Radioactivity

$^{235}_{92}U$ --> $^{231}_{90}Th + ^{4}_{2}\alpha + \gamma_d$ (0.17 MeV)

mass-energy conversion

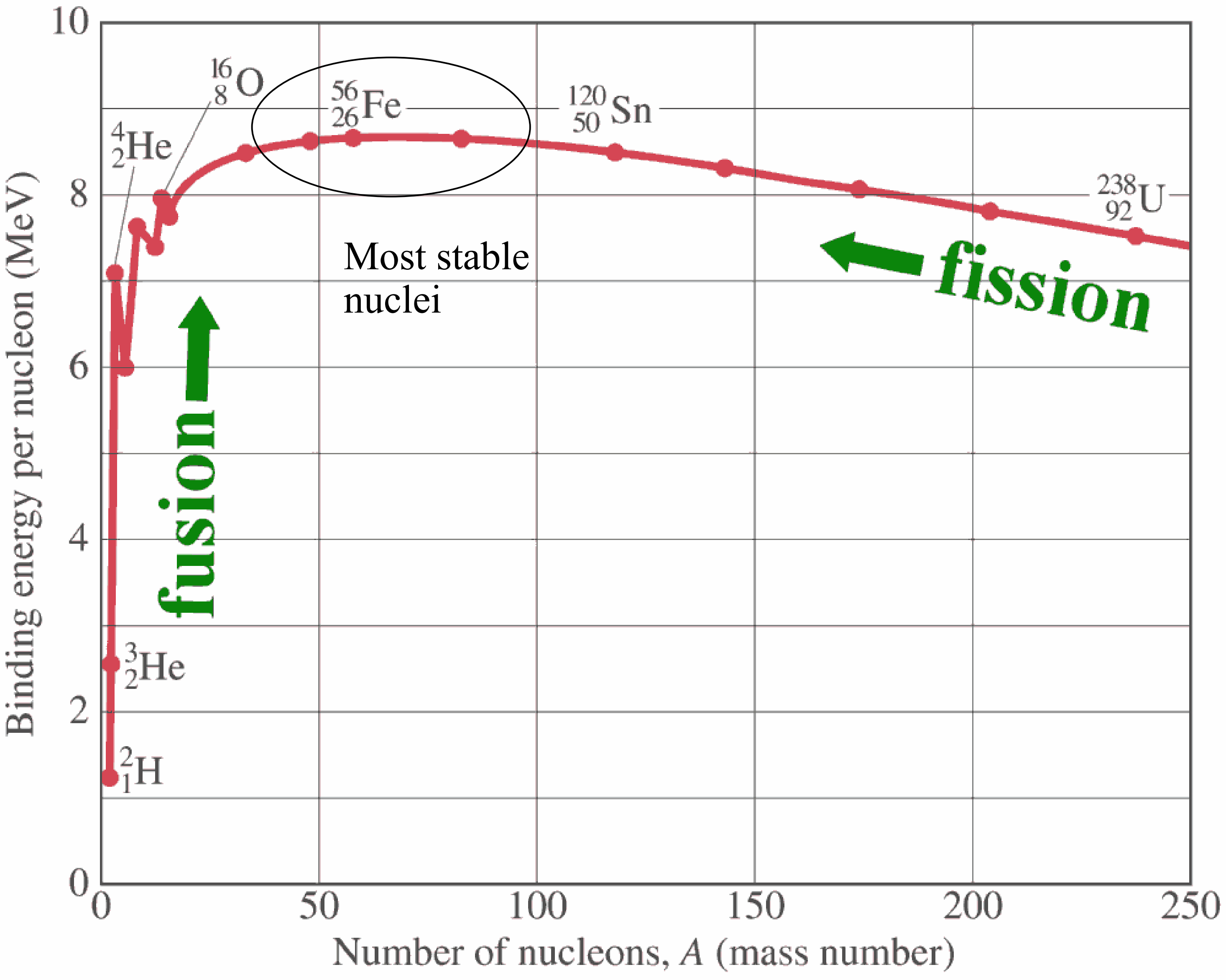
Man-made nuclear reaction

$^{27}_{13}Al + ^{4}_{2}\alpha = ^{30}_{14}Si + ^{1}_{1}H$

energy-mass or mass-energy conversion

In nuclear reactions:

- Sum of mass numbers is conserved
- Sum of atomic numbers is conserved
- Sum of mass and energy (not the mass alone) is conserved
  (931.5 Mev / amu)
- Momentum is conserved


10
8
6
4
2
0
Binding energy per nucleon (MeV)
0
50
100
150
200
250
Number of nucleons, A (mass number)
$^{2}_{1}H$
$^{3}_{2}He$
$^{4}_{2}He$
$^{16}_{8}O$
$^{56}_{26}Fe$
$^{120}_{50}Sn$
$^{238}_{92}U$
Most stable nuclei
fusion
fission

Fusion: 2 or more light nuclei fuse to form a heavier nucleus

Solar: $4\ {}^{1}_{1}\mathrm{H} \rightarrow 2\ {}^{4}_{2}\mathrm{He} + 2\ \underset{\nearrow \atop \text{positrons}}{{}^{0}_{1}\mathrm{e}}$ Mass decrease of 0.0265 amu corresponding to 24.7 MeV

Fusion reactions are called «thermonuclear» because of the very high temperature required to trigger and sustain the reactions.

$$\left.\begin{array}{l} {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \quad (3.27\ \mathrm{MeV}) \\ {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{H} + {}^{2}_{1}\mathrm{H} \quad (4.03\ \mathrm{MeV}) \end{array}\right\} \text{Deuterium – Deuterium Fusion}$$

$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \quad (17.59\ \mathrm{MeV}) \quad \text{Deuterium – Tritium Fusion}$$

See the articles on “OdtuClass”.

Fission: A heavy nucleus is split into 2 or more lighter nuclei

Fission can be triggered by a neutron which does not experience a repulsive force as it approaches the nucleus.

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{236}_{92}\mathrm{U}^{*} \rightarrow {}^{140}_{55}\mathrm{Cs} + {}^{93}_{37}\mathrm{Rb} + 3\,{}^{1}_{0}\mathrm{n} + \lambda_{d}$$

Slow Thermal

Excited

Fission Products

Fast Prompt

- Prompt energy - released at fission event
- Delayed energy - released during
  1. Radioactive decay of fission fragments (such as Cs-140 and Rb-93)
  2. Non-fission capture of excess neutrons

Delayed energy is a major concern in reactor control.

Prompt energy

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^{*} \rightarrow ^{140}_{55}Cs + ^{93}_{37}Rb + 3\,^{1}_{0}n + \lambda_d$$

Mass of reactants: 235.043924 amu + 1.008665 amu = 236.052589 amu

Mass of products: 92.91699 amu + 139.90910 amu + 3 (1.008665) amu

= 235.852085 amu

Δm = 0.200509 amu

ΔE = (931.5 MeV/amu) (0.200509 amu) = 186.8 MeV

There is also roughly 10 MeV of γ_d released

Delayed energy

$$^{93}_{37}\text{Rb} \overset{\beta^-}{\rightarrow} {}^{93}_{38}\text{Sr} \overset{\beta^-}{\rightarrow} {}^{93}_{39}\text{Y} \overset{\beta^-}{\text{-->}} {}^{93}_{40}\text{Zr} \overset{\beta^-}{\rightarrow} {}^{93}_{41}\text{Nb}$$

$$^{140}_{55}\text{Cs} \overset{\beta^-}{\rightarrow} {}^{140}_{56}\text{Ba} \overset{\beta^-}{\rightarrow} {}^{140}_{57}\text{La} \overset{\beta^-}{\rightarrow} {}^{140}_{58}\text{Ce}$$

$$\Delta E_{Rb\ \text{decay}} = \left(931.5\ \frac{\text{MeV}}{\text{amu}}\right)\left[\left(m_{Nb-93} + 4\ m_{\beta^-}\right) - \left(m_{Rb-93}\right)\right] = 7.84\ \text{MeV}$$

$$\Delta E_{Cs\ \text{decay}} = \left(931.5\ \frac{\text{MeV}}{\text{amu}}\right)\left[\left(m_{Cs-140} + 3\ m_{\beta^-}\right) - \left(m_{Ce-140}\right)\right] = 1.89\ \text{MeV}$$

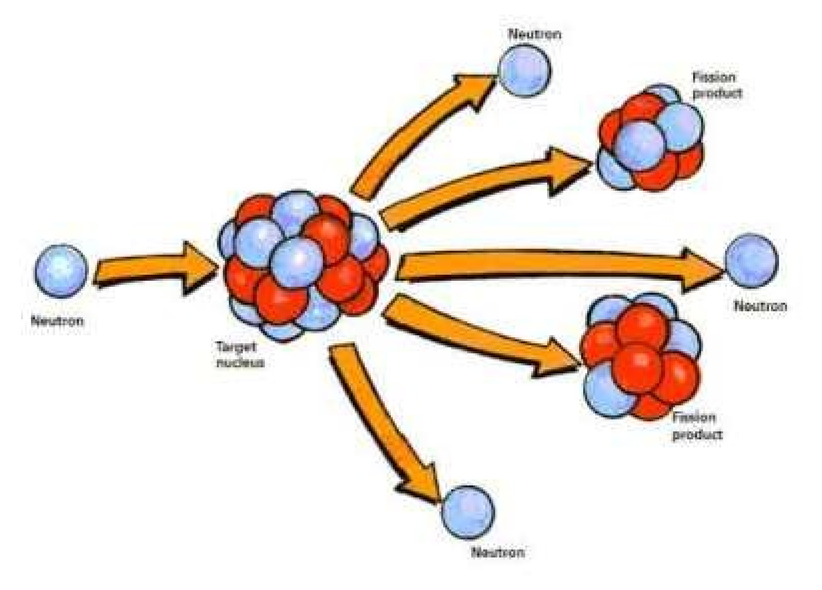
$$\Delta E_{Total\ \text{decay}} = 9.73\ \text{MeV}$$

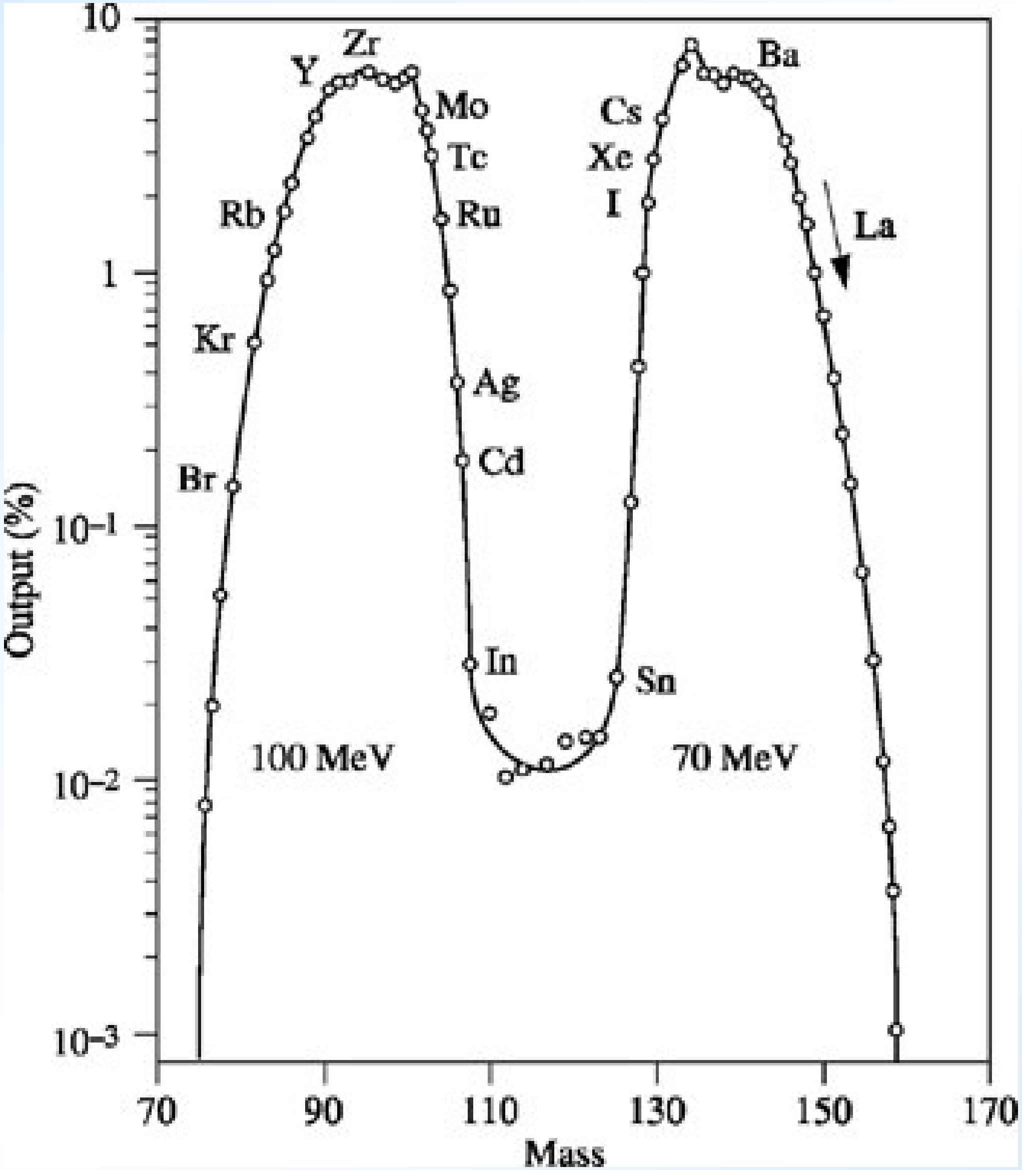
Other U-235 fission reactions:

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{140}_{54}Xe + {}^{91}_{38}Sr + 2\,{}^{1}_{0}n \qquad \Delta E = 196\ MeV $$

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{137}_{56}Ba + {}^{97}_{36}Kr + 2\,{}^{1}_{0}n \qquad \Delta E = 193.6\ MeV $$

There are numerous fission reactions releasing a distribution of energies.




Output (%)
10
1
10−1
10−2
10−3
Mass
70
90
110
130
150
170
Zr
Y
Mo
Tc
Ru
Rb
Kr
Ag
Cd
Br
In
Sn
Cs
Xe
I
Ba
La
100 MeV
70 MeV

Average energy distribution for energy released per fission via a **thermal** neutron absorbed by U-235:

Prompt energy:	167 MeV	kinetic energy of fission fragments
	5 MeV	kinetic energy of fission netrons (**fast** neutrons)
	5 MeV	γ_{fission} energy
	10 Mev	γ_{capture} energy
Total	187 MeV	

Delayed energy:	7 MeV	β- energy from fission fragment decays
	6 MeV	γ_{decay} energy
	10 Mev	kinetic energy of neutrinos
Total	23 MeV	

On the average, 200 MeV released/usable from fission of U-235

10 MeV of neutrinos not counted

$$\frac{\left(200\ \frac{\text{MeV}}{\text{nuclei U-235}}\right)\left(6.022\ 10^{23}\ \frac{\text{nuclei}}{\text{gmol U-235}}\right)}{\left(235.0439\ \frac{\text{g}}{\text{gmol U-235}}\right)} = 5.13\ 10^{23}\ \frac{\text{MeV}}{\text{g U-235}}$$

$$= 2.276\ 10^{4}\ \frac{\text{kWh}}{\text{g U-235}}$$

$$= 8.190\ 10^{10}\ \frac{\text{J}}{\text{g U-235}}$$

$$= 0.948\ \frac{\text{MW-day}}{\text{g U-235}}$$

Fuel: all uranium, plutonium, thorium isotopes, some are most fissionable

Fuel material: fuel + alloying and chemical compounds and mixtures

Fissile materials: U-233, U-235, Pu-239

Fission is possible if energy of an absorbed neutron is sufficiently large.

All fissionable isotopes cannot be fissioned because of accumulation of fission products, such as Xe and Kr (poisons), that absorbs neutrons and eventually drop neutron flux below critical level.

Fuel burn-up rates vary from 1000 to 100 000 MW-day/ton depending on the fuel.

Fission Timeline

1938 : Fission discovered in Germany by Otto Hahn & Fritz Strassmann

Jun. 16, 1939 : Lise Meitener & Otto Robert Frisch published a theoretical interpretation of Hahn & Strassmann experiments in Nature (10 days after Hahn & Strassmann publication)

Apr. 17, 1939 : Frederich Joliet, Hans von Halban & Lew Kowarski publish paper dealing with possibility of nuclear chain reaction

Aug. 2, 1939 : Albert Einstein wrote a letter to President F.D. Roosevelt drawing attention to the possibility of an atomic bomb

1940 : Edwin Mc Millan & Glenn Seaborg discover Plutonium

Dec 1942 : First nuclear reactor went critical (self sustaining chain)

beneath the stands of Univ. of Chicago stadium

Fermi chain reaction

Core was 9 m wide, 9.5 m long, 6 m high

52 tons of natural uranium & 1350 tons of graphite

Cadmium rods are used for control

0.5 W power for few minutes

1943 : Town of Los Alamos constructed for atomic research & weapons construction

Jul. 16, 1945 : 1st nuclear explosion, plutonium bomb, Trinity site in New Mexico

1952 : Fusion weapon, Hydrogen bomb (much less radioactive debris)

Jun. 1, 1954 : 1st nuclear power plant

Obninsk Nuclear Power station, near Moscow

Rated power 5 MW, 30 MW_{th}

Graphite moderated, water cooled

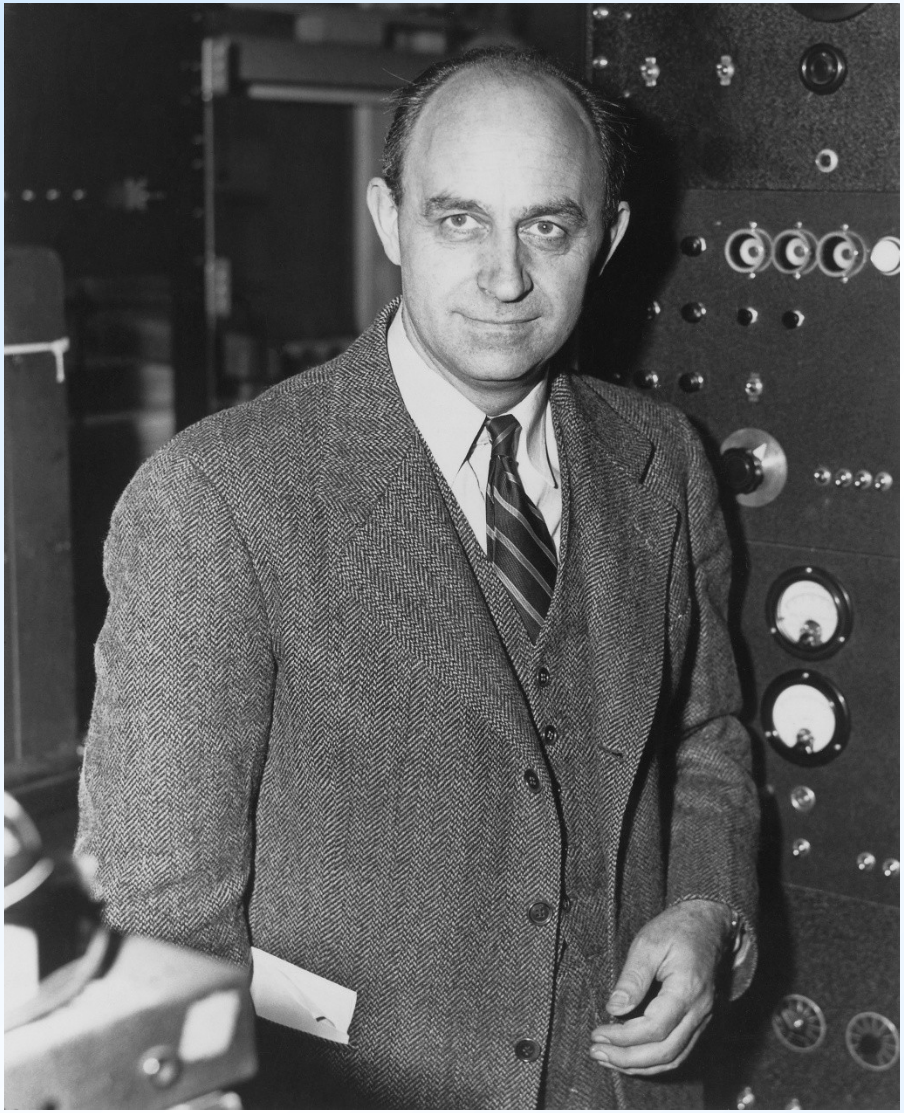
Shut down April 29, 2002

Jun. 21, 1954 : USS Nautilus launched

Pressurized water reactor (Westinghouse)

10 MW_m

Decommissioned March 3, 1980

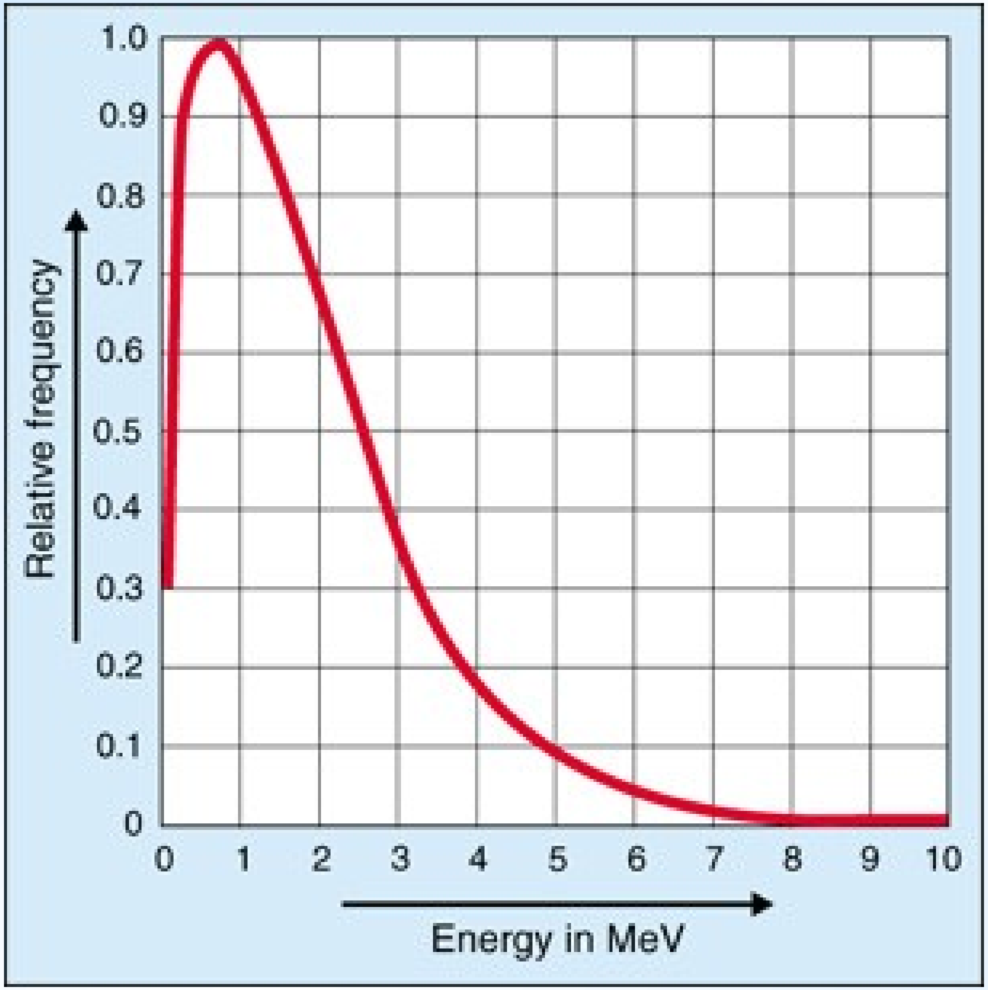


Enrico Fermi

Italian physicist

1901 - 1954

Neutron Energies



$$E_n = \frac{1}{2} m_n V_n^2$$

$m_n = 1.00866$ amu

$0.075 \leq E_n \leq 17$ MeV

When travelling through matter, neutrons collide with nuclei and are decelerated, mainly by light nuclei - scattering

Fast neutrons > 100 keV

Slow neutrons < 1 eV

Neutrons

Fast neutrons > 100 keV

Slow (thermal) neutrons < 1 eV

Prompt neutrons: Newly released fission netrons; within 10^{-14} seconds; fast; carry about 2 % of fission energy.

Delayed neutrons: released during radioactive decay of fission fragments;

0.645 % of fission neutrons for U-235; less for Pu-239 and U-233

Thermal neutrons: Fission neutrons scattered (slowed) my materials in reactor core

Lowest energy a neutron can reach is that which puts it in thermal equilibrium with the surrounding envireonment

A material called **moderator** is put in the core to slow down the fission neutrons.

Neutrons become thermalized in the moderator.

A moderator has small nuclei with high neuron scattering & low neutron absorption probability (cross section). H-1, H-2 in water (D_2O – heavy water) , graphite, Be, BeO

$$E_n = \frac{1}{2} m_n V_n^2 = k\,T \quad , \quad k = \frac{R}{N_A} = \frac{\text{Universal gas constant}}{\text{Avagadro's number}}$$

$$k = 1.3805\ 10^{-23}\ \text{J/K} = 8.617\ 10^{-11}\ \text{MeV/K}$$

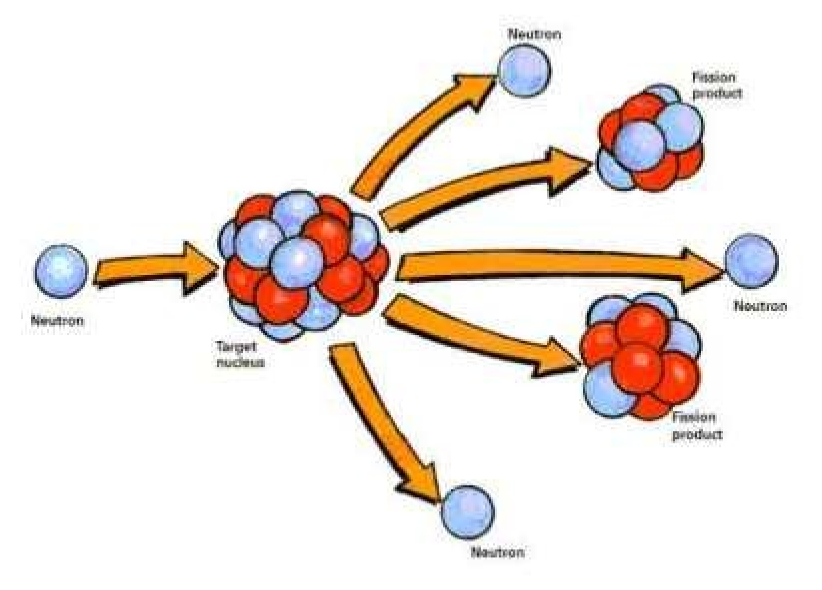
Atomic density: $$n = \frac{\text{particles}}{\text{cm}^2} = \frac{\rho\, N_A}{M} = \frac{\left(\frac{\text{g}}{\text{cm}^3}\right)\left(\frac{\text{particles}}{\text{gmol}}\right)}{\left(\frac{\text{g}}{\text{gmol}}\right)}$$

Other U-235 fission reactions:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^{*} \rightarrow ^{140}_{54}Xe + ^{91}_{38}Sr + 2\ ^{1}_{0}n \qquad \Delta E = 196\ MeV$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^{*} \rightarrow ^{137}_{56}Ba + ^{97}_{36}Kr + 2\ ^{1}_{0}n \qquad \Delta E = 193.6\ MeV$$

There are numerous fission reactions releasing a distribution of energies.



Nuclear Cross Section and Nuclear Reaction Probability:

Reaction probability is dependent on type of reaction, the nucleus and nucleus energy, and interacting particle (neutron) energy.

Probability of a reaction (fission, absorption, scattering, etc) is expressed in terms of the microscopic cross section, σ. Unit is barn.

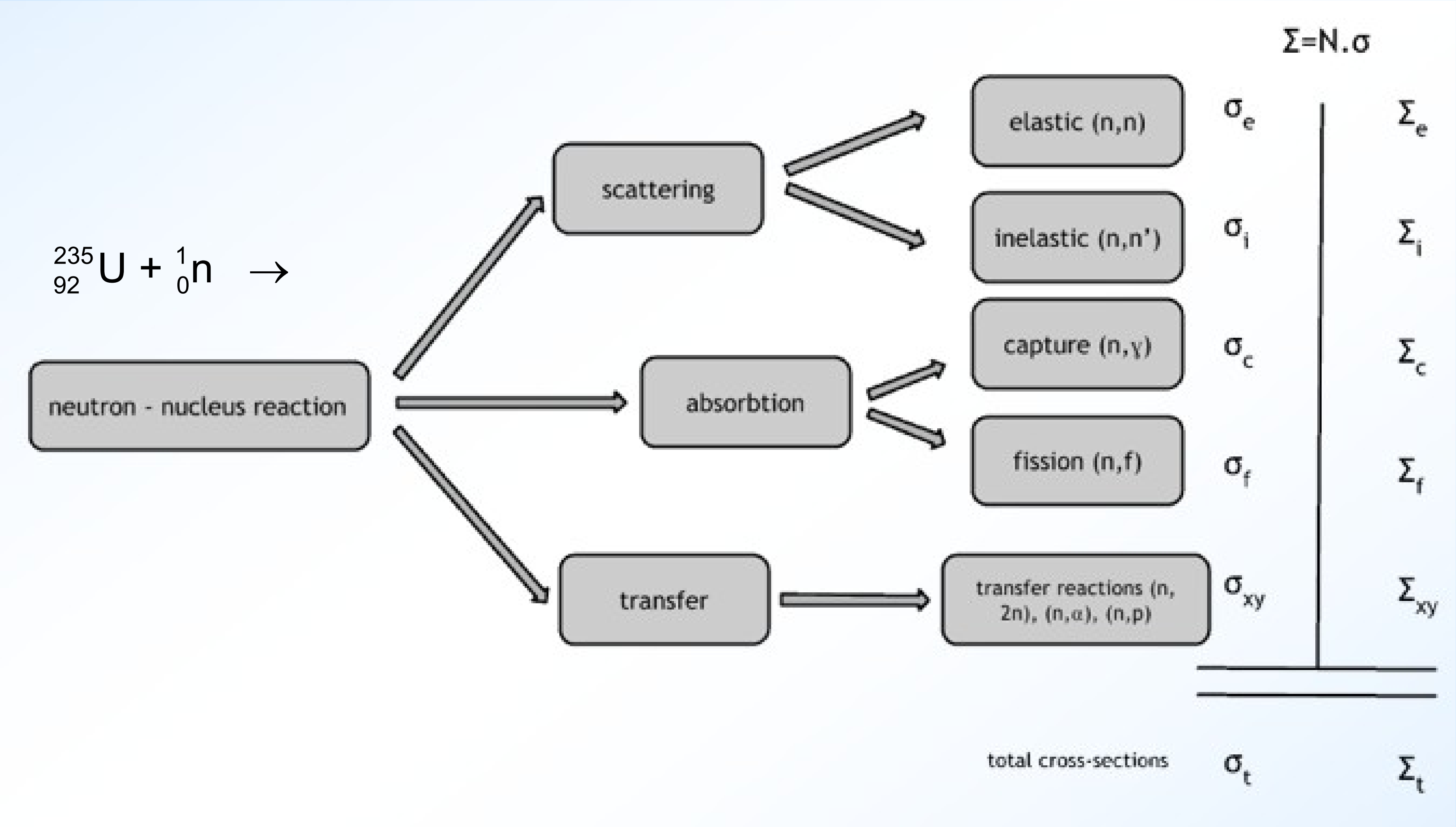
Probability is essentially an effective area of the nucleus.

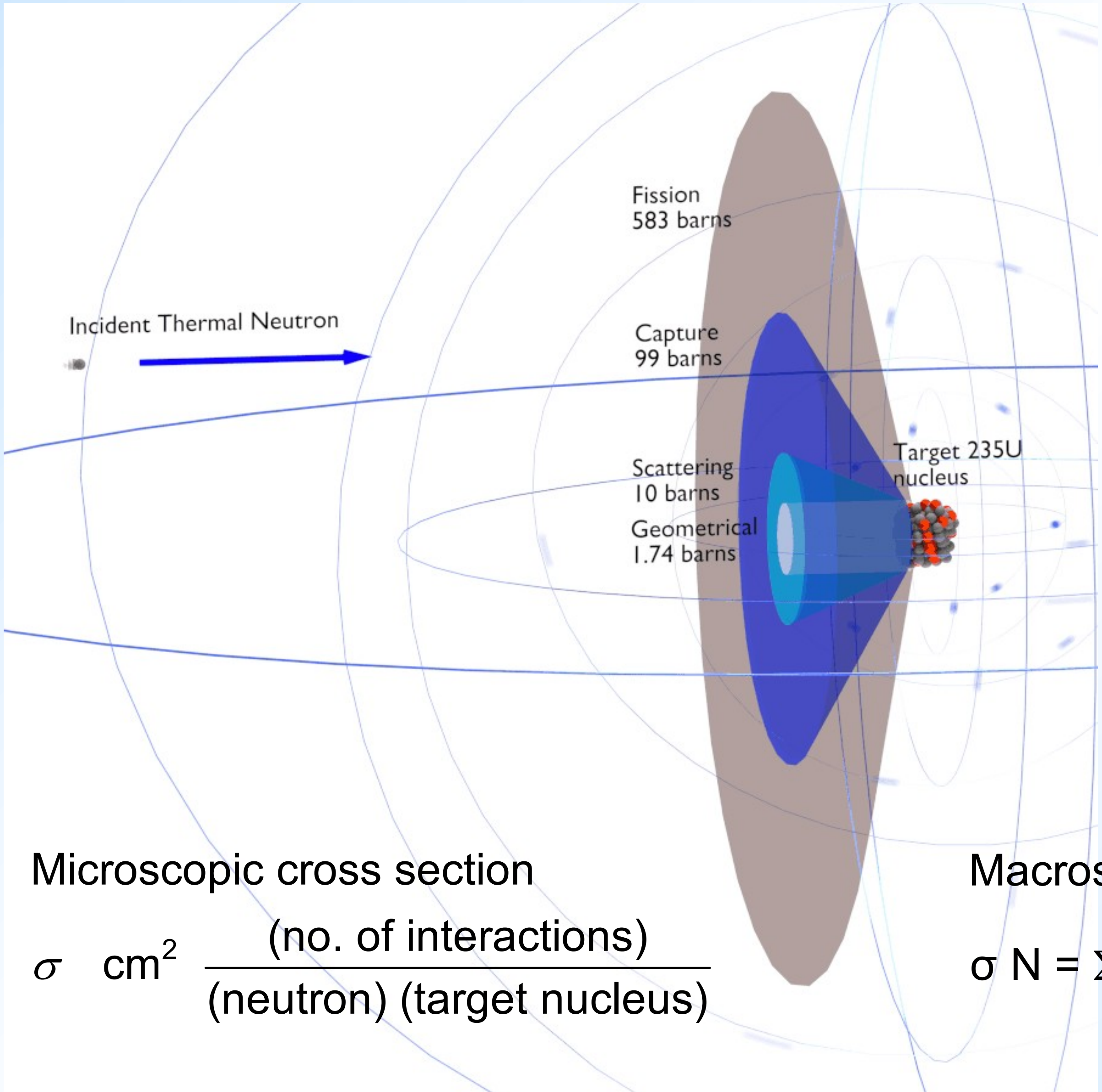
1 barn = 10^{-24} cm^2, which is average cross sectional area of nuclei.

Nuclei have radii approximately 1/10000 of atoms radii. So. neutrons have small targets.

Macroscopic cross section = $\Sigma = N\sigma$ where N is the number of nuclei.

$^{235}_{92}U + ^{1}_{0}n \rightarrow$





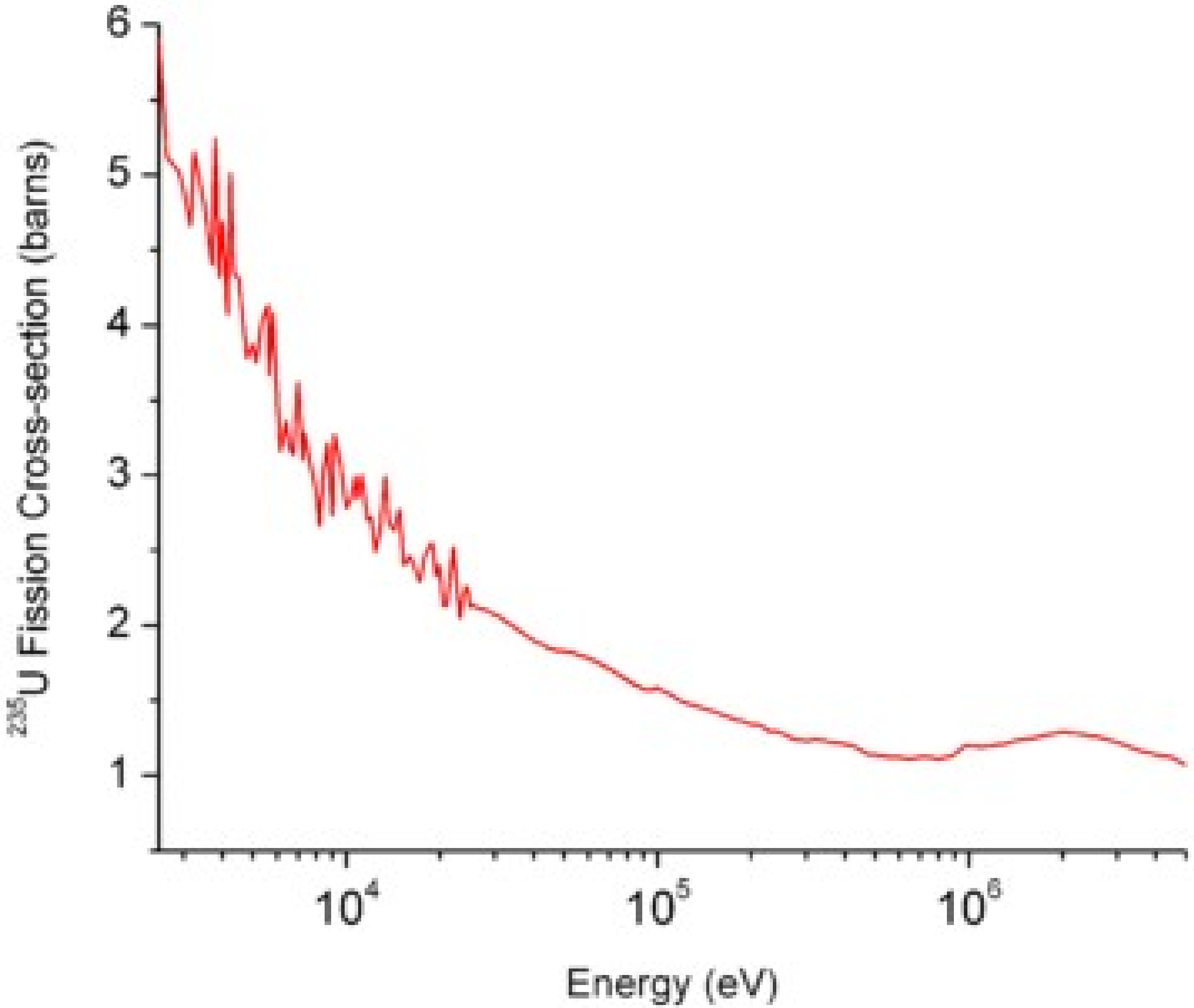
In general, **the cross-section** is an effective area that quantifies **the likelihood** of certain interaction between an incident object and a target object.

Microscopic cross section

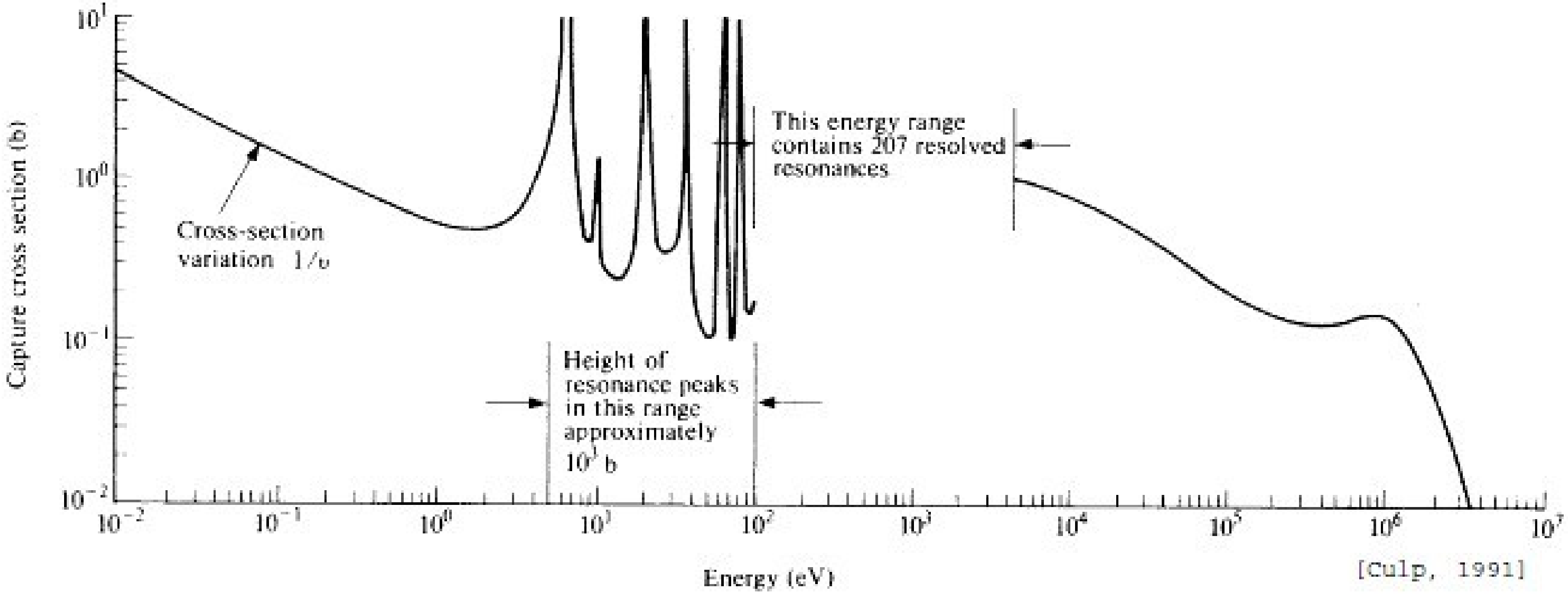
$$\sigma \quad \text{cm}^2 \quad \frac{\text{(no. of interactions)}}{\text{(neutron) (target nucleus)}}$$

Macroscopic cross section

$$\sigma\, N = \Sigma \quad \text{cm}^2 \quad \frac{\text{(no. of interactions)}}{\text{(neutron)}}$$


6
5
4
3
2
1
^{235}U Fission Cross-section (barns)
10^4
10^5
10^6
Energy (eV)

Neutron Cross Section



[Culp, 1991]

FIGURE 3.2
The microscopic neutron-absorption cross section for U-238. (*From Steam: Its Generation and Use, 1972.*)

Neutron Flux, Φ, in neutrons/m^2.s

$\Phi = n\,V =$ (neutron velocity, m/s) (neutron density, neutrons/cm^3)

It is a function of spatial location in the core and kinetic energy of neutrons.

Neutron reaction rate = $\bar{\phi}\ N\ \bar{\sigma} = \bar{\phi}\ \bar{\Sigma}$

$\bar{\phi}$ = Average neutron flux, neutrons/cm^2.s

N = Number of nuclei

$\bar{\sigma}$ = Average neutron cross section for the given reaction

$$cm^2 \quad \frac{\text{no of reactions}}{\text{(neutron) (nucleus)}}$$

Example 6

Determine the reactor fuel loading in kg of U-235, in a 1200 MWe power reactor (like PWR at Akkuyu) operating at a thermal efficiency of 33 %, and average neutron flux of 6 10^{17} neutrons/m^2.s in the core, and an average fission cross section of 365 barns.

If the reactor is fuelled with 2.3 % enriched UO_2, determine the fuel loading.

The **mole** is the unit of measurement in the International System of Units (SI) for amount of a substance.

It is defined as the amount of a chemical substance that contains as many elementary entities, e.g., atoms, molecules, ions, electrons, or photons, as there are atoms in 12 grams of carbon-12 (^{12}C), the isotope of carbon with relative atomic mass 12 by definition.

This number is expressed by the Avogadro constant, which has a value of $6.022140857(74)\times10^{23}$ mol^{-1}. The mole is one of the base units of the SI, and has the unit symbol **mol**.

$$\text{Reactor Power} = \dot{W}_{th} = \frac{\dot{W}_e}{\eta} = \frac{1200\ \text{MW}_e}{0.33\ \text{MW}_e/\text{MW}_{th}} = 3636.36\ \text{MW}_{th}$$

$$\text{Fission Rate} = \left(\frac{3636.36\ \text{MW}_{th}}{200\ \text{MeV}/\text{fission}}\right)\left(6.242\ 10^{15}\ \frac{\text{MeV}/\text{s}}{\text{kW}_{th}}\right) = 1.135\ 10^{20}\ \text{Fissions}/\text{s}$$

$$\text{Fission Rate} = (\text{Neutron Flux})\ (\text{Atom density})\ (\text{Fission cross section}) = \phi\ N_{U\text{-}235}\ \sigma_f$$

$$1.135\ 10^{20} = \left(6\ 10^{17}\ \text{neutrons}/\text{m}^2.\text{s}\right)\ (365\ \text{barns})\ \left(10^{-28}\ \text{m}^2/\text{barn}\right)\ N_{U\text{-}235}$$

Solve for $N_{U\text{-}235}$ $\quad N_{U\text{-}235} = 5.1826\ 10^{27}$ atoms of U-235

$$m_{U\text{-}235} = \frac{\left(N_{U\text{-}235}\right)\ \left(M_{U\text{-}235}\right)}{N_A} = \frac{\left(5.1826\ 10^{27}\ \text{U-235 atoms}\right)\ \left(235.0439\ {}^{\text{kg}}\!\big/\!{}_{\text{kg mol U-235}}\right)}{6.0225\ 10^{26}\ {}^{\text{atoms}}\!\big/\!{}_{\text{kg mol}}}$$

$$= 2022.7 \text{ kg U-235}$$

Fuel loading: 99 766 kg of 2.3 % enriched Uranium