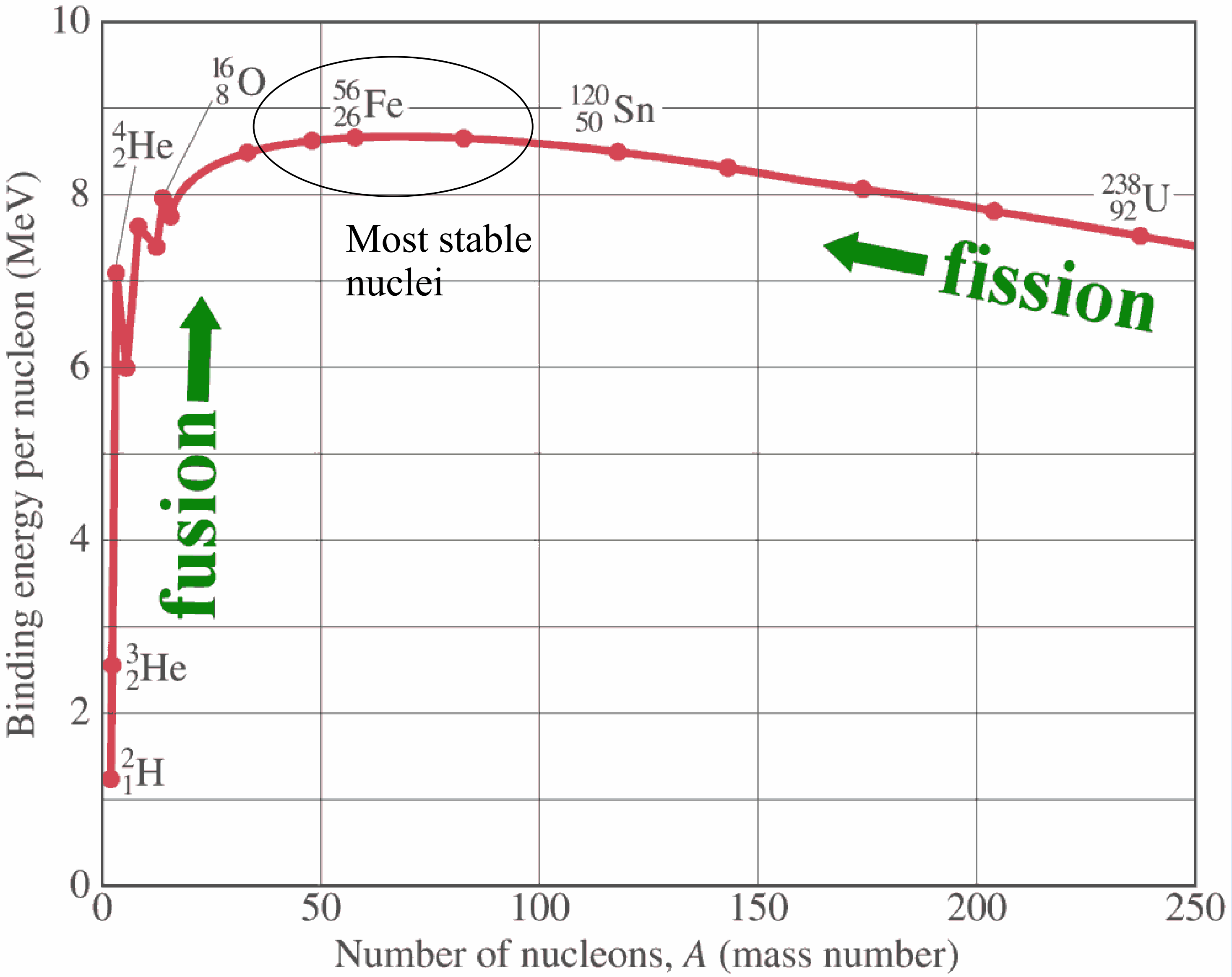
BE / nucleon = 1821.758 / 238 = 7.65 MeV/nucleon

The excess binding energy is released in any nuclear reaction in which heavy-mass nucleus is broken into two intermediate-mass nuclei – **fission**.

The same is true for two light-mass nuclei combined into a heavier nucleus – **fusion**.

Radioactive decay is the release of binding energy as a nucleus spontaneously decays (disintegrates) to a slightly lighter and more stable nucleus, accompanied by emission of particles, electromagnetic radiation (gamma rays), or both.





10
8
6
4
2
0
0
50
100
150
200
250
Binding energy per nucleon (MeV)
Number of nucleons, A (mass number)
16 8 O
56 26 Fe
120 50 Sn
238 92 U
4 2 He
3 2 He
2 1 H
Most stable nuclei
fusion
fission

Four Fundamental Forces of Nature

Kind of Interaction	Source	Range	Relative Strength
Strong Nuclear	Nucleons	> 10^{-15} m	1
Electromagnetic	Charged particles	Infinite ($1/r^2$)	10^{-2}
Weak Nuclear	Electron Decay	Short	10^{-15}
Gravitational	Mass	Infinite ($1/r^2$)	10^{-58}

Strong Nuclear Force:

The strong interaction is carried by a particle called the gluon and is responsible for quarks binding together to form hadrons, such as protons and neutrons. As a residual effect, it creates the nuclear force that binds the latter particles to form atomic nuclei.

Electromagnetic Force:

The electromagnetic force, also called the Lorentz force, acts between charged particles, like negatively charged electrons and positively charged protons. Opposite charges attract one another, while like charges repel.

Electromagnetic forces are transferred between charged particles through the exchange of massless, force-carrying bosons called photons, which are also the particle components of light. The electromagnetic force, carried by the photon, creates electric and magnetic fields, which are responsible for the attraction between orbital electrons and atomic nuclei which holds atoms together, as well as chemical bonding and electromagnetic waves, including visible light, and forms the basis for electrical technology.

Weak Nuclear Force:

The weak force, also called the weak nuclear interaction, is responsible for particle decay. This is the literal change of one type of subatomic particle into another. Physicists describe this interaction through the exchange of force-carrying particles called bosons. Specific kinds of bosons are responsible for the weak force, electromagnetic force and strong force. In the weak force, the bosons are charged particles called W and Z bosons. When subatomic particles such as protons, neutrons and electrons come within 10^-18 meters, or 0.1% of the diameter of a proton, of one another, they can exchange these bosons. As a result, the subatomic particles decay into new particles,

Gravitational Force:

Gravity is not an attraction or a force. Instead, it's a consequence of objects bending space-time, says Einstein. A large object works on space-time a bit like how a large ball placed in the middle of a sheet affects that material, deforming it and causing other, smaller objects on the sheet to fall toward the middle.

Though gravity holds planets, stars, solar systems and even galaxies together, it turns out to be the weakest of the fundamental forces, especially at the molecular and atomic scales.

Nuclear Reaction: occurs in a nucleus

Conversion of mass to energy or energy to mass

Two kinds - Occurs all by itself (radioactivity)

- Caused by man (bombardment) → Generate isotopes / Fission / Fusion

Conservations - Atomic number (charge), Z

- Mass number (no. of nucleons), A
- Mass / energy, together
- Momentum

There is a probability of occurrence for any reaction

Example 2

An exothermic reaction occurs when common aluminum (Al-27, mass = 26.99153 amu) is bombarded with high-energy α-particles (helium atom, mass = 4.00260 amu) resulting in Si-30 (a heavy isotope if silicon; mass = 29.9736 amu; the most common isotope of silicon is Si-28). During the reaction, a small particle is emitted. Write the reaction and calculate the change in mass. Find out what the emitted particle is.

In nuclear reactions:

- Sum of mass numbers is conserved (not the mass alone)
- Sum of atomic numbers is conserved
- Sum of mass and energy is conserved
- Momentum is conserved

$$^{27}_{13}Al + ^{4}_{2}\alpha = ^{30}_{14}Si + ^{A}_{Z}X \quad \left.\begin{aligned} A &= (27 + 4) - 30 = 1 \\ Z &= (13 + 2) - 1 = 1 \end{aligned}\right\} \; ^{1}_{1}H$$

$^{4}_{2}\alpha \rightarrow \; ^{4}_{2}He$

Mass of H-1 atom: 1.007825 amu

Change in mass: 26.98153 + 4.0026 – (29.9736 + 1.007825) = 0.0027 amu

- This mass appears as kinetic energy of the products.
- Momentum should be conserved.
- KE's of the products are immediately converted to thermal energy.

Radioactive decay (Radioactivity)

Unstable isotopes are radioactive - radioisotopes

Most naturally occurring isotopes are stable

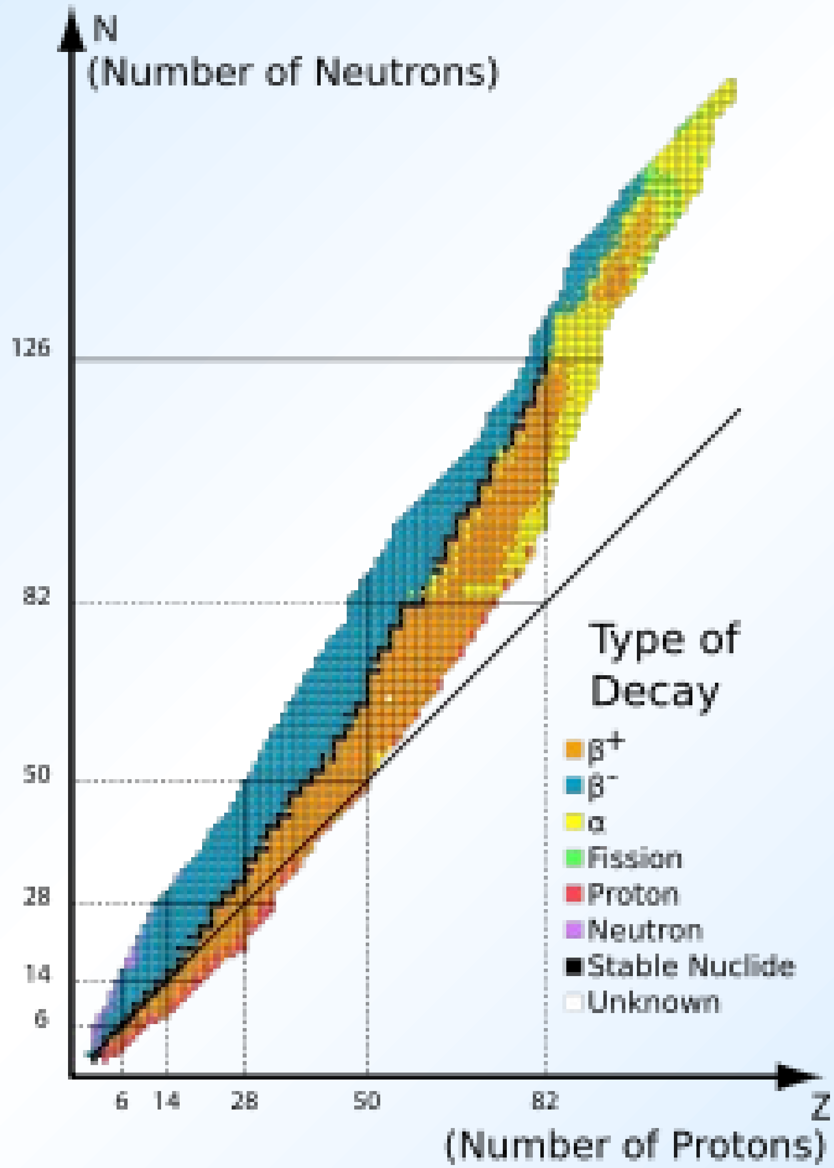
Radioactivity → spontaneous disintegration

Emission of one or more small particles (α, β, and γ) and radiation from the "parent" nucleus which changes into the "daughter" nucleus

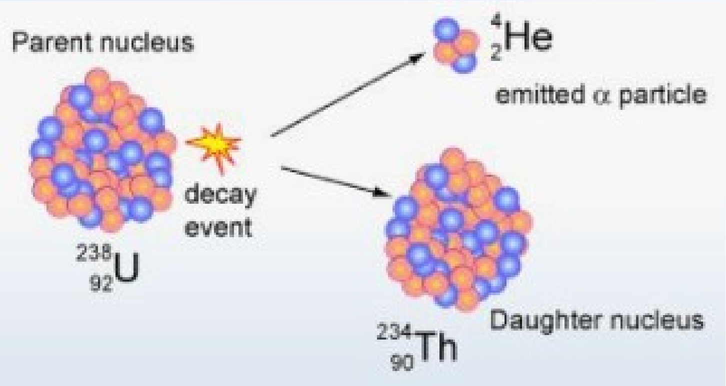
Radioactivity results in a net mass decrease → exothermic

Energy released is in the form of (1) kinetic energy; (2) γ radiation; and both (1) and (2)

Artificially generated radioisotopes may emit or undergo α, β, and γ as well as positrons, orbital electron absorption, K-capture, and neutrons and neutrinos.


N
(Number of Neutrons)
126
82
50
28
14
6
6
14
28
50
82
Z
(Number of Protons)
Type of Decay
β+
β-
α
Fission
Proton
Neutron
Stable Nuclide
Unknown

Alpha Decay - α



α-particle is the nucleus of He-4 atom -- $^4_2\alpha$

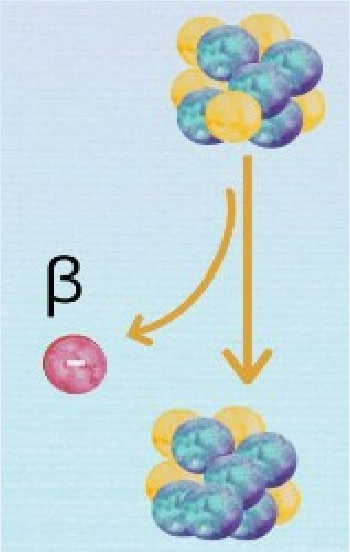
There are 150 radioisotopes that emit α-particles.

The decay that is important with respect to power production is

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2\alpha + \gamma\text{'s} \qquad t_{1/2} = 4.51\ 10^9 \text{ years}$$

- Daughter nucleus has a mass number 4 less than the parent nucleus
- **Half life**, $t_{1/2}$, is the time required for half of radioactive atoms to decay
- The resulting nucleus may be stable or unstable (radioactive)
- α-particles are mono-energetic, 4-6 MeV - very high kinetic energy but low penetration power; So not a biological hazard unless ingested.

Beta Decay - β



β-particle is an electron (negatron)

There are 450 radioisotopes that emit β-particles.

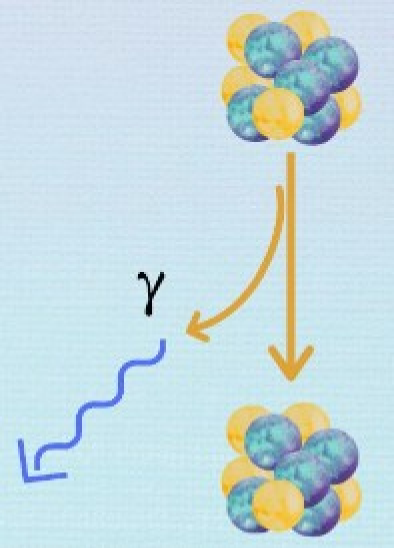
The decay that is important in medical applications is

$$^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + {}^{0}_{-1}\beta + \gamma_d + \nu$$

$$t_{1/2} = 5.3 \text{ years}$$

- One neutron in the nucleus changes into a proton and an electron + one neutrino is emitted.

Gamma Decay - γ



It is an electromagnetic radiation (photon)

Very short wavelength → high frequency

High energy

Similar to x rays

- x rays come from the orbital change of electrons
- γ rays come from the nucleus

Usually accompanies α and β decay

Positron (antielectron) Decay – β^+

$$^{55}_{27}\text{Co} \rightarrow\ ^{55}_{26}\text{Fe} + {}^{0}_{+1}\beta + \gamma_d + \nu \qquad t_{1/2} = 18 \text{ days}$$

- Proton changes into a neutron and emits a positron and neutrino
- Annihilation with β^- results in 2 γ_a (annihilation gamma rays), each having a rest mass of an electron (0.51 MeV)
  - γ_a's travel in opposite direction to conserve momentum
  - Mass is completely converted to energy
- Reaction is reversible
  - If a high energy γ ray (E_γ > 1.02 MeV) passes near a nucleus, it can be converted into an electron and a positron in a γ-ray reaction known as pair production

K-capture (electron capture)

- Nucleus absorbs one of the nearest orbiting electrons (K shell) and a proton is converted into a neutron

- Nucleus generally left in an unstable (excited) state

$$ {}^{57}_{27}Co + {}^{0}_{-1}e \rightarrow {}^{57}_{26}Fe + \gamma_d + x \text{ rays} \qquad t_{1/2} = 270 \text{ days} $$

Units of Radioactivity

Becquerel (Bq) = 1 disintegration per second

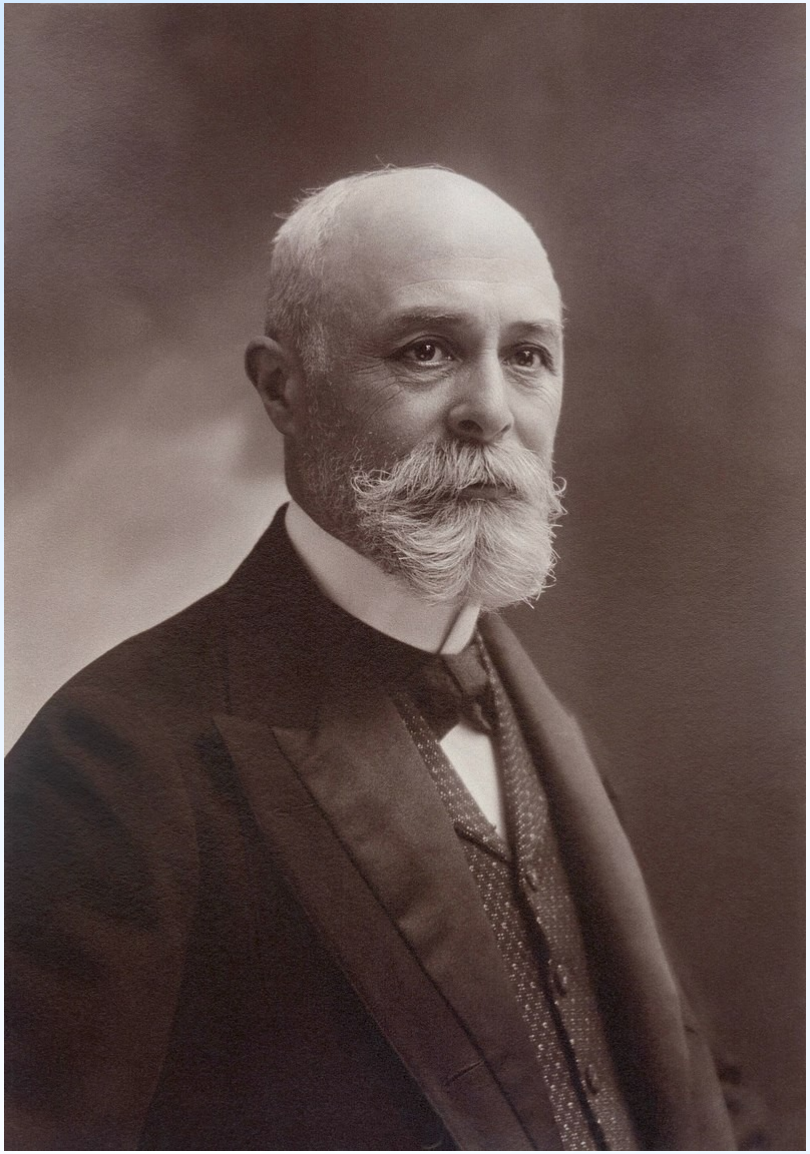
Curie (Ci) = decay rate of 1 g of pure radium-266 = $3.7\ 10^{10}$ Bq

Rutherford = 10^{6} Bq

Maria Salomea Skłodowska-Curie

Polish physicist and chemist

1867 - 1934



Antoine Henri Becquerel

French physicist

1852 - 1908

Radioactivity

Decay rate is proportional to the number of radioactive nuclei: $\frac{dN}{dt} \propto N$

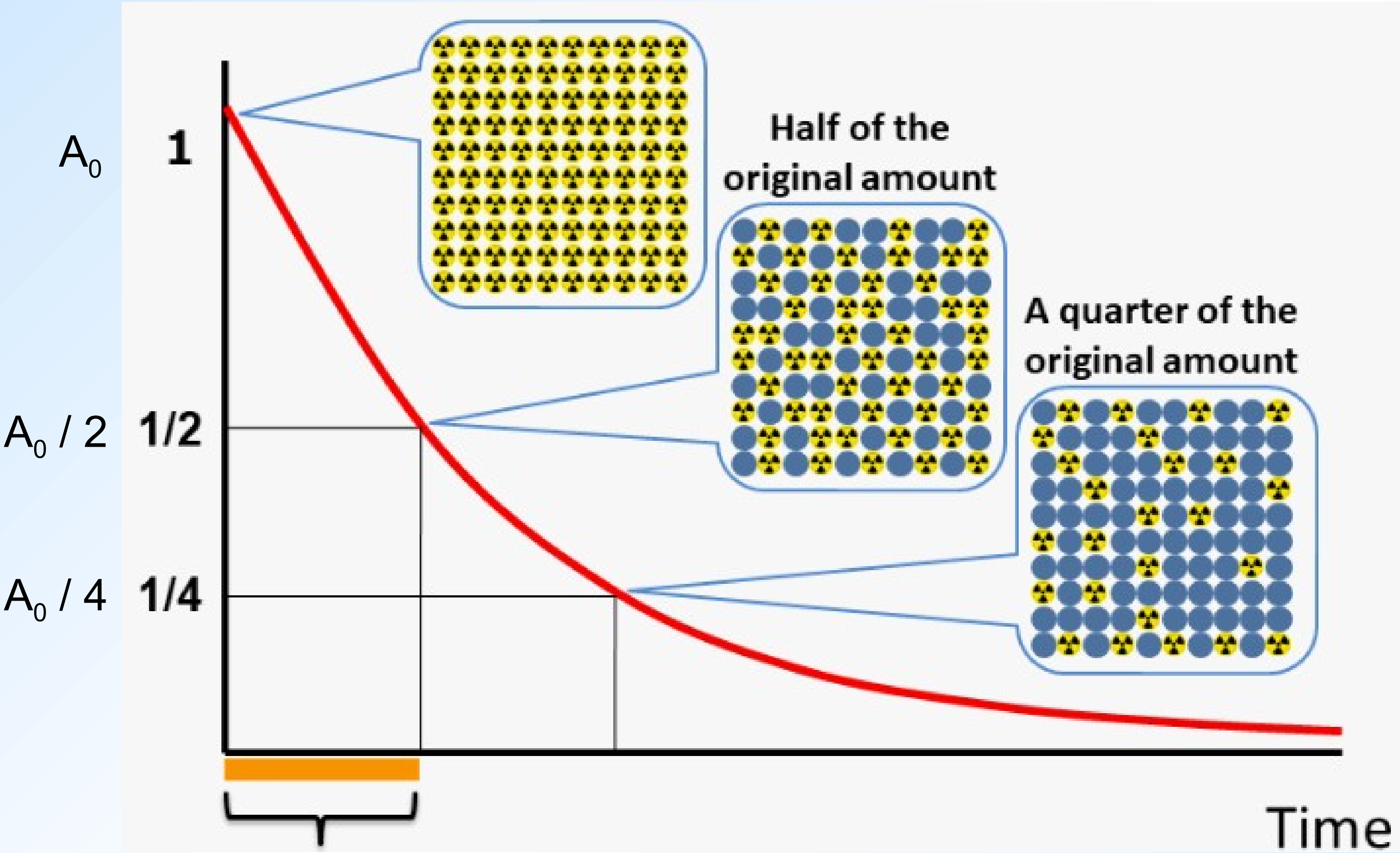
$$\left.\begin{aligned} \frac{dN}{dt} &= -\lambda N \\ N(t) &= N_0\, e^{-\lambda t} \end{aligned}\right\}$$

where λ is constant of proportionality called decay constant

Activity = Decay rate = - dN/dt = λ N $\quad A = A_0\, e^{-\lambda t}$

Half life is the time required for half of the radioactive nuclei to decay:

$$N = \frac{N_0}{2} = N_0\, e^{-\lambda t_{1/2}} \quad \Rightarrow \quad t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$



Time required for the amount of radionuclides to reduce to half
= (physical) half-life

Example 3

Radium-226 decays into Radon gas. Compute

- The decay constant;
- The initial activity of 1 gram of Radium-226; and
- Activity after 100 years.

$^{226}_{88}Ra$ 226.0254 amu (g/mol) , $t_{1/2}$ = 1600 years , α decay

Avogadro number: $N_A = 6.02214\ 10^{23}$ atoms (nuclei) per mol

Hint: Find the number of nuclei in 1 gram of Ra-226

$$\lambda = \frac{\ln(2)}{t_{1/2}} \qquad A(t) = A_0\ e^{-\lambda t}$$

(a)

Decay constant: $\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{(1600)\,(365)\,(24)\,(3600)} = 1.37372\ 10^{-11}\ s^{-1}$

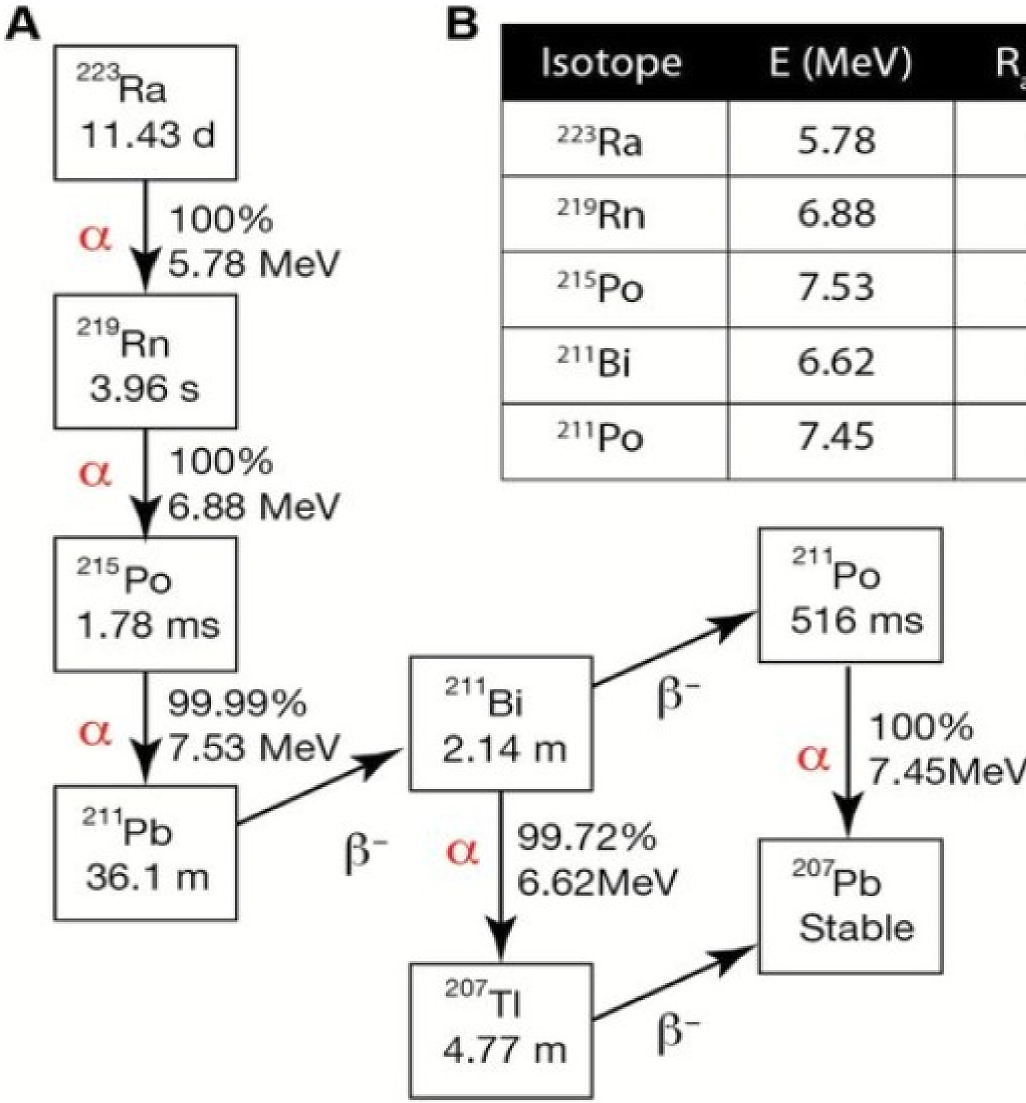
(b)

No of nuclei = $N_{Ra\text{-}226} = \frac{(1\ g)\ (6.02214\ 10^{23}\ \text{nuclie/mol})}{226.0254\ g/mol} = 2.66436\ 10^{21}$ nuclei

Initial activity = $A_0 = \lambda\ N_{Ra\text{-}226} = 3.66\ 10^{10}$ Bq (disintegrations per second)

(c)

$A(t) = A_0\ e^{-\lambda t} = 3.66\ 10^{10}\ e^{-(1.37\ 10^{-11})\,(100)\,(365)(24)(3600)} = 3.5\ 10^{10}$ Bq



B

Isotope	E (MeV)	R_{air} (cm)	R_{tissue} (μm)
^{223}Ra	5.78	4.5	45.85
^{219}Rn	6.88	5.9	59.54
^{215}Po	7.53	6.7	68.17
^{211}Bi	6.62	5.5	56.20
^{211}Po	7.45	6.6	67.09

Note that the activity of 1 g Ra-226 is very small compared to the total number of nuclei (10^{10} disintegrations/second vs 10^{21} nuclei). Therefore, the activity can be treated as constant. This is true for any isotope with a long half life.

Example 4

Determine the activity, in Bq, of the three uranium isotopes found in 100 kg of uranium nitrate (U_3N_4) when natural uranium is used.

The three naturally occurring uranium isotopes are:

$^{234}_{92}U$ 0.0055 %, 234.0409 amu (g/mol) , $t_{1/2} = 2.4\ 10^5$ years

$^{235}_{92}U$ 0.7200 %, 235.0439 amu (g/mol) , $t_{1/2} = 7.1\ 10^8$ years

$^{238}_{92}U$ 0.992745 %, 238.0508 amu (g/mol) , $t_{1/2} = 4.51\ 10^9$ years

$$\begin{Bmatrix}\text{Nuclear mass}\\ \text{of compound}\end{Bmatrix} \cong \left\{3\ \frac{\text{kg mol U}}{\text{kg mol } U_3N_4}\right\}\left\{238\ \frac{\text{kg U-238}}{\text{kg mol U-238}}\right\} + \left\{2\ \frac{\text{kg mol } N_2}{\text{kg mol } U_3N_4}\right\}\left\{28\ \frac{\text{kg } N_2}{\text{kg mol } N_2}\right\} = 770\ \frac{\text{kg } U_3N_4}{\text{kg mol } U_3N_4}$$

If the isotopic masses of U-238 and N-2 are used, then

$$\begin{Bmatrix}\text{Nuclear mass}\\ \text{of compound}\end{Bmatrix} = M_{U_3N_4} = 770.0867\ \frac{\text{kg } U_3N_4}{\text{kg mol } U_3N_4}$$

Number of radioactive nuclei in 100 kg U_3N_4:

$$N = f\left\{\frac{\left[\text{mass of } U_3N_4\right]\left[\dfrac{\text{nuclei}}{\text{kg mol}}\right]\left[\dfrac{\text{kg mol U}}{\text{kg mol } U_3N_4}\right]}{\begin{bmatrix}\text{molecular mass}\\ \text{of } U_3N_4\end{bmatrix}}\right\}$$

f is the fraction of each isotope

Number of radioactive nuclei in 100 kg U_3N_4:

$$N = f\left\{\frac{\left[\text{mass of } U_3N_4\right]\left[\frac{\text{nuclei}}{\text{kg mol}}\right]\left[\frac{\text{kg mol U}}{\text{kg mol } U_3N_4}\right]}{\left[\begin{array}{l}\text{molecular mass}\\ \text{of } U_3N_4\end{array}\right]}\right\}$$

f is the fraction of each isotope

f values are different for U-238, U-235 and U-234.

$$N = f\left\{\frac{\left[100 \text{ kg } U_3N_4\right]\left[6.0225\ 10^{26}\ \frac{\text{nuclei}}{\text{kg mol}}\right]\left[3\ \frac{\text{kg mol U}}{\text{kg mol } U_3N_4}\right]}{\left[770\ \frac{\text{kg } U_3N_4}{\text{kg mol } U_3N_4}\right]}\right\}$$

$$= f\left(2.336\ 10^{26}\ \text{nuclei U}\right) \text{ in 100 kg of } U_3N_4$$

Activity:

$$A = N\,\frac{\ln(2)}{t_{1/2}} = f\left(2.336\ 10^{26}\right)\frac{\ln(2)}{(t_{1/2}\ \text{in y})\ (8766\ \text{h/y})\ (3600\ \text{s/h})}$$

$$= (5.1529\ 10^{18})\left(\frac{f}{t_{1/2}\ \text{in y}}\right)$$

$$A_{U\text{-}234} = (5.1529\ 10^{18})\left(\frac{0.000055}{2.4\ 10_5}\right) = 1.2517\ 10^{9}\ \text{Bq}\ \ (\text{U-234 disintegrations per s})$$

$$A_{U\text{-}235} = (5.1529\ 10^{18})\left(\frac{0.0072}{7.1\ 10^{8}}\right) = 0.0523\ 10^{9}\ \text{Bq}\ \ (\text{U-235 disintegrations per s})$$

$$A_{U\text{-}238} = (5.1529\ 10^{18})\left(\frac{0.99274}{4.51\ 10^{9}}\right) = 1.1342\ 10^{9}\ \text{Bq}\ \ (\text{U-238 disintegrations per s})$$

Total activity is the sum: $A = 2.4389\ 10^{9}$ Bq = 65.92 mCi (milli Curies)

During radioactive decay, the sum of mass and energy must be conserved.

The energy distribution among the product particles can be determined using conservation of momentum.

$$E_{total} = 931.5 \frac{MeV}{amu} \left\{ \begin{bmatrix} \text{parent} \\ \text{nuclear} \\ \text{mass} \end{bmatrix} - \begin{bmatrix} \text{particle} \\ \text{mass} \end{bmatrix} - \begin{bmatrix} \text{daughter} \\ \text{nuclear} \\ \text{mass} \end{bmatrix} \right\}$$

$$E_{KE,total} = E_{total} - E_{\gamma_d} = \frac{1}{2} m v^2 + \frac{1}{2} M V^2$$

E_{γ_d}: Total emitted γ-ray energy
Experimentally determined
Negligible

$m v^2$: Light nucleus

$M V^2$: Heavy nucleus

- Neglecting E_γ
- Assume initial kinetic energy of parent nucleus to be zero
- Conservation of momentum requires m v = M V

$$E_{KE,total} = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{m^2 v^2}{M}\right) = \frac{1}{2} m v^2 \left(1 + \frac{m}{M}\right)$$

Similarly: $$E_{KE,total} = \frac{1}{2} M V^2 \left(1 + \frac{M}{m}\right)$$

$$E_{KE,heavy} = \frac{1}{2} M V^2 = \frac{E_{KE,\ total}}{\left(1 + \frac{M}{m}\right)} \qquad E_{KE,light} = \frac{1}{2} m v^2 = \frac{E_{KE,\ total}}{\left(1 + \frac{m}{M}\right)}$$

m << M, thus, the light nucleus carries most of the momentum.

Example 5

Uranium undergoes α decay with emission of a 0.17 MeV gamma ray. What is the kinetic energy of the product nucleus and the α particle?

$$^{235}_{92}U \; \text{-->} \; ^{231}_{90}Th + {}^{4}_{2}\alpha + \gamma_d \; (0.17 \text{ MeV})$$

$^{235}_{92}U$	235.0439 amu (g/mol)
$^{231}_{90}Th$	231.0347 amu (g/mol)
$^{4}_{2}He$	4.0026 amu (g/mol)

Total kinetic energy:

$$E_{KE,total} = (931.5 \text{ MeV/amu}) \underbrace{\left[(m_{U\text{-}235} - 92\, m_e) - (m_{Th\text{-}231} - 90\, m_e) - (m_{He\text{-}4} - 2\, m_e)\right]}_{\text{Mass change}} - E_{\gamma_d}$$

Note that the masses of the electrons (m_e) cancel out.

Substitute the given values:

$$E_{KE,total} = (931.5 \text{ MeV/amu})\,(6.6\ 10^{-3} \text{ amu}) - 0.17 \text{ MeV} = 5.9779 \text{ MeV}$$

This total kinetic energy is shared by the product nuclei, Th-231 nucleus and He-4 nucleus (α particle):

$$E_{KE,total} = E_{KE,Th\text{-}231} + E_{KE,\alpha} = \frac{1}{2}\, m_{Th\text{-}231}\, V^2_{Th-231} + \frac{1}{2}\, m_\alpha\, V^2_\alpha = 5.9779 \text{ MeV}$$

$$E_{KE,total} = E_{KE,Th\text{-}231} + E_{KE,\alpha} = \frac{1}{2} m_{Th\text{-}231} V_{Th-231}^2 + \frac{1}{2} m_\alpha V_\alpha^2 = 5.9779 \text{ MeV}$$

Conservation of momentum: $m_{Th\text{-}231} V_{Th-231} = m_\alpha V_\alpha$

Two equations, two unknowns:

$$E_{KE,Th\text{-}231} = 0.102 \text{ MeV}$$

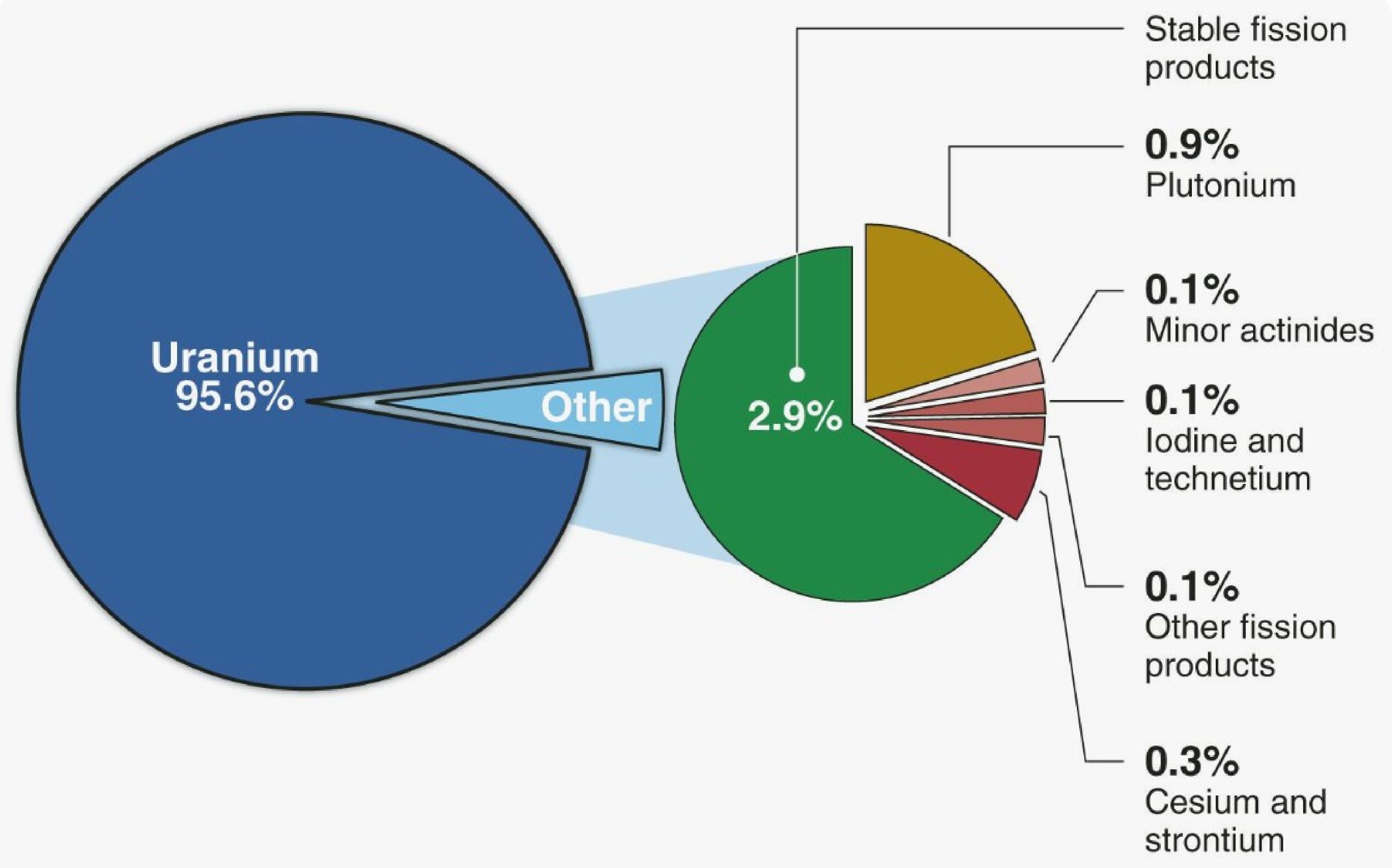
$$E_{KE,He\text{-}4} = 5.876 \text{ MeV}$$

There are two sources of radioisotopes:

- Naturally occurring, including that produced by cosmic radiation
- Manufactured in fission reactors and particle accelerators

Long-lived Isotopes		
Symbol	**Half life**	**Decay**
$^{87}_{37}Ra$	$5\ 10^{10}$ y	β^-
$^{40}_{19}K$	$1.28\ 10^{9}$ y	K-capture, β^-
$^{50}_{23}V$	$6.0\ 10^{15}$ y	K-capture
$^{96}_{40}Zr$	$3.6\ 10^{17}$ y	β^-
$^{115}_{49}In$	$6.0\ 10^{14}$ y	β^-
$^{138}_{57}La$	$1.1\ 10^{11}$ y	K-capture, β^-, β^+

Long-lived Isotopes		
Symbol	**Half life**	**Decay**
$^{142}_{58}Ce$	$5.0\ 10^{16}$ y	β^-
$^{148}_{62}Sn$	$8.0\ 10^{15}$ y	α
$^{209}_{83}Bi$	$2\ 10^{18}$ y	α
$^{232}_{90}Th$	$1.41\ 10^{10}$ y	α
$^{238}_{92}U$	$4.51\ 10^{9}$ y	α


Uranium
95.6%
Other
2.9%
Stable fission products
0.9%
Plutonium
0.1%
Minor actinides
0.1%
Iodine and technetium
0.1%
Other fission products
0.3%
Cesium and strontium

Most naturally occurring isotopes are produced by the decay of 3 parent isotopes

$$ {}^{232}_{90}\mathrm{Th} \rightarrow \dots \rightarrow {}^{208}_{82}\mathrm{Pb} + 6\,({}^{4}_{2}\alpha) + 4\,({}^{0}_{-1}\beta) $$

$$ {}^{235}_{92}\mathrm{U} \rightarrow \dots \rightarrow {}^{207}_{82}\mathrm{Pb} + 7\,({}^{4}_{2}\alpha) + 4\,({}^{0}_{-1}\beta) $$

$$ {}^{238}_{92}\mathrm{U} \rightarrow \dots \rightarrow {}^{206}_{82}\mathrm{Pb} + 8\,({}^{4}_{2}\alpha) + 6\,({}^{0}_{-1}\beta) $$

U-238 goes through 14 stages of decay to reach a stable nuclide.

Included in the decay chain are:

- Radium-266, $t_{1/2}$ = 1690 y, isolated by M. Curie in 1902
- Radon-222, $t_{1/2}$ = 3.82 d, noble gas

Cosmic radiation are high energy neutrons, protons, nuclei, … which produces isotopes from atoms in the atmosphere such as

- Carbon-14 (radiocarbon) and
- Hydrogen-3 (tritium)

Manufactured isotopes – from fission reactors and accelerators

$$ {}^{59}_{2}\mathrm{Co} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{60}_{27}\mathrm{Co}^{*} \rightarrow {}^{60}_{27}\mathrm{Co} + \gamma $$

Excited state

Capture γ

$$ {}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + {}^{0}_{-1}\beta + 2\,\gamma \qquad t_{1/2} = 5.27 \text{ years} $$

Use: Cancer radiotherapy; testing welds and castings, leveling devices, thickness gauges, etc

Radioisotope Thermal Generators (RTG)

A radioisotope thermoelectric generator (RTG, RITEG) is an electrical generator that uses an array of thermocouples to convert the heat released by the decay of a suitable radioactive material into electricity by the Seebeck effect.

RTGs have been used as power sources in satellites, space probes and such unmanned remote facilities as a series of lighthouses that the former Soviet Union erected inside the Arctic Circle. RTGs are usually the most desirable power source for robotic or unmaintained situations that need a few hundred watts (or less) of power for durations too long for fuel cells, batteries, or generators to provide economically and in places where solar cells are impractical. Safely using RTGs requires containing the radioisotopes long after the productive life of the unit.

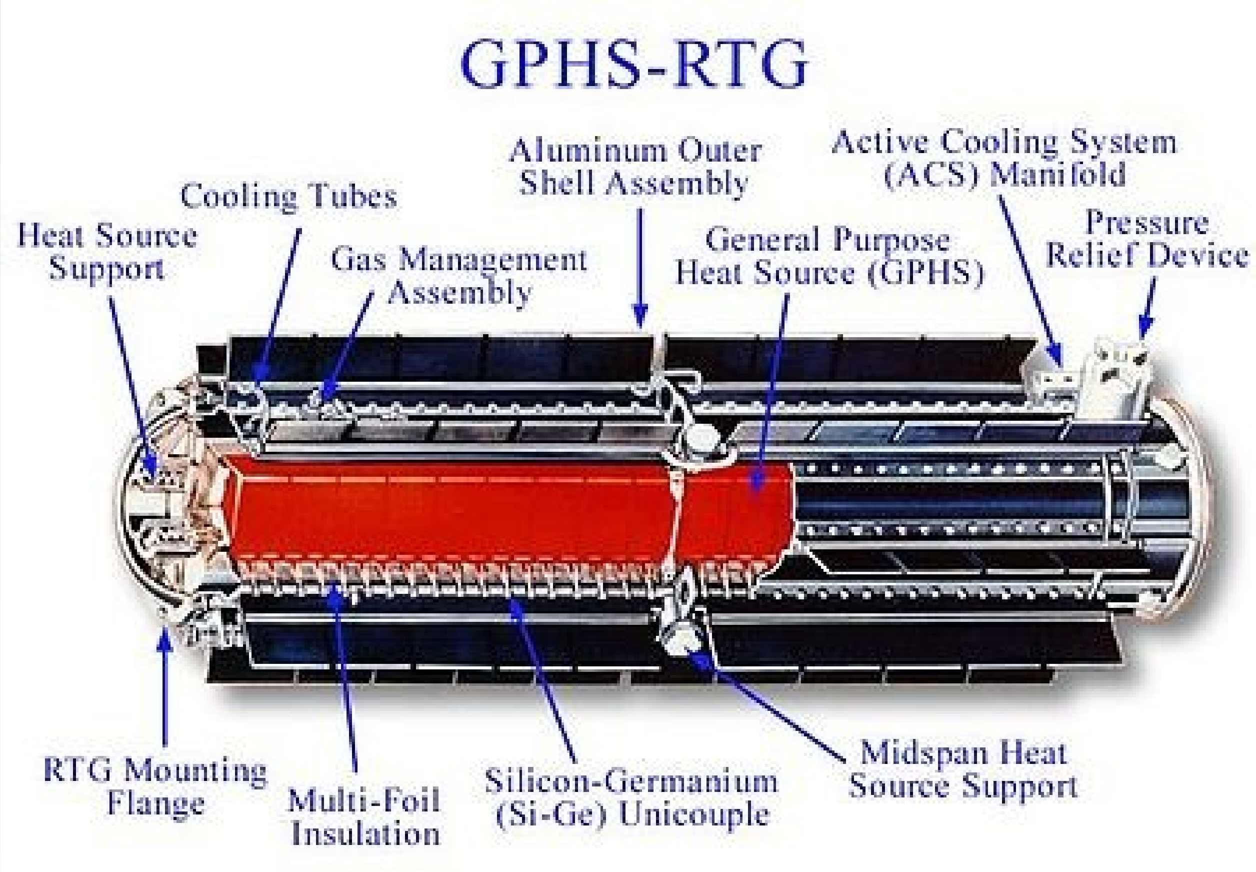


Diagram of an RTG used on the Cassini probe

See https://en.wikipedia.org/wiki/Radioisotope_thermoelectric_generator

See the book chapter on OdtuClass, «Radioisotope Power Production»

Cassini–Huygens is an unmanned spacecraft sent to the planet Saturn. It is a flagship-class NASA–ESA–ASI robotic spacecraft. *Cassini* is the fourth space probe to visit Saturn and the first to enter orbit, and its mission is ongoing as of 2016. It has studied the planet and its many natural satellites since arriving there in 2004.

Development started in the 1980s. Its design includes a Saturn orbiter and a lander for the moon Titan. The lander, called *Huygens*, landed on Titan in 2005. The two-part spacecraft is named after astronomers Giovanni Cassini and Christiaan Huygens.

The spacecraft launched on October 15, 1997 aboard a Titan IVB/Centaur and entered orbit around Saturn on July 1, 2004, after an interplanetary voyage that included flybys of Earth, Venus, and Jupiter. On December 25, 2004,*Huygens* separated from the orbiter and reached Saturn's moon Titan on January 14, 2005. It entered Titan's atmosphere and descended to the surface. It successfully returned data to Earth, using the orbiter as a relay. This was the first landing ever accomplished in the outer Solar System.

The fuel is Pu-238

$t_{1/2}$ = 87.7 years

SUMMARY

Isotopes of chemical elements - over 3000

Stable isotopes - proton/neutron balance in the nucleus

Unstable isotopes or radioisotopes:

- Naturally occurring or man-made
- Decay or disintegrate spontaneously (all by itself)
- With a half life (decay constant) – wide range
- To become stable, eventually
- With various decay modes: α, β, γ, others

As a result of decay – mass is converted to energy

first appears as kinetic energy of products

then, most of it is converted to thermal energy

Isotope	Z	N	half-life	DM	DE keV	Mode of formation
Tritium (^{3}H)	1	2	12.3 y	β^-	19	Cosmogenic
Beryllium-10	4	6	1,387,000 y	β^-	556	Cosmogenic
Carbon-14	6	8	5,700 y	β^-	156	Cosmogenic
Fluorine-18	9	9	110 min	β^+, EC	633/1655	Cosmogenic
Aluminium-26	13	13	717,000 y	β^+, EC	4004	Cosmogenic
Chlorine-36	17	19	301,000 y	β^-, EC	709	Cosmogenic
Potassium-40	19	21	1.24×10^9 y	β^-, EC	1330 /1505	Primordial
Calcium-41	20	21	99,400 y	EC		Cosmogenic
Cobalt-60	27	33	5.3 y	β^-	2824	Synthetic
Krypton-81	36	45	229,000 y	β^+		Cosmogenic
Strontium-90	38	52	28.8 y	β^-	546	Fission product
Technetium-99	43	56	210,000 y	β^-	294	Fission product
Technetium-99m	43	56	6 hr	γ,IC	141	Synthetic
Iodine-129	53	76	15,700,000 y	β^-	194	Cosmogenic

Iodine-131	53	78	8 d	β^-	971	Fission product
Xenon-135	54	81	9.1 h	β^-	1160	Fission product
Caesium-137	55	82	30.2 y	β^-	1176	Fission product
Gadolinium-153	64	89	240 d	EC		Synthetic
Bismuth-209	83	126	2.01×10^{19}y	α	3137	Primordial
Polonium-210	84	126	138 d	α	5307	Decay product
Radon-222	86	136	3.8 d	α	5590	Decay product
Thorium-232	90	142	1.4×10^{10} y	α	4083	Primordial
Uranium-235	92	143	7×10^{8}y	α	4679	Primordial
Uranium-238	92	146	4.5×10^{9} y	α	4267	Primordial
Plutonium-238	94	144	87.7 y	α	5593	Synthetic
Plutonium-239	94	145	24,110 y	α	5245	Synthetic
Americium-241	95	146	432 y	α	5486	Synthetic
Californium-252	98	154	2.64 y	α/SF	6217	Synthetic

Use of Radioisotopes

- In medicine – diagnosis, imaging, therapy
- In industry – irradiation treatment of materials
- In archeology – dating of artifacts and fossils
- In geology – tracing, well logging, mineralogy
- In engineering – measuring and testing, radiography, electricity generation
- In agriculture – animal husbandry, nutrition studies, food irradiation, diet additives, milk production, plant physiology, soil fertility, uptake of fertilizers
- In astronomy - solar energy from nuclear processes, atomic clocks
- In botany - transport of fluids, photosynthesis research
- In zoology – mutations, destruction of life by radiation
- Others