Electromagnetic Energy

Speed of light $c = \lambda \nu = \dfrac{c_0}{n}$

λ: wavelength (m, μm)

ν: frequency (1/s) = c / λ

c: speed of light in a medium, m/s

c_0: speed of light in vacuum, m/s

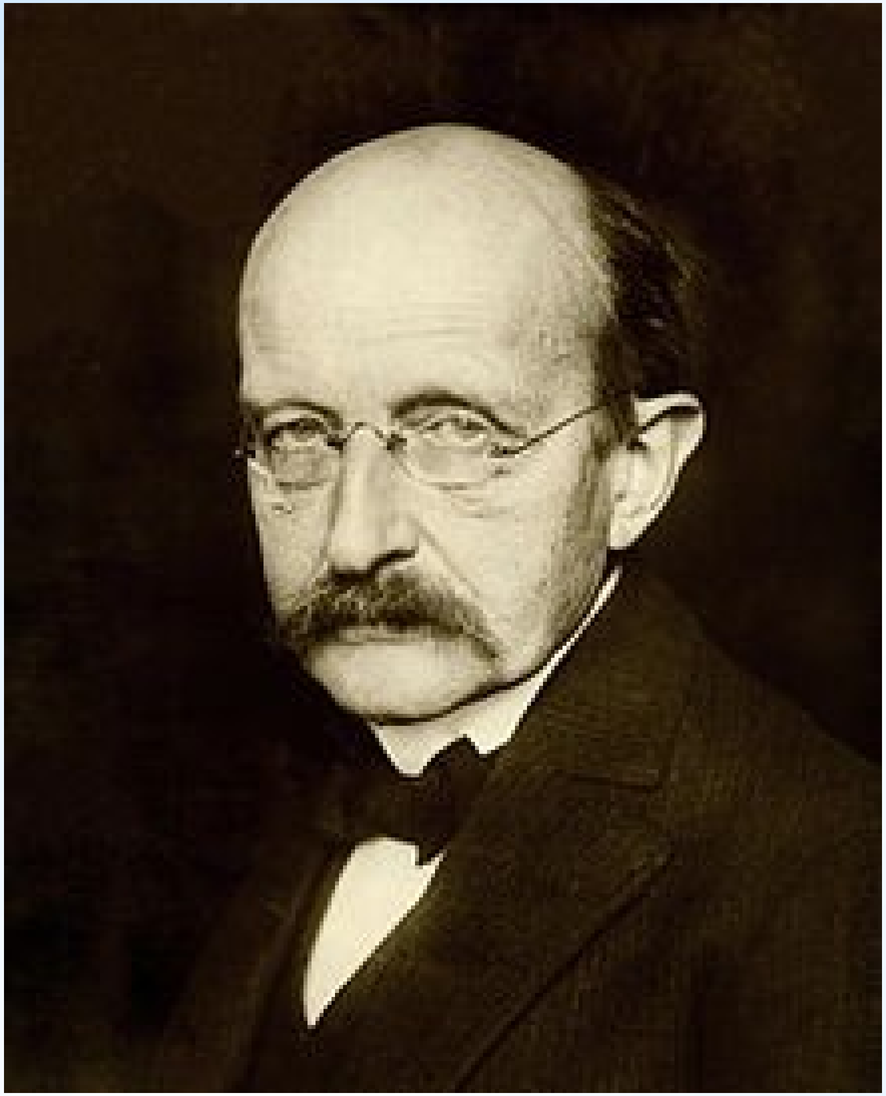
n: refractive index of the medium

Energy of a photon $E_p = h \nu = \dfrac{h c}{\lambda}$ h: Plank's constant = $6.626 \; 10^{-34}$ J.s

Thermal radiation: 0.1 to 100 μm , Solar radiation: 0.1 to 3 μm

Note: Read "Cosmic Numbers – The Numbers That Define Our Universe"

by James D. Stein, Basic Books, 2011

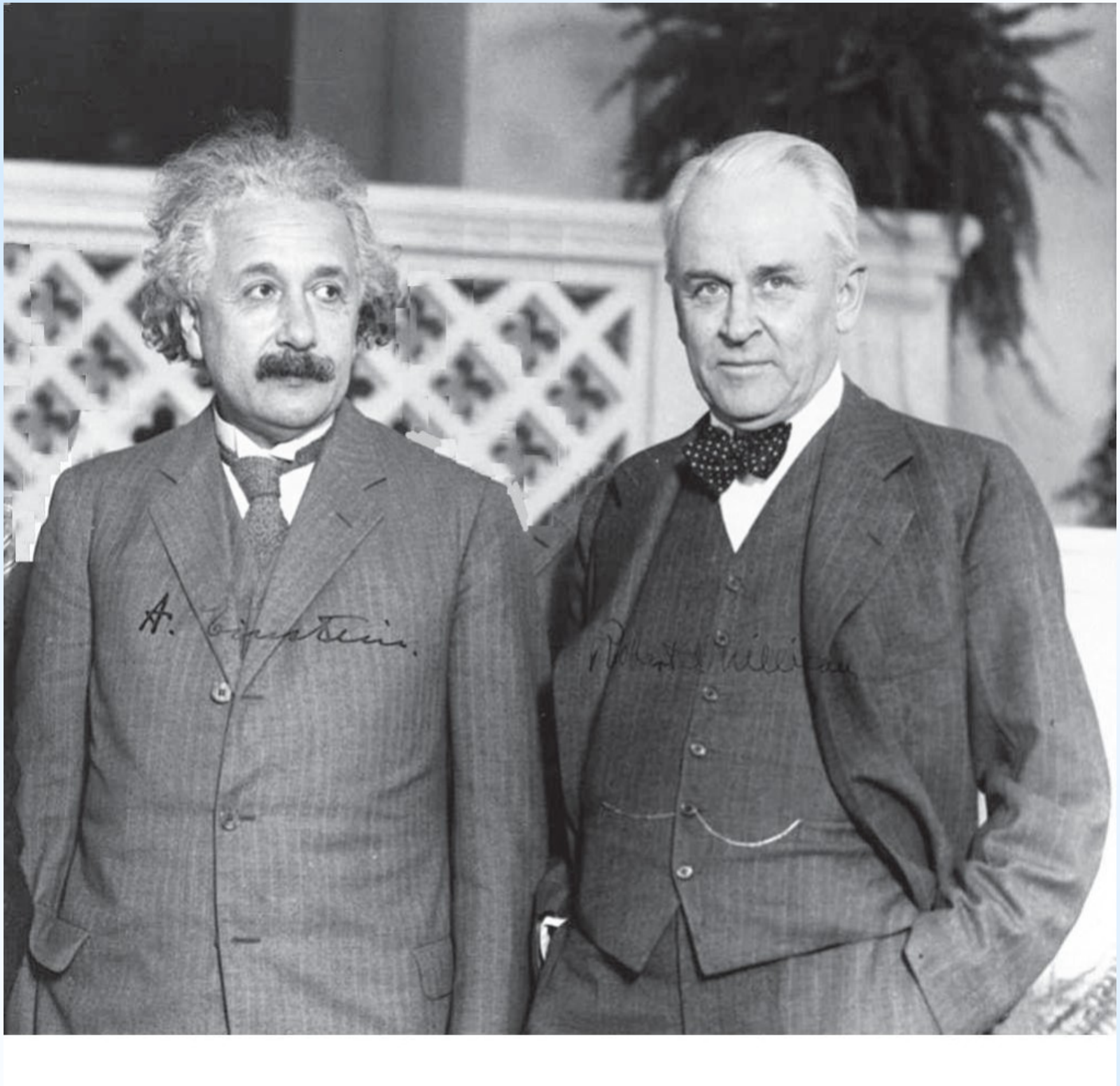


Max Karl Ernst Ludwig Planck

German Physicist

1858 - 1947

In 1905, while employed as a patent examiner at the Swiss Patent Office, Albert Einstein wrote five papers, published in Annalen der Physik, that initiated the twentieth century revolution in science. For general public, Einstein is mostly known for his theory of relativity. Therefore, when the Swedish Academy announced in 1922 that Einstein had won the Nobel Prize “for services to theoretical physics and especially for the discovery of the law of the photoelectric effect,” referring to his paper «On a Heuristic Viewpoint Concerning the Production and Transformation of Light», the public was surprised. In hindsight, the Nobel Committee was correct: His paper on photoelectric effect is considered the boldest, the most revolutionary, and the most original. Although its predictions were fully verified by experiments, for many years, several prominent physicists did not accept Einstein’s concept of photons.



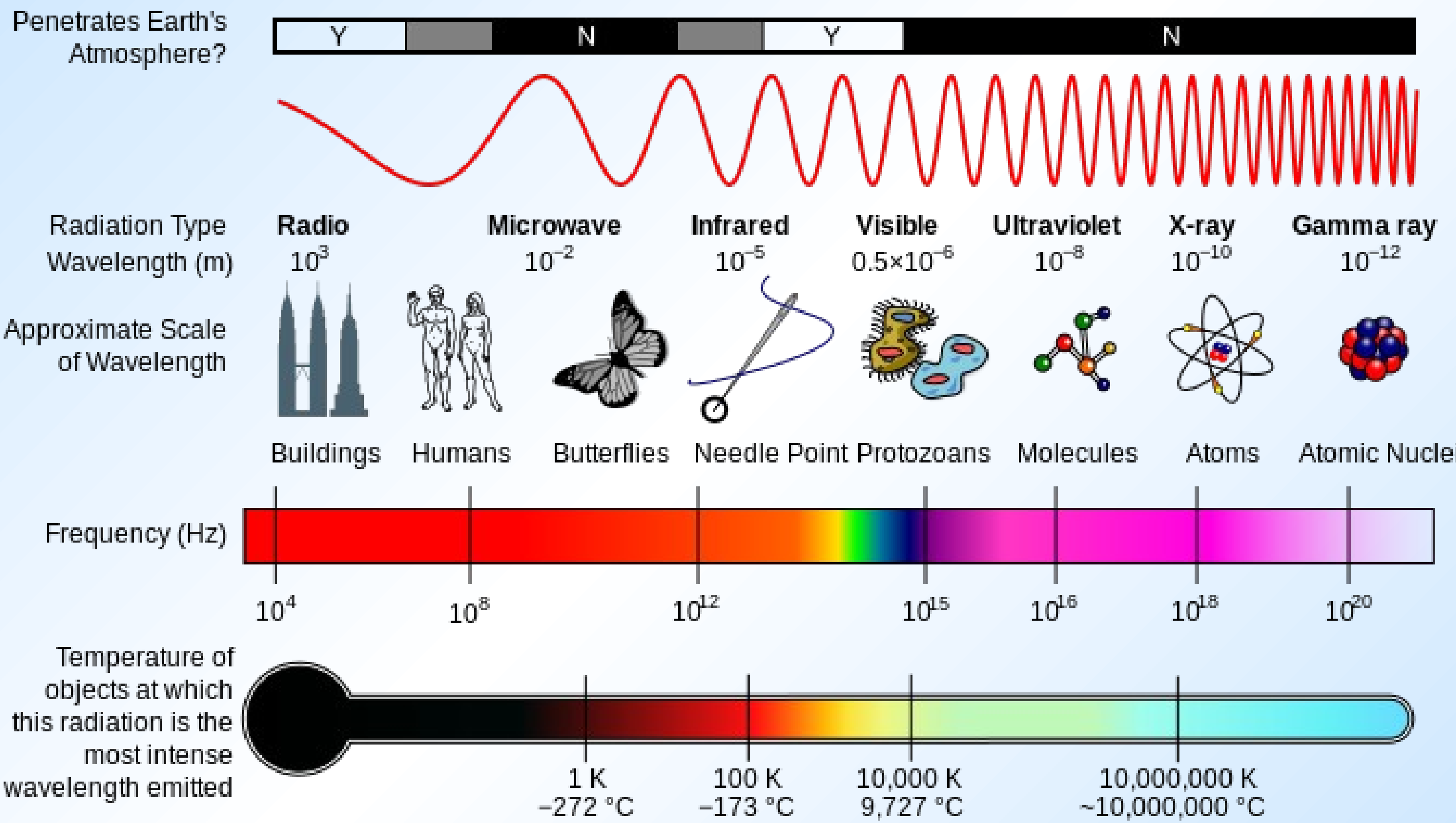
Albert Einstein and Robert Millikan.

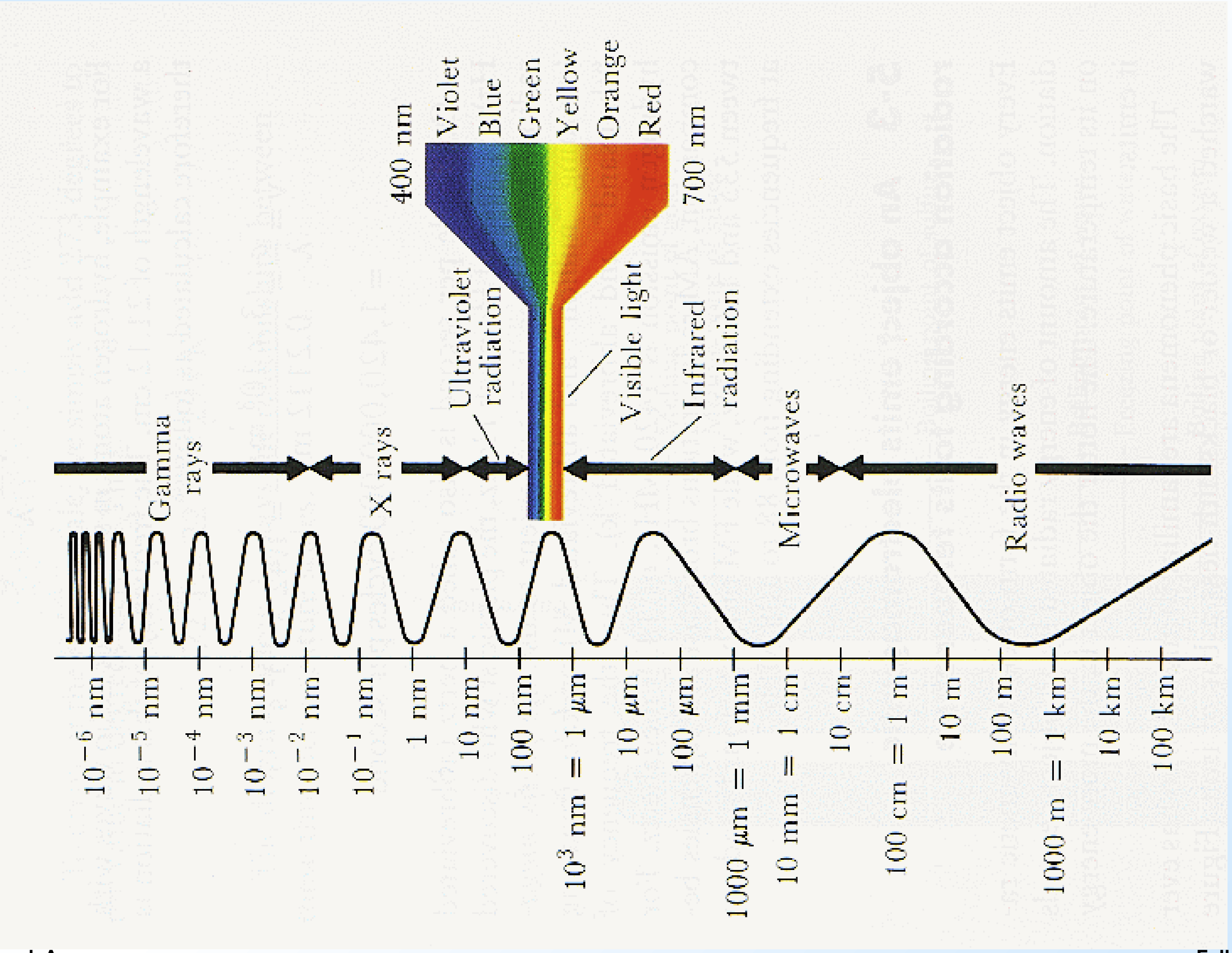
They won a Nobel Prize for their contributions to the photoelectric effect.

The nature of light is twofold. It can act as a particle (a photon) at times, which explains why light travels in straight lines. It can function like a wave at times, explaining how light bends (or diffracts) around an object.

Scientists embrace the evidence that supports the dual nature of light (despite the fact that it defies our common sense!).

Modern physics is wholly based on quantum theory as it explains the nature and behaviour of matter and energy at the atomic and subatomic levels. Quantum physics and quantum mechanics are terms used to describe nature and behaviour of matter and energy at that level.


Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Wavelength (m)
Radio
10^3
Microwave
10^-2
Infrared
10^-5
Visible
0.5×10^-6
Ultraviolet
10^-8
X-ray
10^-10
Gamma ray
10^-12
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^12
10^15
10^16
10^18
10^20
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
–272 °C
100 K
–173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C


400 nm
Violet
Blue
Green
Yellow
Orange
Red
700 nm
Gamma rays
X rays
Ultraviolet radiation
Visible light
Infrared radiation
Microwaves
Radio waves
10⁻⁶ nm
10⁻⁵ nm
10⁻⁴ nm
10⁻³ nm
10⁻² nm
10⁻¹ nm
1 nm
10 nm
100 nm
10³ nm = 1 μm
10 μm
100 μm
1000 μm = 1 mm
10 mm = 1 cm
10 cm
100 cm = 1 m
10 m
100 m
1000 m = 1 km
10 km
100 km

Approximate Wavelength:
390-455 nanometers

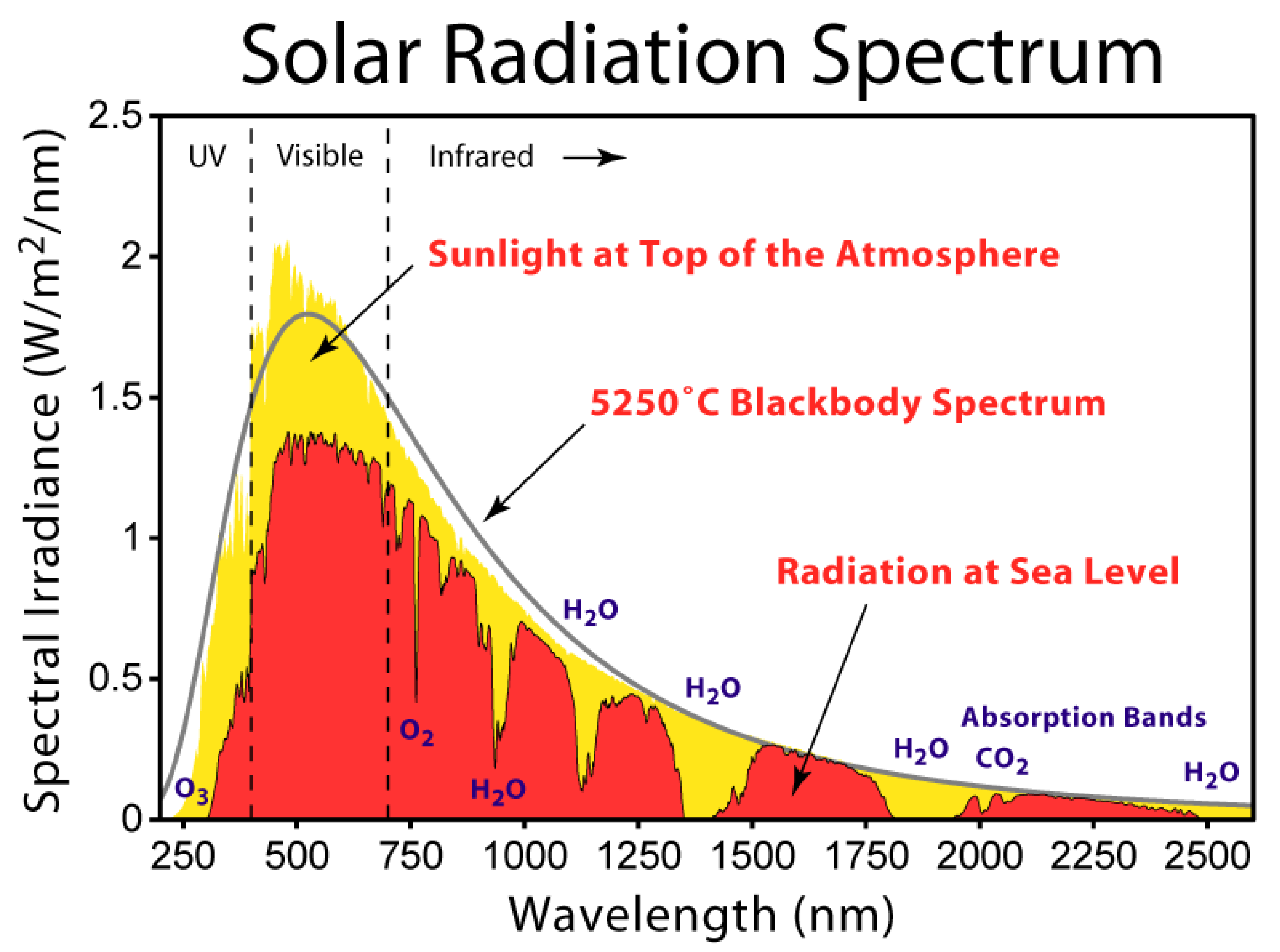
Approximate Wavelength:
455-492 nanometers

Approximate Wavelength:
492-577 nanometers

Approximate Wavelength:
577-597 nanometers

Approximate Wavelength:
597-622 nanometers

Approximate Wavelength:
622-780 nanometers



Solar Radiation Spectrum
Spectral Irradiance (W/m²/nm)
2.5
2
1.5
1
0.5
0
UV
Visible
Infrared
Sunlight at Top of the Atmosphere
5250°C Blackbody Spectrum
Radiation at Sea Level
H₂O
H₂O
O₂
H₂O
O₃
H₂O
Absorption Bands
CO₂
H₂O
250
500
750
1000
1250
1500
1750
2000
2250
2500
Wavelength (nm)

Solar Data

Mean distance from the Earth	149 600 000 km (Astronomical unit, au)
Diameter	1 392 000 km (109 times that of Earth)
Volume	1 300 000 times that of Earth
Mass	1 993 10^{27} kg (332 000 time that of Earth)
Density at the center	> 100 10^{3} kg/m^{3} (Over 100 times that of water)
Pressure at the center	Over 1 billion atm.
Temperature at the center	About 15 000 000 K
Temperature at the surface	6 000 K
Radiation energy	380 10^{21} kW
Earth receives	170 10^{12} kW

Sun Earth Relationships

Sun:

Diameter: 865,000 miles (1,392,000 km, 109 times earth)
Mass: 2 x 10^{30} kg (330,000 times earth)
Density: 1.41 g/cm^3
Gravity: 274 m/s^2 (28 g)

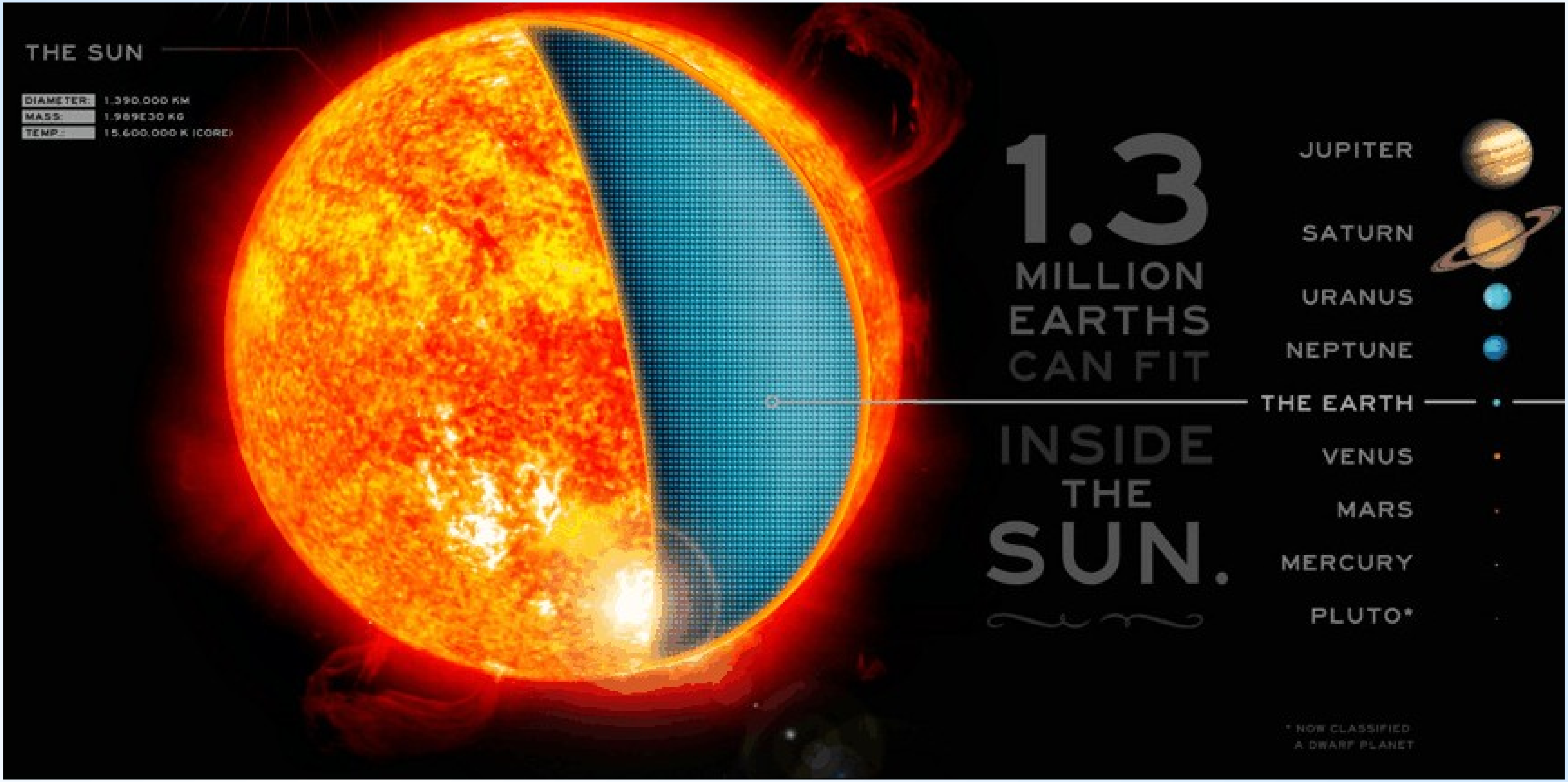
Surface Temperature: 10,000 F (5800 K)

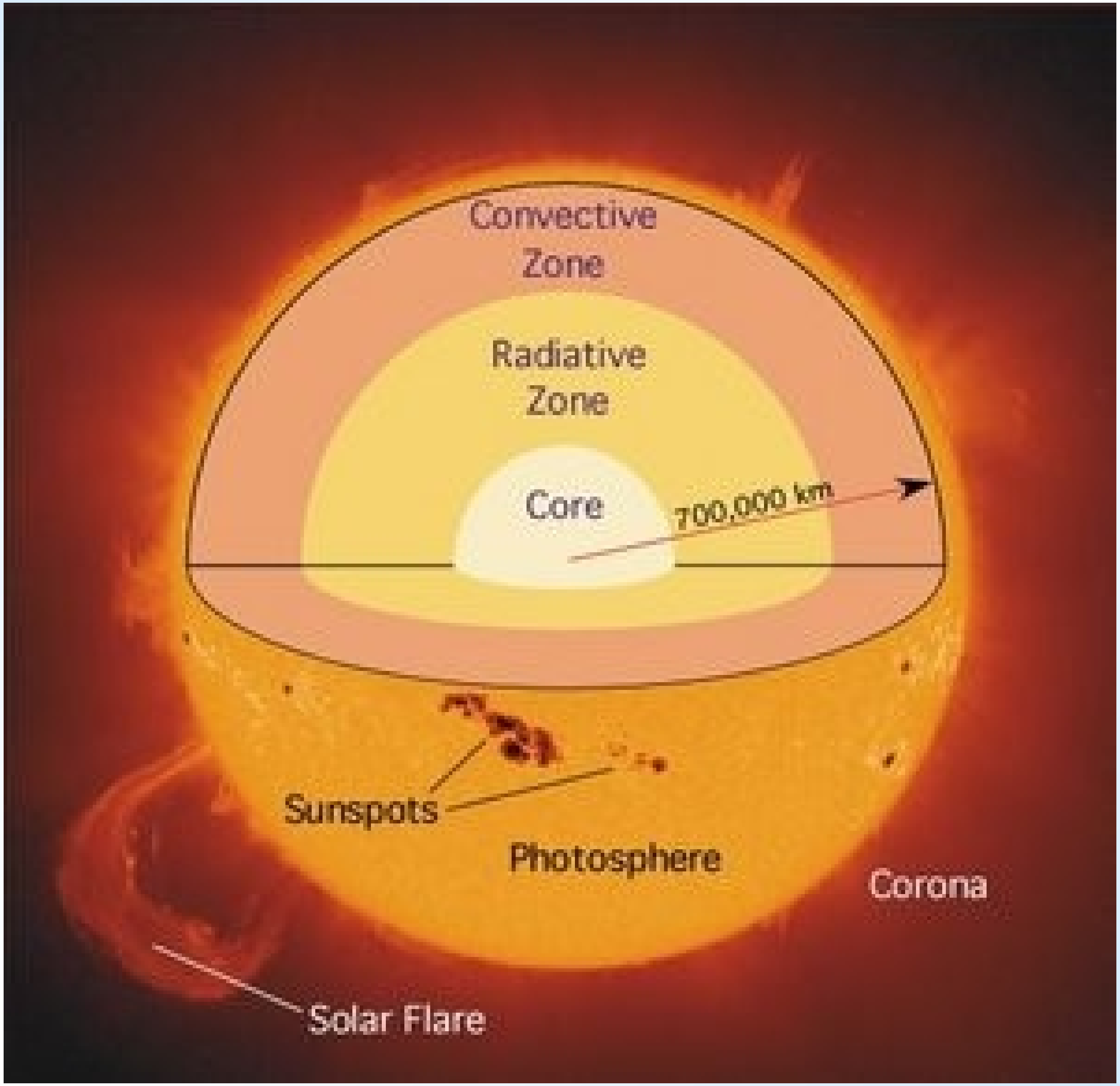
Earth:

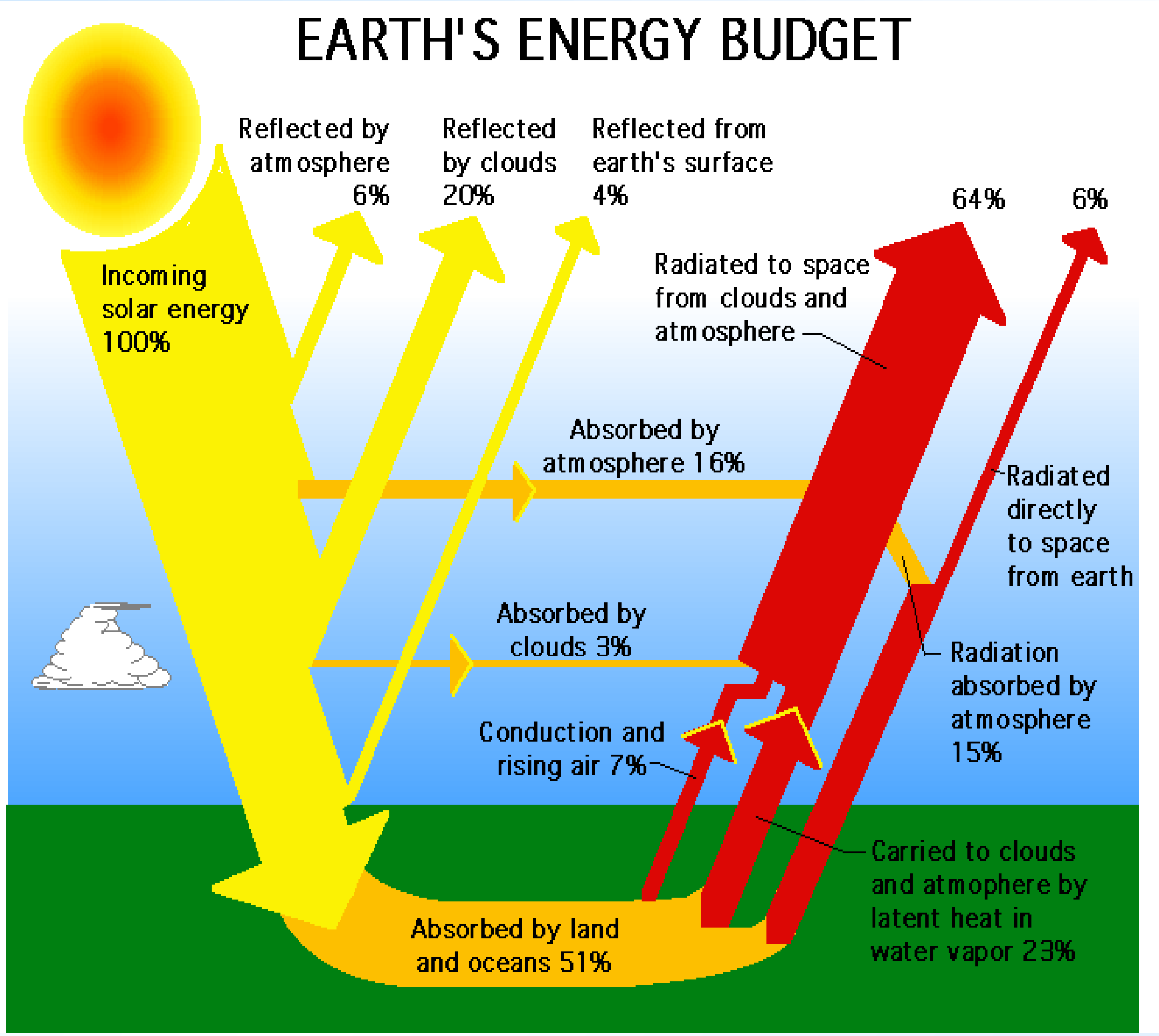
Diameter: 7,930 miles (12,756 km)
Mass: 5.97 x 10^{24} kg
Density: 5.52 kg/cm^3
Gravity: 9.81 m/s^2 (1 g)

Typical Surface Temperature: 68 F (300K)

Earth's Orbit Around Sun: 1 year
Earth's Rotation about its Polar Axis: 1 day


THE SUN
DIAMETER: 1.390.000 KM
MASS: 1.989E30 KG
TEMP.: 15.600.000 K (CORE)
1.3 MILLION EARTHS CAN FIT INSIDE THE SUN.
JUPITER
SATURN
URANUS
NEPTUNE
THE EARTH
VENUS
MARS
MERCURY
PLUTO*
* NOW CLASSIFIED A DWARF PLANET


Convective
Zone
Radiative
Zone
Core
700,000 km
Sunspots
Photosphere
Corona
Solar Flare



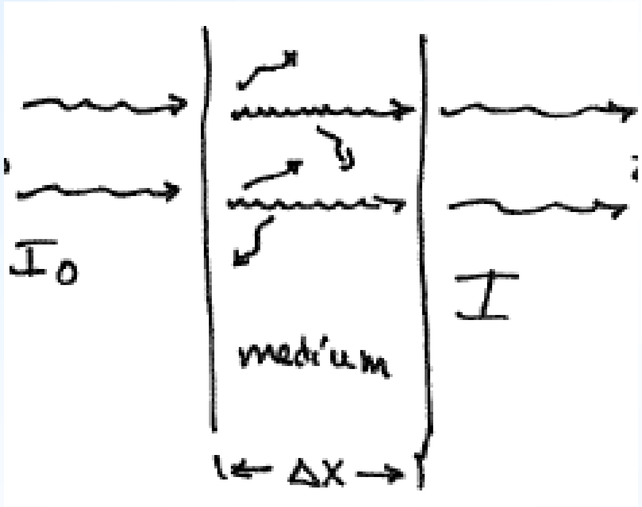
Rate of terrestrial energy per unit surface area in W/m^2, $J/s.m^2$

- Radiation (ışınım)
- Irradiation (ışınlama)
- Radiance (parlaklık)
- Irradiance (parlama)
- Insolation
- Energy flux

Wavelengths of 0.29 – 2.5 μm are of interest

Terrestrial (below the atmosphere) Radiation

- Varies daily and seasonally (day of the year and time of the day)
- Local solar radiation intensities strongly dependent on latitude (location)
- Solar radiation attenuated in atmosphere by scattering and absorption (atmospheric conditions)



I = electromagnetic energy flux, W/m^2.μm

$$\Delta I_{lost} \propto I_0 \, \Delta x \quad => \quad dI \propto I \, dx$$

$$\frac{dI}{I} \propto k_\lambda \, dx \quad => \quad I_\lambda(x) = I_{\lambda,0} \, e^{-k_\lambda x}$$

k_λ = monochromatic extinction coefficient

property of the medium

affected by scattering and absorption

Monochromatic transmission: $T_\lambda(x) = \dfrac{I_\lambda(x)}{I_{\lambda,0}} = e^{-k_\lambda x}$

Spectral transmission: $T(x) = \int_0^\lambda T_\lambda(x)\, d\lambda$

Scattering --> air molecules, water vapor, dust

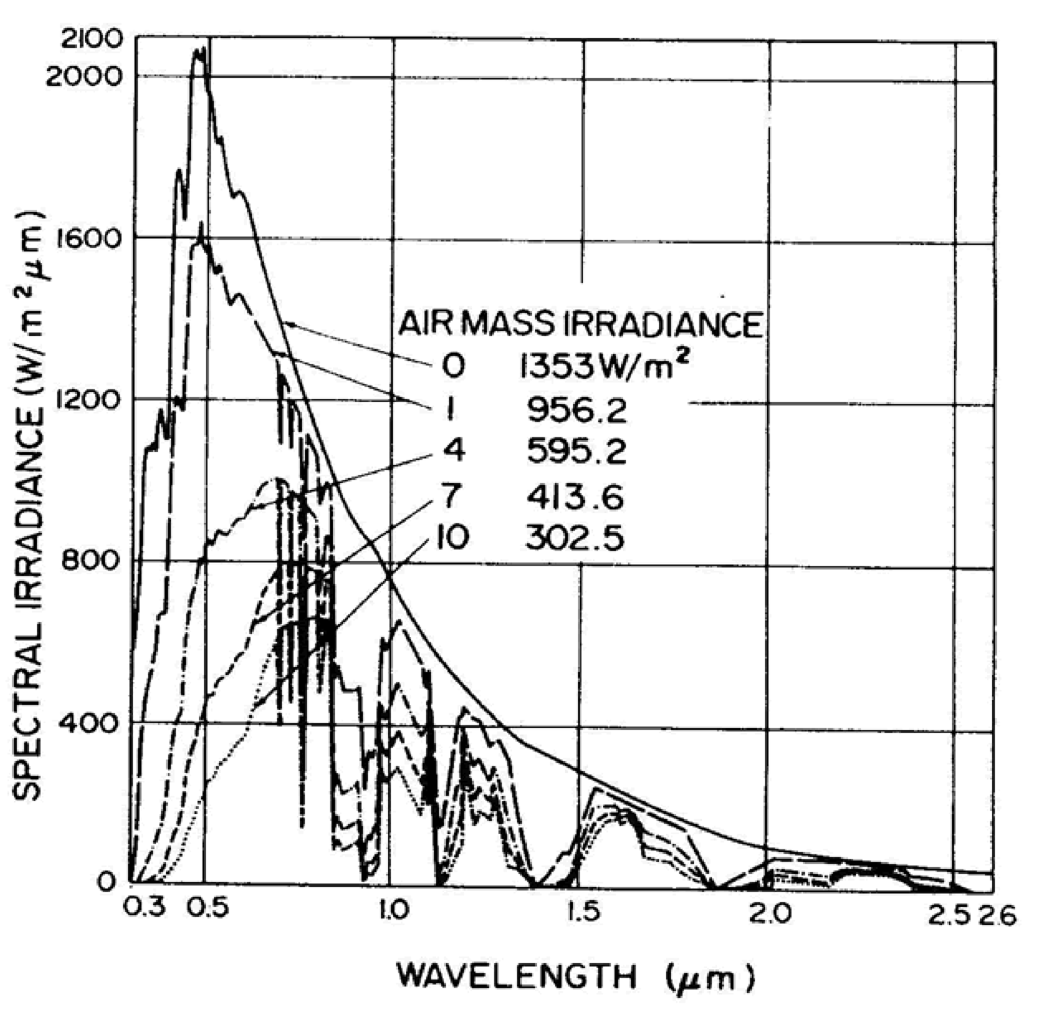
mostly shorter wavelengths - reason why the sky is blue

Absorption --> (i) Ozone (short wavelength – complete absorption at $\lambda < 0.29$ μm)

(ii) H_2O (long wavelength – 1.0, 1.4, 1.8 μm)

(iii) CO_2 (no absorption in short and visible wavelength)

Extraterrestrial and Terrestrial Solar Radiation



Air mass, m_a

- = 0 extraterrestrial
- = 1 sea level, sun at zenith
- = 2 sea level zenith angle 60 °

Solar Constant:

$$S = \int_0^\infty (\text{Spectral irradiance})\, d\lambda$$

$$S = 1366 \text{ W/m}^2$$

Elliptical orbit: ± 1 % variation

Sun spots: more variation

Extraterrestrial Radiation

Two sources of variation in extraterrestrial radiation must be considered:

1. The variation in the radiation emitted by the sun. For engineering purposes, in view of the uncertainties and variability of atmospheric transmission, the energy emitted by the sun can be considered to be fixed.
2. The variation of the sun-earth distance, that leads to variation of extraterrestrial radiation flux in the range of plus/minus 3.3 %.

A simple equation with accuracy adequate for most engineering calculations is:

$$G = SC\left(1 + 0.033\cos\left(\frac{360\ N}{365}\right)\right)$$

Extraterrestrial radiation incident on the plane normal to the radiation

Solar constant 1366 W/m^2

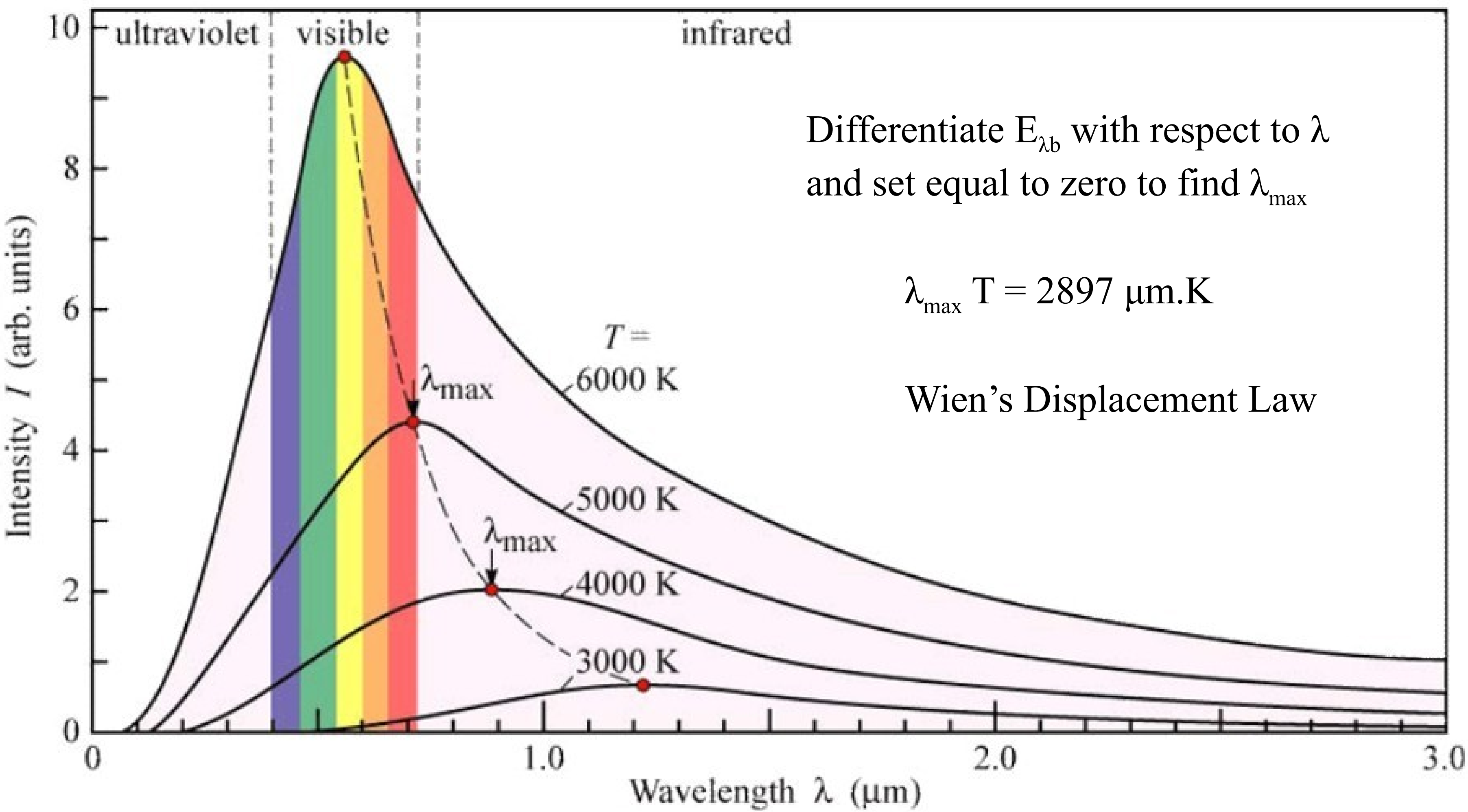
Nth day of the year

Ludwig Eduard Boltzmann

Austrian Physicist

1844 - 1906

Black Body Radiation



Differentiate $E_{\lambda b}$ with respect to λ and set equal to zero to find λ_{max}

$$\lambda_{max}\, T = 2897\ \mu m.K$$

Wien's Displacement Law

Wilhelm Wien

German Physicist

1864 - 1928

Differentiate $E_{\lambda b}$ with respect to λ and set equal to zero to find λ_{max}, the wavelength that corresponds to maximum emittance.

Wien's displacement law: $\lambda_{max} T = 2897$ μm.K

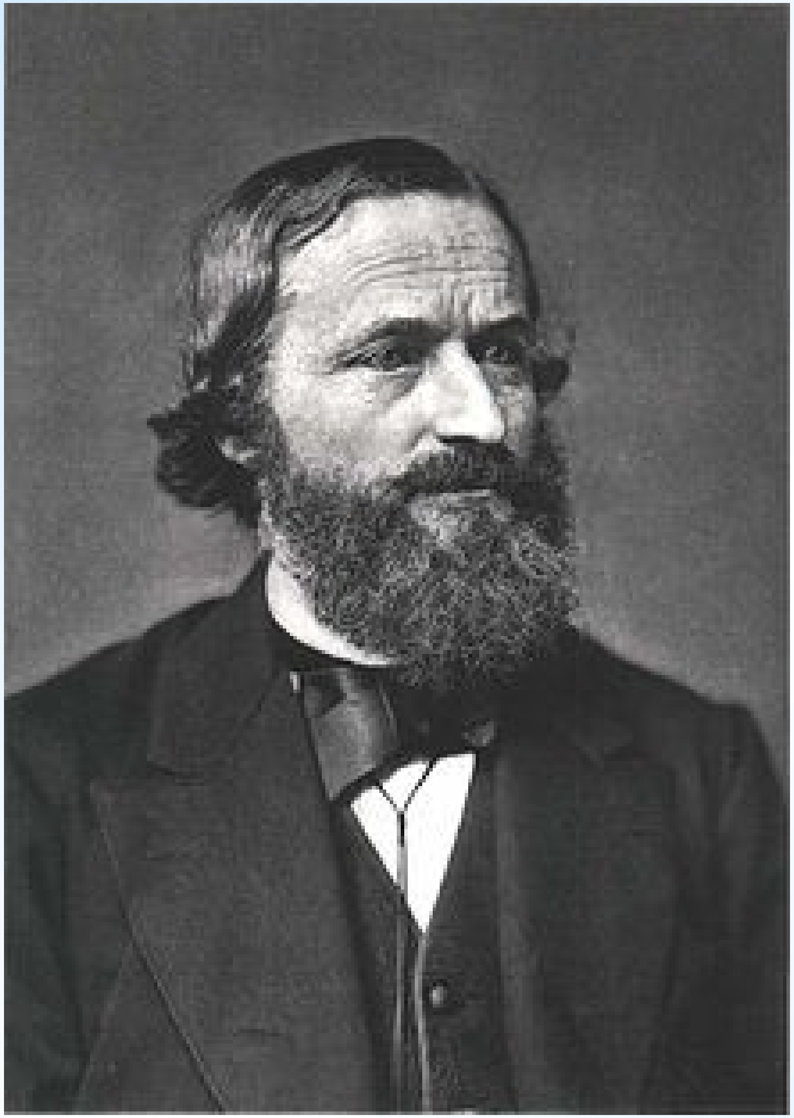
Real surfaces have some reflectance and emit less radiation than a black body at the same temperature. The emissivity of a real surface can be related to a black body using emittance, ε.

Total emissive power at a given temperature: $E = \varepsilon E_b$

For a real surface, spectral emittance, ε_λ, is equal to the spectral absorptance, α_λ

$$\varepsilon_\lambda = \alpha_\lambda$$

This is Kirchhoff's law of thermal radiation.



Gustav Robert Kirchhoff

German Physicist

1824 - 1887

Terrestrial Radiation

While the solar radiation incident on the Earth's atmosphere is relatively constant, the radiation at the Earth's surface varies widely due to:

- atmospheric effects, including absorption and scattering;
- local variations in the atmosphere, such as water vapor, clouds, and pollution;
- latitude of the location; and
- the day of the year and
- the time of the day.

Terrestrial Radiation

Composed of three parts:

1. **Beam** (direct) radiation, I_{BN}
2. **Diffuse** (diffuse-scattered) radiation; sky radiation, I_{DS}

   Radiation felt when standing in the shade on a sunny day

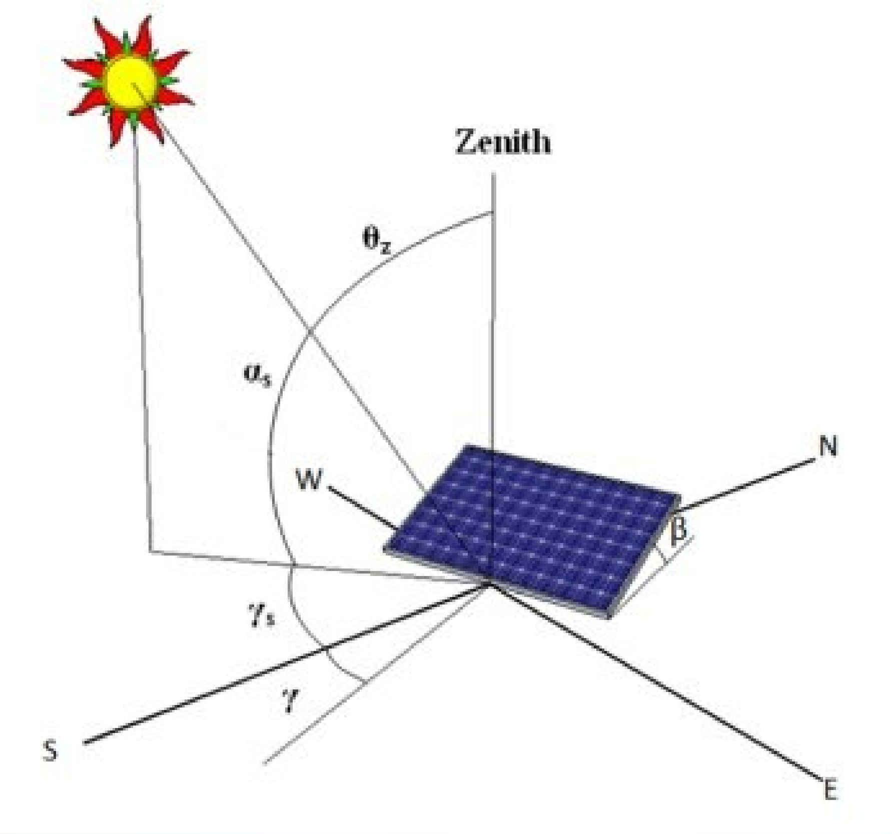
   Not the same as radiation emitted by the atmosphere

3. **Reflected** radiation (from surroundings, like the ground), I_R

Incident radiation can be represented in terms of dimensionless air mass (ratio)

$$m_a = \text{ratio of optical thickness}$$

$$= \frac{\text{thickness through which radiation passes}}{\text{zenith thickness}}$$



m_a is time dependent.

$m_a = 0$ extraterrestrial

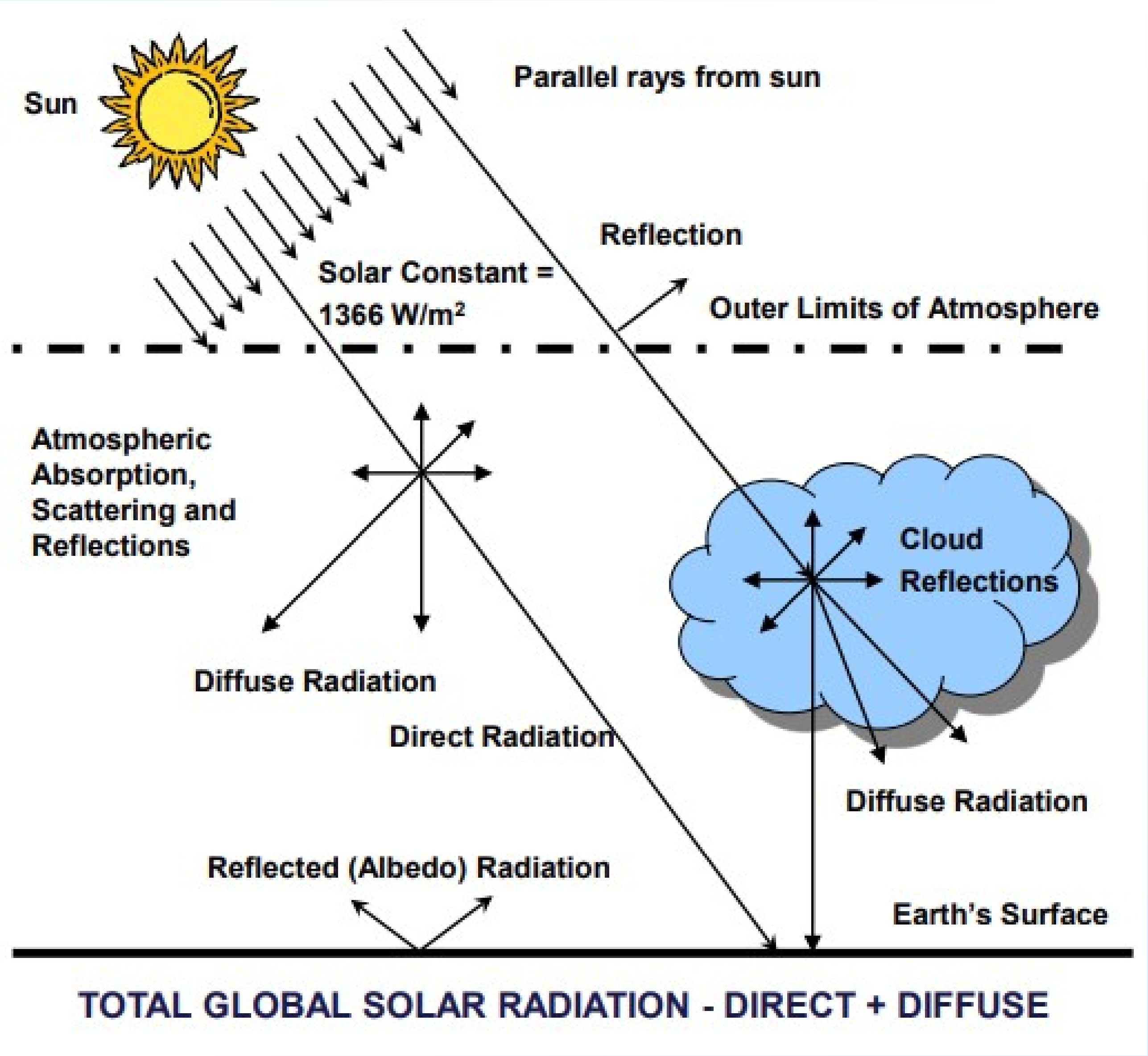
$= 1$ sea level, sun at zenith, $\theta_z = 0$

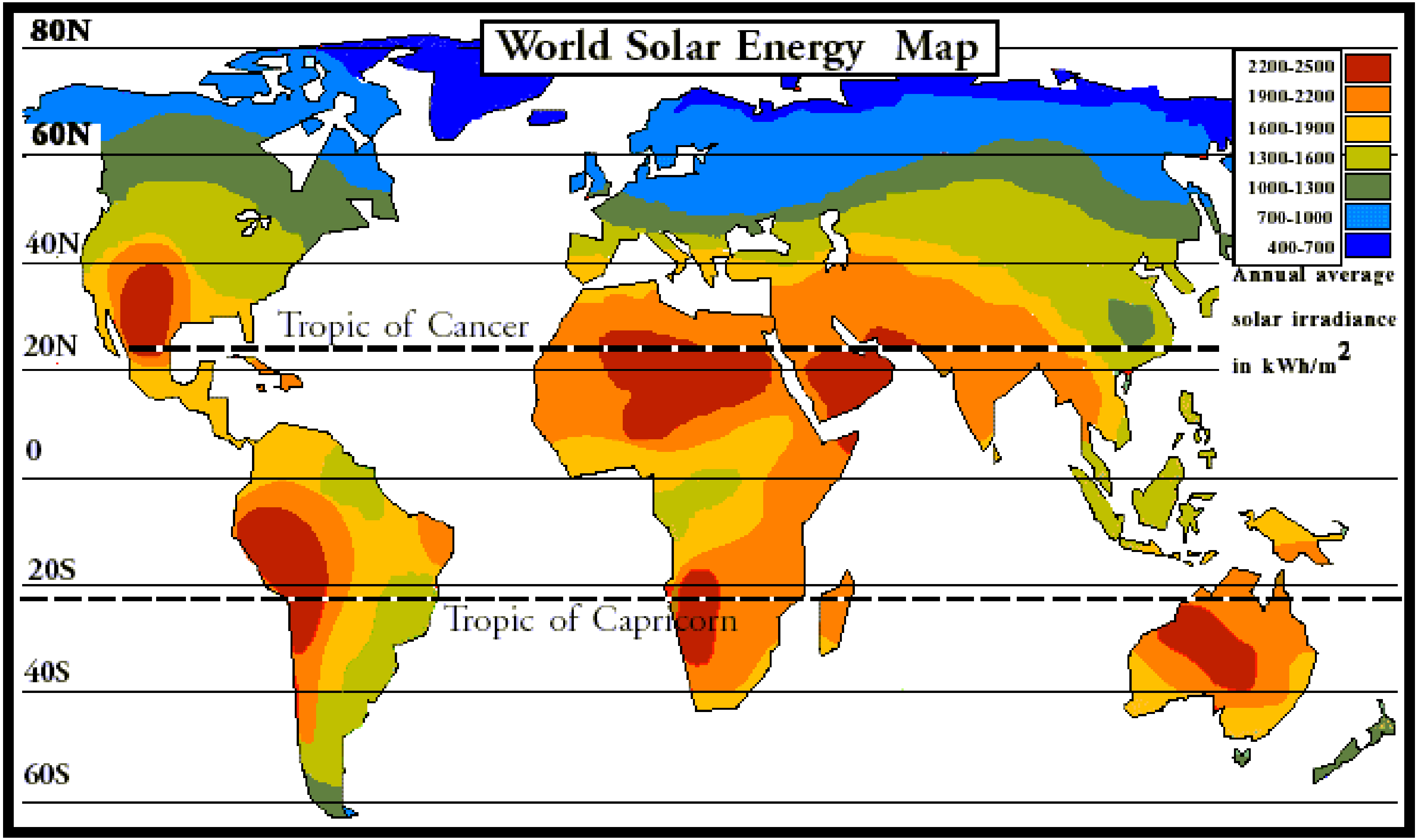
$= 2$ sea level zenith angle, $\theta_z = 60°$

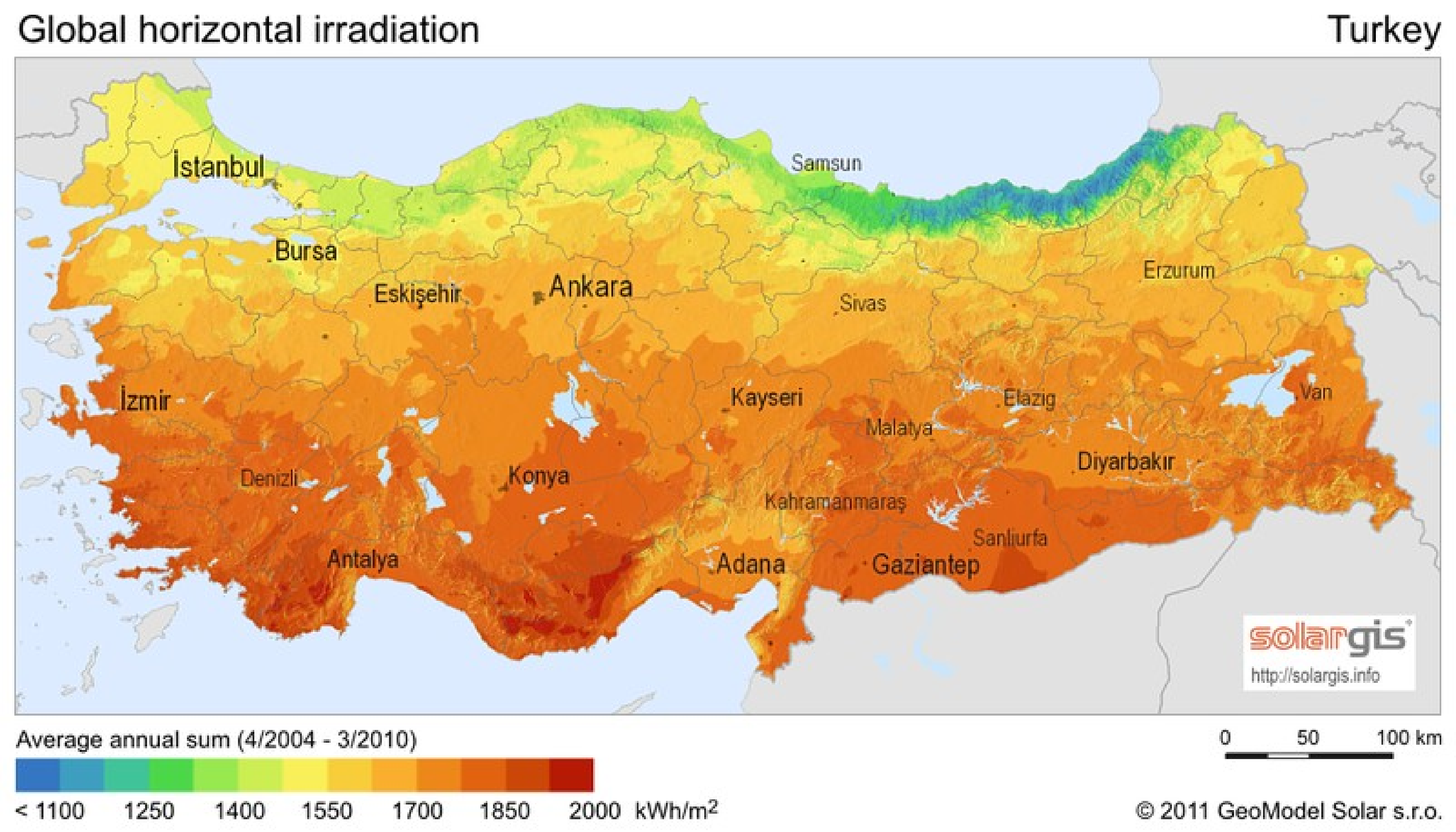
$$m_a = \frac{1}{\cos(\theta_z)} \quad , \quad 0 \le \theta_z \le 70^\circ$$

Beyond 70°, the curvature of the earth's atmosphere rapidly decreases intensity

Diffuse component is similar to beam component, but drifted slightly to shorter wavelengths; scattering occurs mostly at shorter wavelengths. Spectral energy distribution from an overcast sky is similar to that from a clear sky.


Sun
Parallel rays from sun
Reflection
Solar Constant =
1366 W/m²
Outer Limits of Atmosphere
Atmospheric
Absorption,
Scattering and
Reflections
Cloud
Reflections
Diffuse Radiation
Direct Radiation
Diffuse Radiation
Reflected (Albedo) Radiation
Earth's Surface
TOTAL GLOBAL SOLAR RADIATION - DIRECT + DIFFUSE


World Solar Energy Map
80N
60N
40N
20N
0
20S
40S
60S
Tropic of Cancer
Tropic of Capricorn
2200-2500
1900-2200
1600-1900
1300-1600
1000-1300
700-1000
400-700
Annual average solar irradiance in kWh/m2



See the article on OdtuClass about how to to calculate global irradiance.

Solar Insolation – Incident or Incoming Solar Radiation

Total energy flux on a surface: $I_{total} = I_{BN} + I_{DS} + I_R$

- I_R: Reflected from surroundings
- I_{DS}: Diffuse-scattered
- I_{BN}: Beam, or direct, normal to the surface

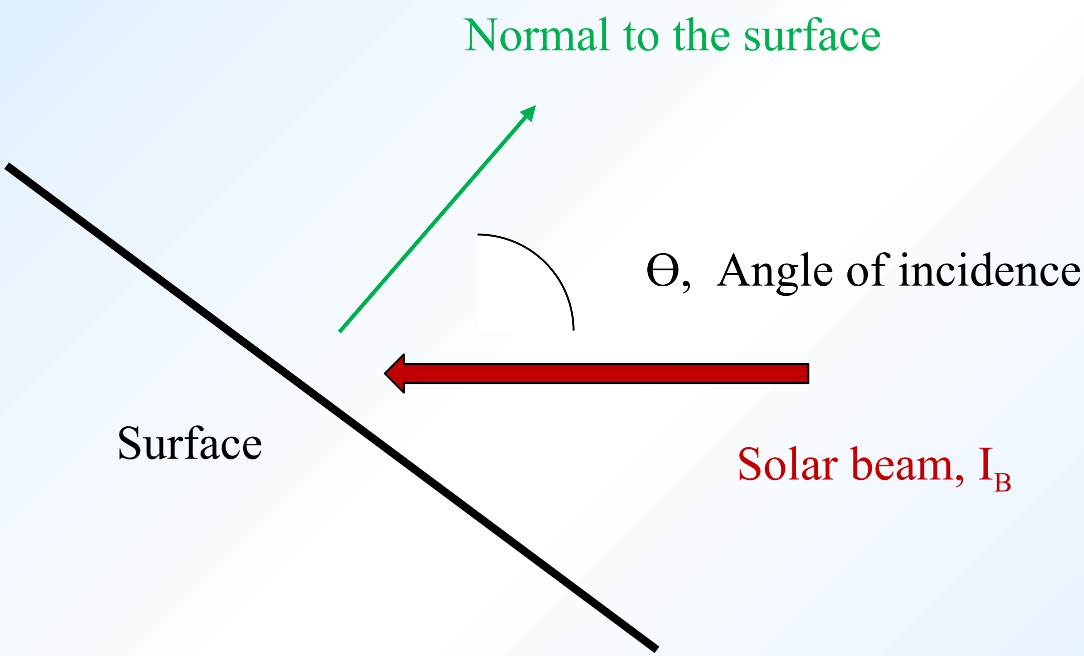
$I_{BN} = I_B \cos(\theta)$

θ is the angle between surface normal and incoming radiation vector; dependent on time of day and day of year

I_B is dependent on atmospheric conditions and thickness of atmosphere; the later varies with time of day and day of year.

I_{DS} can be linearly related to I_B

See: http://re.jrc.ec.europa.eu/pvgis/apps4/pvest.php



$$I_{BN} = I_B \cos(\theta)$$

In order to calculate the solar energy flux at a surface, I_{BN}, we need to calculate the insolation I_B and the angle of the insolation, θ, relative to the surface normal.

Distinguish between these two terms, insolation and irradiance:

Irradiance is the rate of energy that is being delivered to a surface area at any given time. Its units are Watts per square meter.

Insolation is the total amount of energy that has been collected on a surface area within a given time period.

While the irradiance denotes the instantaneous rate in which power is delivered to a surface, the insolation denotes the cumulative sum of all the energy striking the surface for a specified time interval.

I_B, Beam Insolation: $$I_B = A_s\, e^{-\frac{B_s}{\sin(\beta_1)}}$$

A_s: Apparent extraterrestrial solar insolation ($m_a = 0$)

- Varies with time of year
- Look up in a table based on date

B_s: Atmospheric extinction coefficient

- Varies with time of year and water vapor content
- Look up in table based on date

β_1: altitude angle between sun rays and horizontal surface

$$\frac{1}{\sin(\beta_1)} = m_a = \text{atmospheric path length}$$

I_{DS}, Diffuse-Scattered Insolation: $I_{DS} \approx C_S \, I_B \, F_{SS}$

I_{DS} is linearly proportional to I_B when exposed to direct sunlight.

C_S = Clearness Number = I_{DS}/I_B on a horizontal surface

- Varies with time of year and elevation
- Look up in a table
- Averaging could be daily, hourly, or monthly
- Varies widely from 0 to 0.3 - 0.7 depending on weather

$$F_{SS} = \text{shape factor} \approx \frac{1}{2}\left(1 + \cos(\beta_2)\right)$$

Tilt angle of surface (β_2)

I_R, Reflected Insolation

$$I_R = (I_B + I_{DS})\,\rho_g\,F_{wg}$$

Total Insolation at ground level → $(I_B + I_{DS})$

Ground reflectivity → ρ_g

Angle factor from wall to ground → F_{wg}

$$F_{wg} = \frac{1}{2}(1 - \cos\beta_2)$$

Total Insolation:

$$I_{total} = I_{BN} + I_{DS} + I_R$$

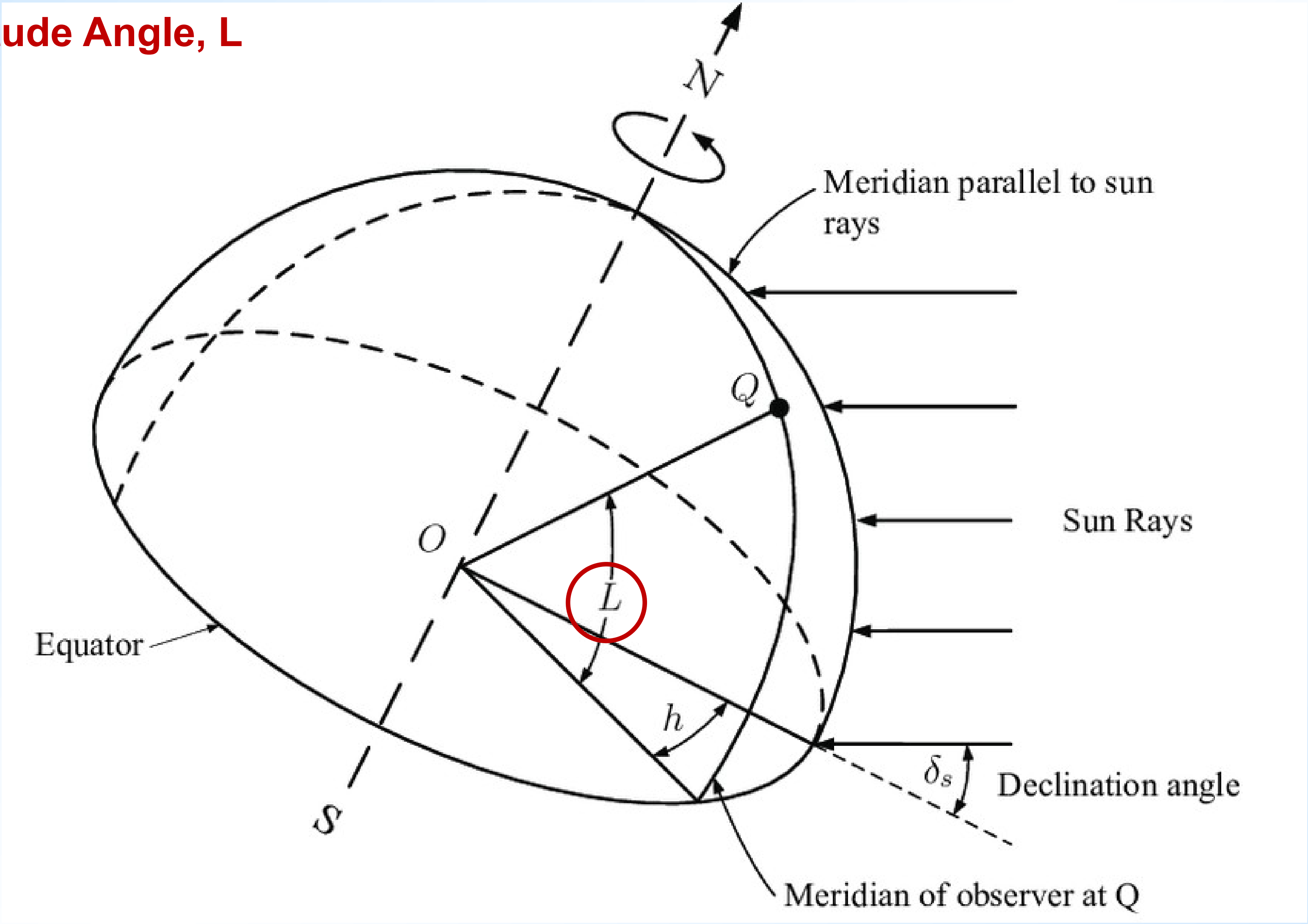
$$= I_B\cos\theta + I_B\,C_s\,F_{ss} + \rho_g\,(I_B + I_B\,C_s\,F_{ss})\,F_{wg}$$

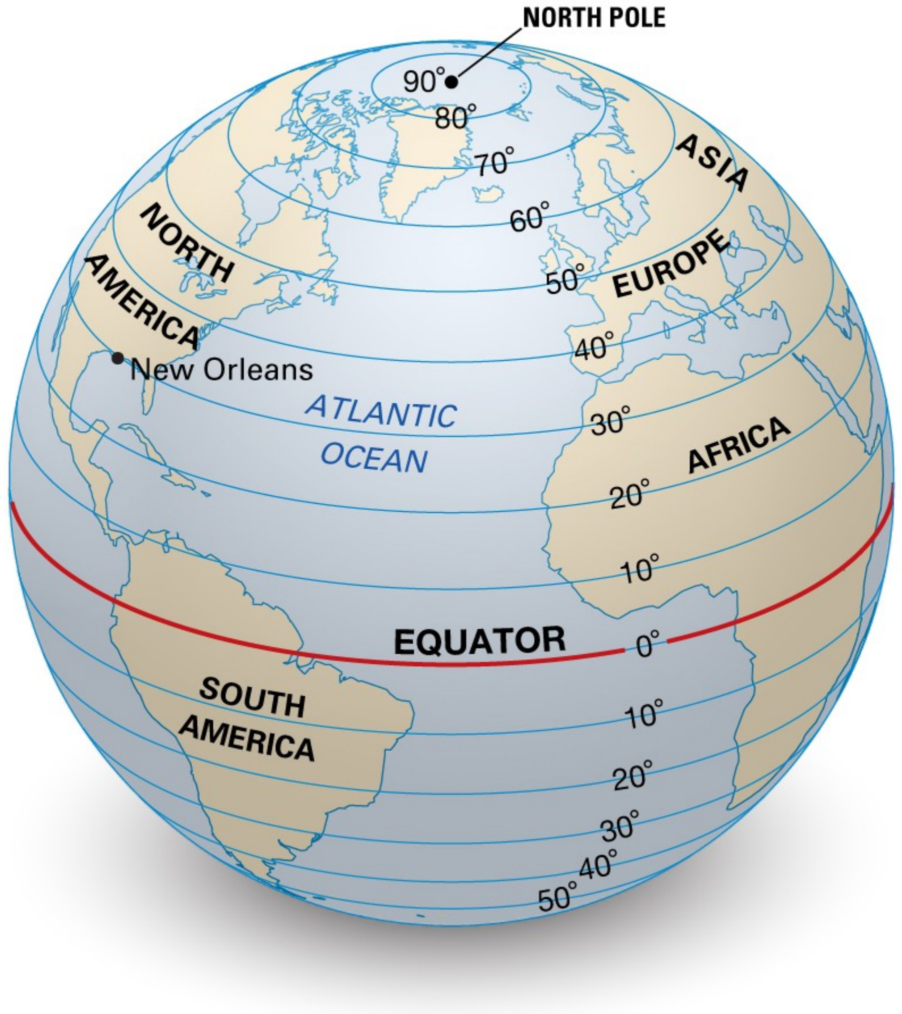
$$= I_B\,(\cos\theta + C_s\,F_{ss} + \rho_g\,F_{wg} + C_s\,\rho_g\,F_{ss}\,F_{wg})$$

$$I_{total} = I_B\left\{\cos\theta + \frac{1}{2}\left[C_s(1+\cos\beta_2) + \rho_g(1-\cos\beta_2)\right] + \frac{1}{4}C_s\,\rho_g\,\sin^2\beta_2\right\}$$

Tilt angle → β_2

Latitude Angle, L

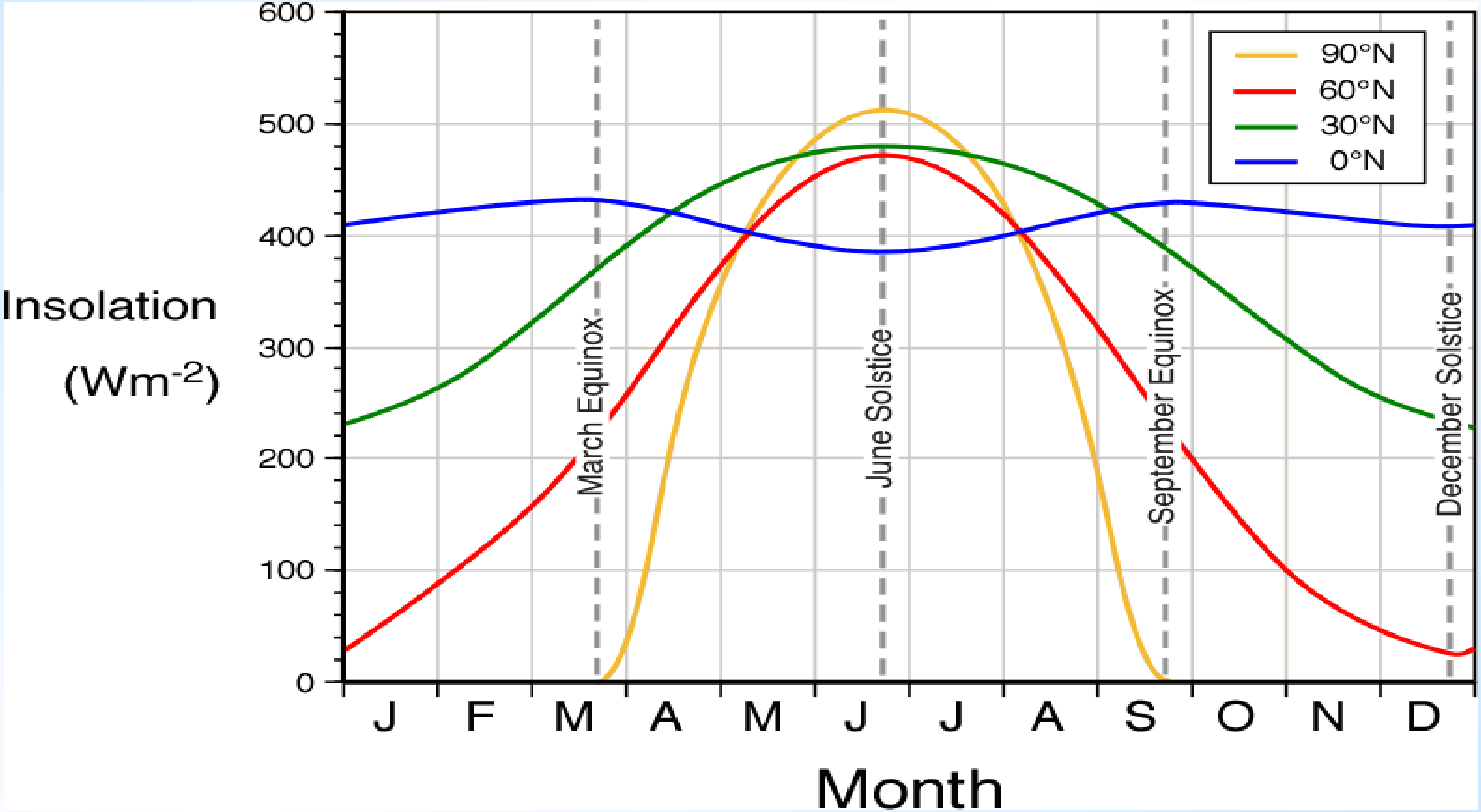




FACTS ABOUT LINES OF LATITUDE

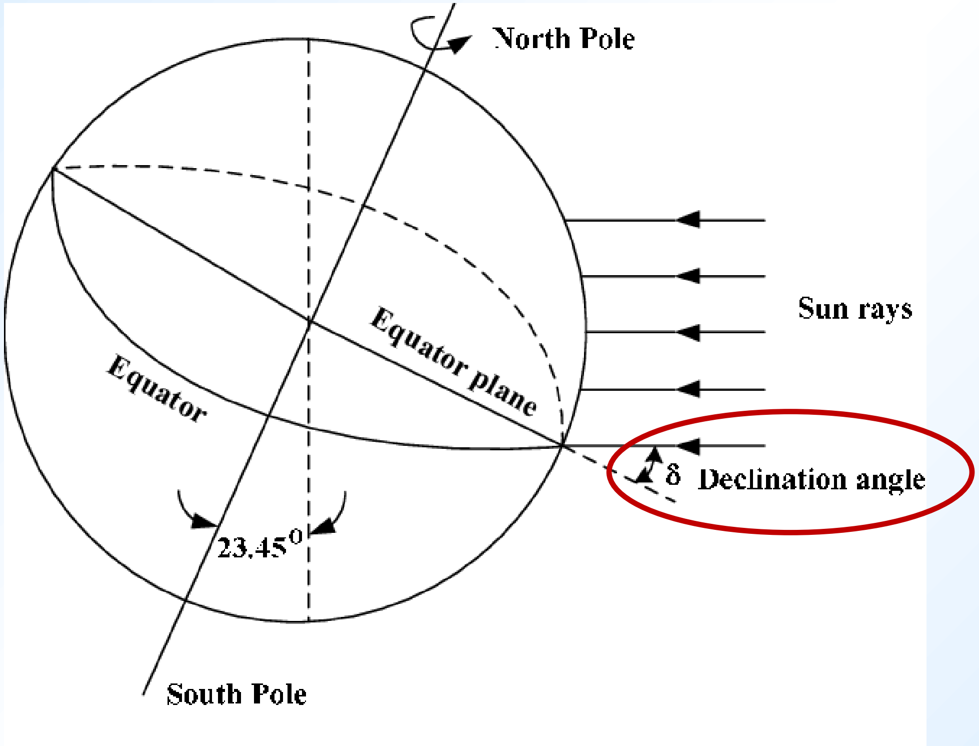
- Are known as parallels.
- Run in an east-west direction.
- Measure distance north or south from the Equator.
- Are parallel to one another and never meet.
- Cross the prime meridian at right angles.
- Lie in planes that cross the Earth's axis at right angles.
- Get shorter toward the poles, with only the Equator, the longest, a great circle.

Insolation by Latitude

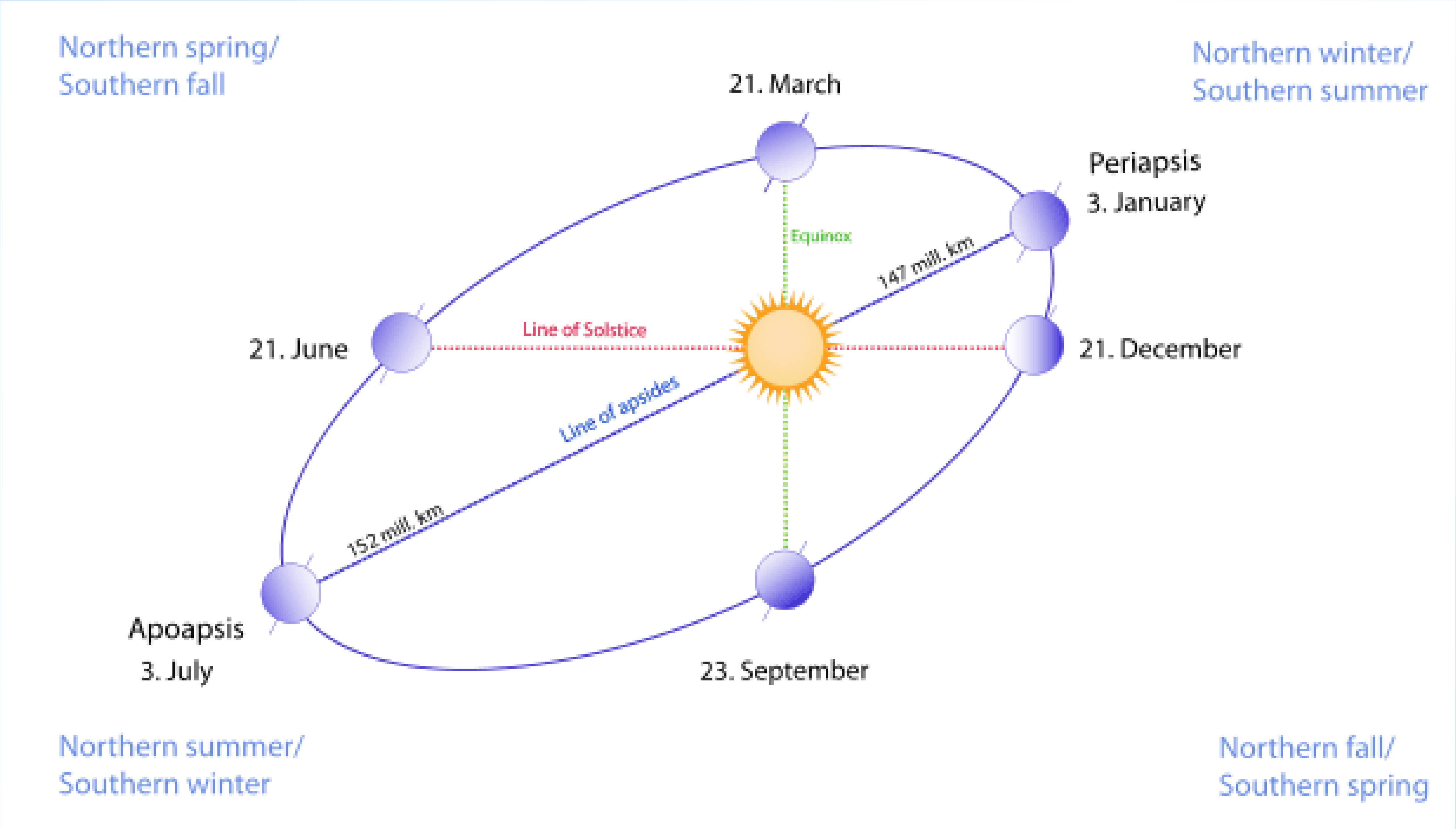


Solar insolation, I_B, depends on atmospheric conditions and air thickness

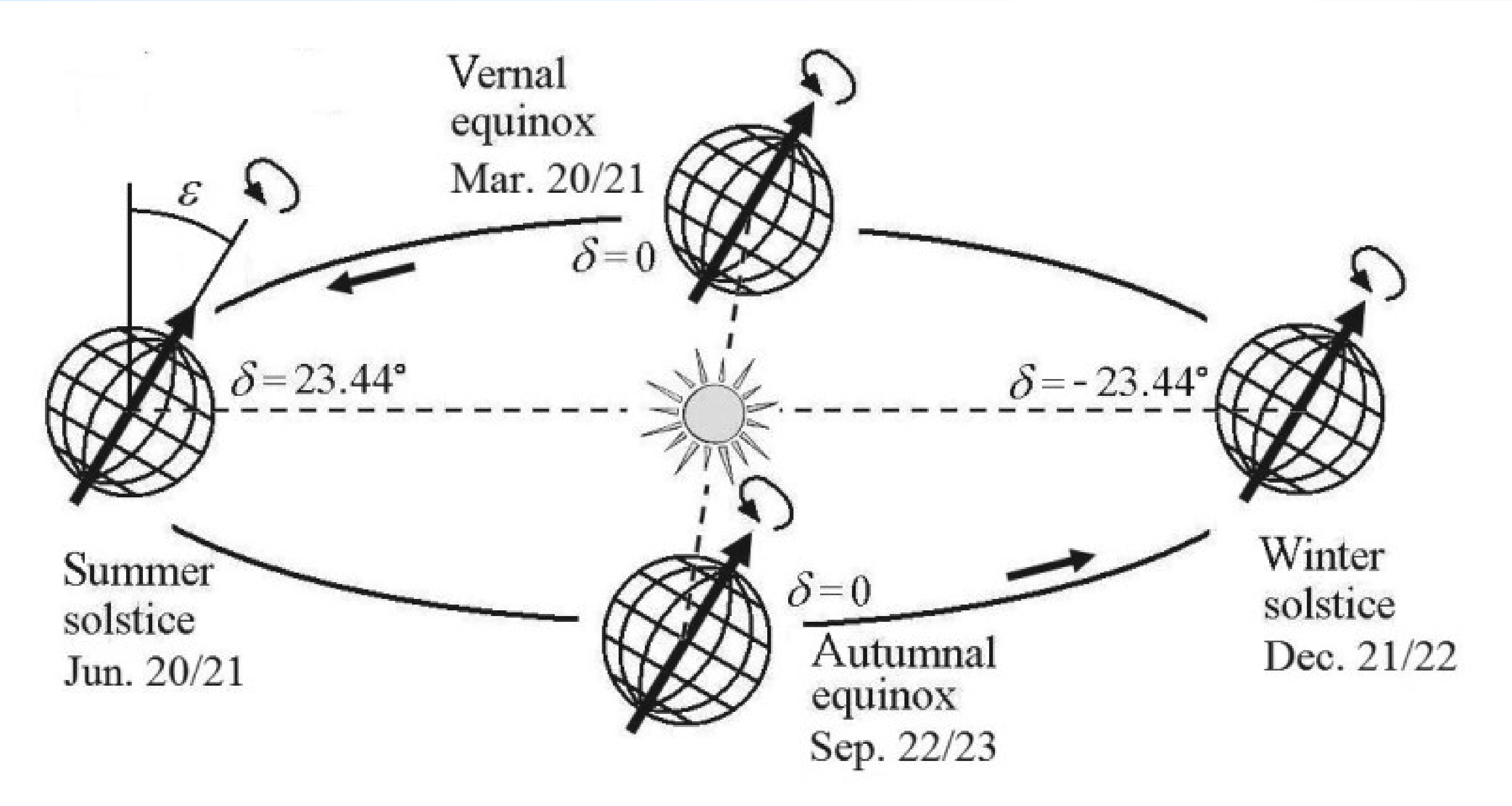
- Atmospheric conditions (humidity dust pollution clouds) vary with the season (time - time of day and time of year) and location.
- Air thickness varies with time and location



δ: Declination angle



Northern spring/
Southern fall
21. March
Northern winter/
Southern summer
Periapsis
3. January
Equinox
147 mill. km
21. June
Line of Solstice
21. December
Line of apsides
152 mill. km
Apoapsis
3. July
23. September
Northern summer/
Southern winter
Northern fall/
Southern spring


Vernal
equinox
Mar. 20/21
ε
$\delta = 0$
$\delta = 23.44°$
$\delta = -23.44°$
Summer
solstice
Jun. 20/21
$\delta = 0$
Autumnal
equinox
Sep. 22/23
Winter
solstice
Dec. 21/22

In order to calculate the solar insolation, I_B, and the angle between surface normal and incoming radiation (the angle of incidence θ) we need to know the following:

- the location on earth

  latitude (angle, L),

  longitude (meridean),

  elevation from sea level,
- **time of the day** (hour angle)
- day of the year (declination angle), and
- orientation of the surface (tilt angle).

We can choose a time interval of minute, hour, day, month, or year.

By the hour

The hour angle is referenced against solar noon; that is, when the sun is at the zenith for that particular day. We measure time using time zones and the assumption that the earth moves around the sun at a constant velocity.

Mean Solar Time **(MST)** = Local solar time presuming the Earth moves at a constant velocity; this is calculated from the local longitude, and is not the same as local time, or local civil time.

Apparent Solar Time **(AST)** = True solar time, relative to the sun`s zenith

$$AST = MST + (\text{Equation of Time})$$

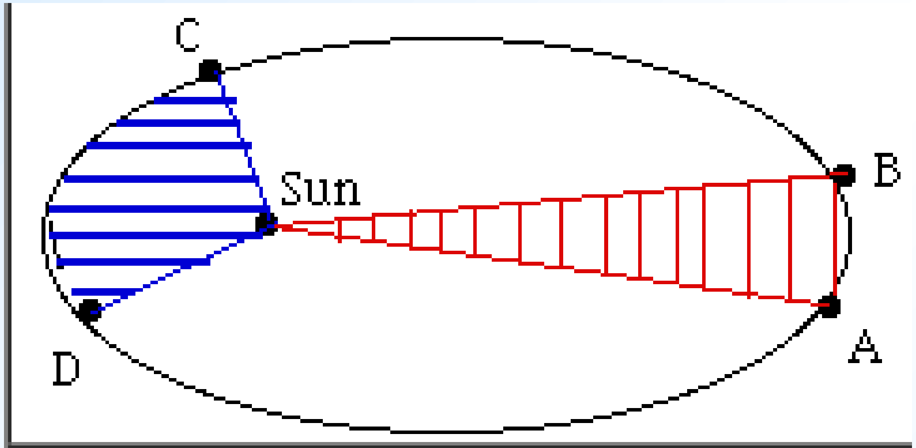
Equation of Time = Not an actual equation. This is a correction factor to account for the variation in the Earth`s velocity around the Sun due to the elliptical orbit. The correction range from =16.3 min in November to -14.4 min in February

The Earth revolves 360° in a 24 hour period. Therefore, 1° of Earth`s rotation (1 longitude degree) is 4 minutes. Imaginary lines running through Earth`s north and south poles define the center of each time zone on 15° increments. These are the Standard Meridians.

$$\text{Apparent (True) solar time} = \text{Mean solar time } (\text{Local clock time})$$
$$+ \text{ Geo time difference}$$
$$+ \text{ Equation of time (EoT)}$$
$$+ \text{ Daylight savings time.}$$

Geo time difference = (4) (Local meridean long. - Standard meridean long.)

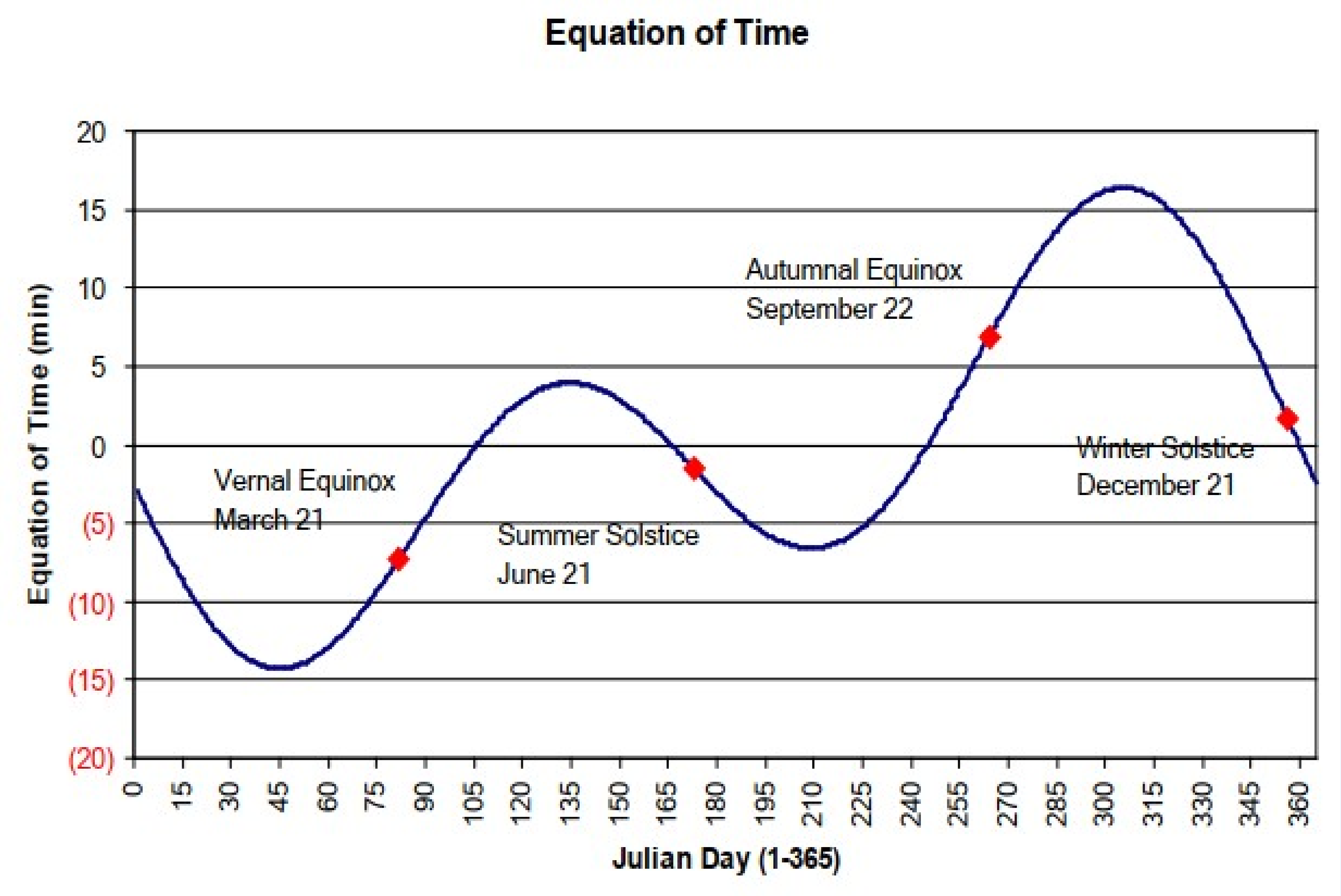

Noon
Midnight
Greenwich Meridian
International Date Line
MONDAY
SUNDAY
15° 0° 15° 30° 45° 60° 75° 90° 105° 120° 135° 150° 165° 180° 165° 150° 135° 120° 105° 90° 75° 60° 45° 30°
+1h 0h -1h -2h -3h -4h -5h -6h -7h -8h -9h -10h -11h -12h+ +11h +10h +9h +8h +7h +6h +5h +4h +3h +2h



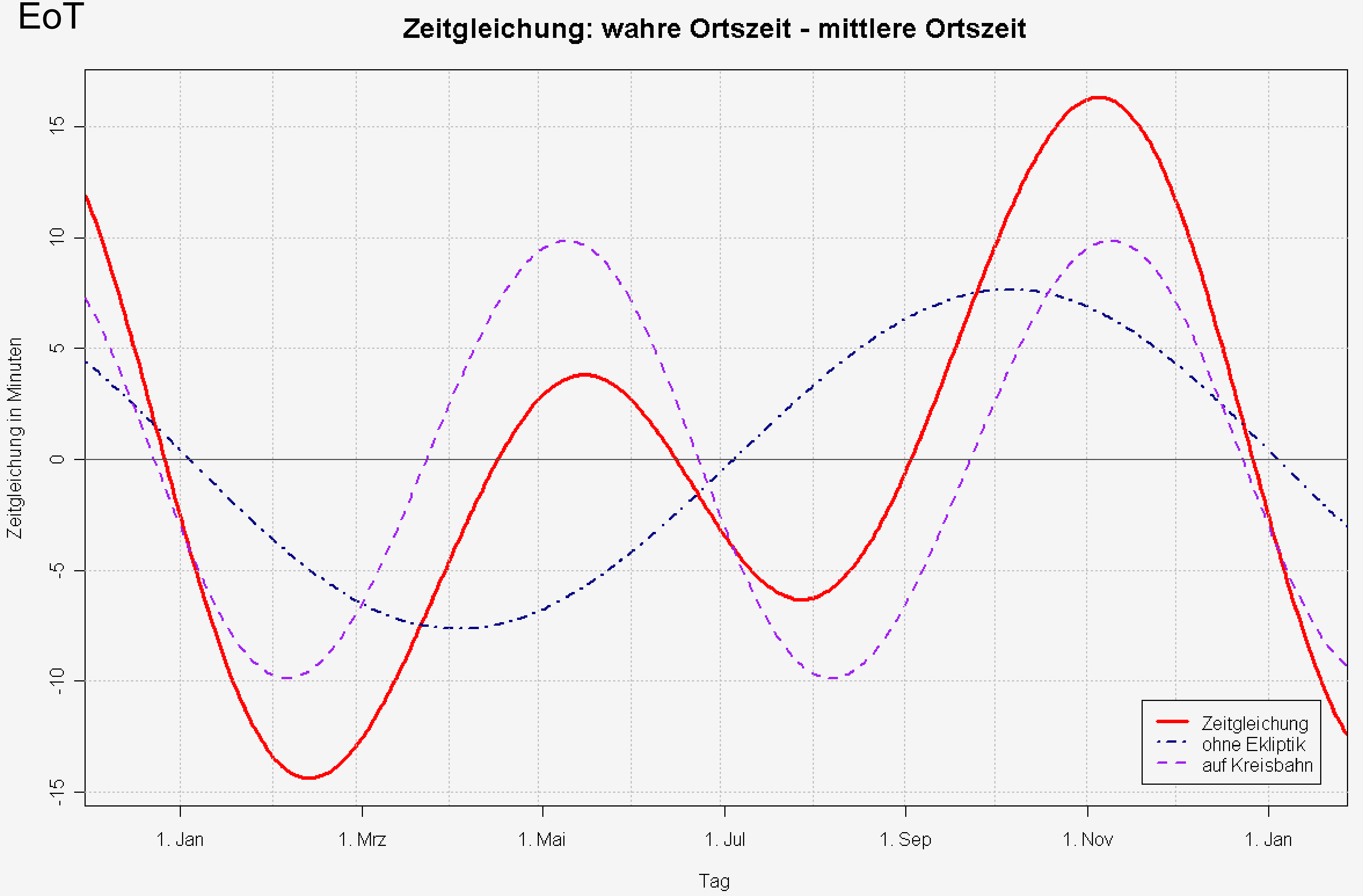
Illustrating Kepler's 2nd law: segments AB and CD take equal times to cover.

$$EoT = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B) \quad \text{in minutes}$$

$$B = \frac{360}{365}(N - 81) \quad \text{in degrees} \qquad N \text{ is day number}$$


Equation of Time
Equation of Time (min)
20
15
10
5
0
(5)
(10)
(15)
(20)
Autumnal Equinox
September 22
Vernal Equinox
March 21
Summer Solstice
June 21
Winter Solstice
December 21
0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360
Julian Day (1-365)

EoT



Zeitgleichung: wahre Ortszeit - mittlere Ortszeit
Zeitgleichung in Minuten
15
10
5
0
-5
-10
-15
1. Jan
1. Mrz
1. Mai
1. Jul
1. Sep
1. Nov
1. Jan
Tag
Zeitgleichung
ohne Ekliptik
auf Kreisbahn

Example 1

What is the local solar time on 1st of May, in Ankara, at 13:00 clock time?

See http://www.mapsofworld.com/lat_long/turkey-lat-long.html for longitude and latitude of Ankara.

For Ankara: Latitude: 39◦ 57’ N

Longtitude (Meridian): 32 ◦ 54’ E

Altitude (measured from sea level)

Solution

N = 31+28+31+30+1= 121

$$B = \frac{360}{365}\left(N - 81\right) = \frac{360}{365}\left(121 - 81\right) = 39.45 \text{ degrees}$$

EoT = 9.87 sin(2 B) - 7.53 cos(B) - 1.5 sin(B) = 2.92 minutes

$$LST = \begin{pmatrix}\text{local}\\ \text{standard}\\ \text{time}\end{pmatrix} + (4)\left[\begin{pmatrix}\text{local}\\ \text{longitude}\end{pmatrix} - \begin{pmatrix}\text{standard}\\ \text{meridian}\\ \text{longitude}\end{pmatrix}\right] + EoT$$

Correction = (4) (32.9 - 45) + 2.92 = - 45.5 minutes

LST = 13:00 - 45.5 => 12:15

Interesting books that you may want to read about keeping time:

«Longtitude»

1995 best-selling book by Dava Sobel

It is about John Harrison, an 18th-century clockmaker who created the first clock (chronometer) sufficiently accurate to be used to determine longitude at sea - an important development in navigation

«İstanbul Muvakkithaneleri – The Clock Rooms of Istanbul»

by Server Dayıoğlu, published in 2010, IBB (İstanbul Büyükşehir Belediyesi)

Read the article on OdtuClass by the same name; author: Salim Aydüz

Read about atomic clocks and the article titled «Leap Second» on OdtuClass

See: A Brief History of Atomic Time | NIST

By fixing a camera toward the southern sky (in the Northern Hemisphere), taking one picture at the same time of the day for a year, then superimposing the pictures of the sunny days, an “8”-like pattern is recovered, which is called the analemma . It is a combined result of equation of time and the variation of declination angle. This is compiled by Greek astronomer Anthony Ayiomamitis from 47 clear-day photos taken in 2003 near the Temple of Apollo, Corinth, Greece.

Several other angles are essential for the solar energy calculations. A diagram showing the orientation of the various angles is given in the next figure. At a given latitude, L, the sun's position can be defined in terms of the altitude angle, β_1, and the azimuth angle, α_1, of the sun. The altitude angle, β_1, is the angle between the sun's rays and the horizontal surface at that location.

α_1: Azimuth angle of sun's rays

α_2: Azimuth angle of normal to the surface

$\delta = \alpha_1 - \alpha_2$ Solar declination angle

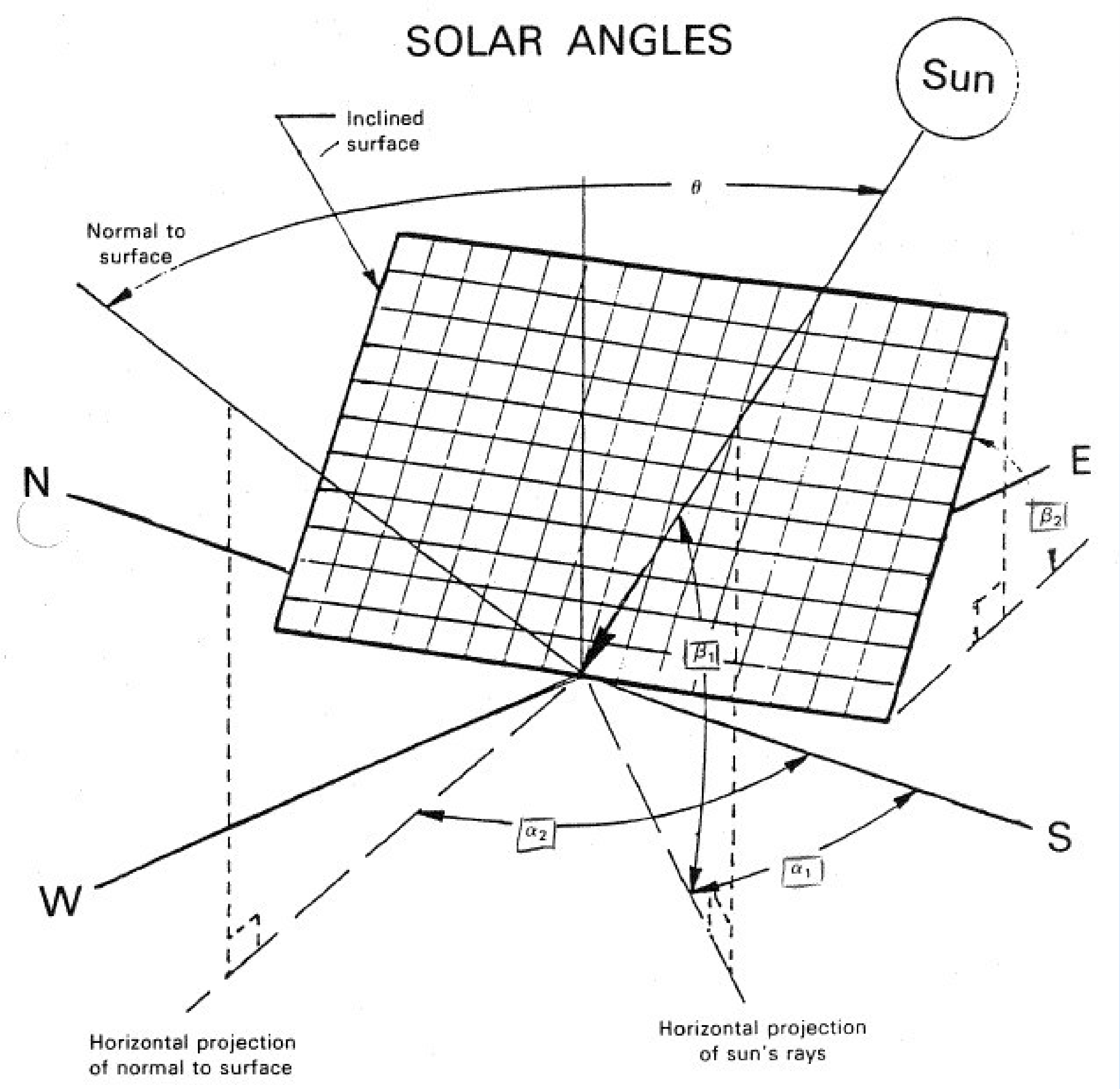
β_1 or β : Altitude angle of sun's rays

β_2 or α: Tilt angle of the surface

Θ: Angle between the normal to the surface and the sun's rays

$$\cos\theta = (\sin\beta_1)\left(\cos\beta_2\right) + (\cos\beta_1)\left(\sin\beta_2\right)\left[\cos(\alpha_1 - \alpha_2)\right]$$

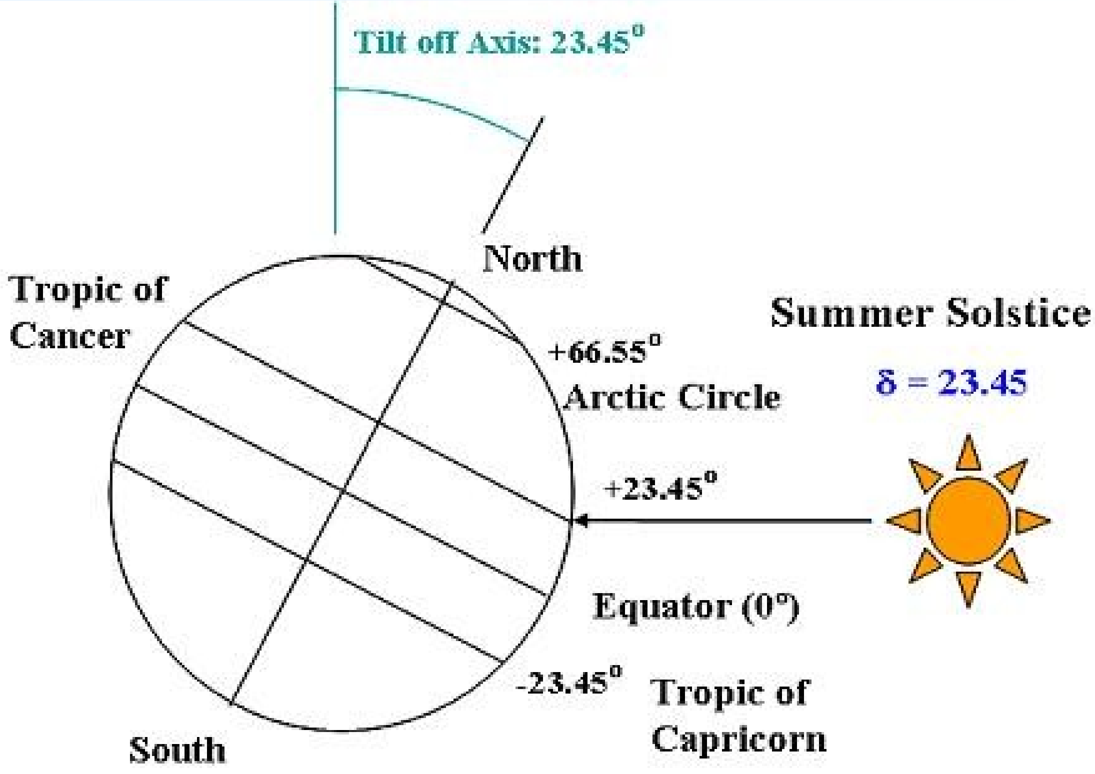
$$\boxed{\cos\theta = (\sin\beta_1)\left(\cos\beta_2\right) + (\cos\beta_1)\left(\sin\beta_2\right)\left(\cos\delta\right)}$$



SOLAR ANGLES
Sun
Inclined surface
θ
Normal to surface
N
E
β2
β1
α2
α1
S
W
Horizontal projection of normal to surface
Horizontal projection of sun's rays

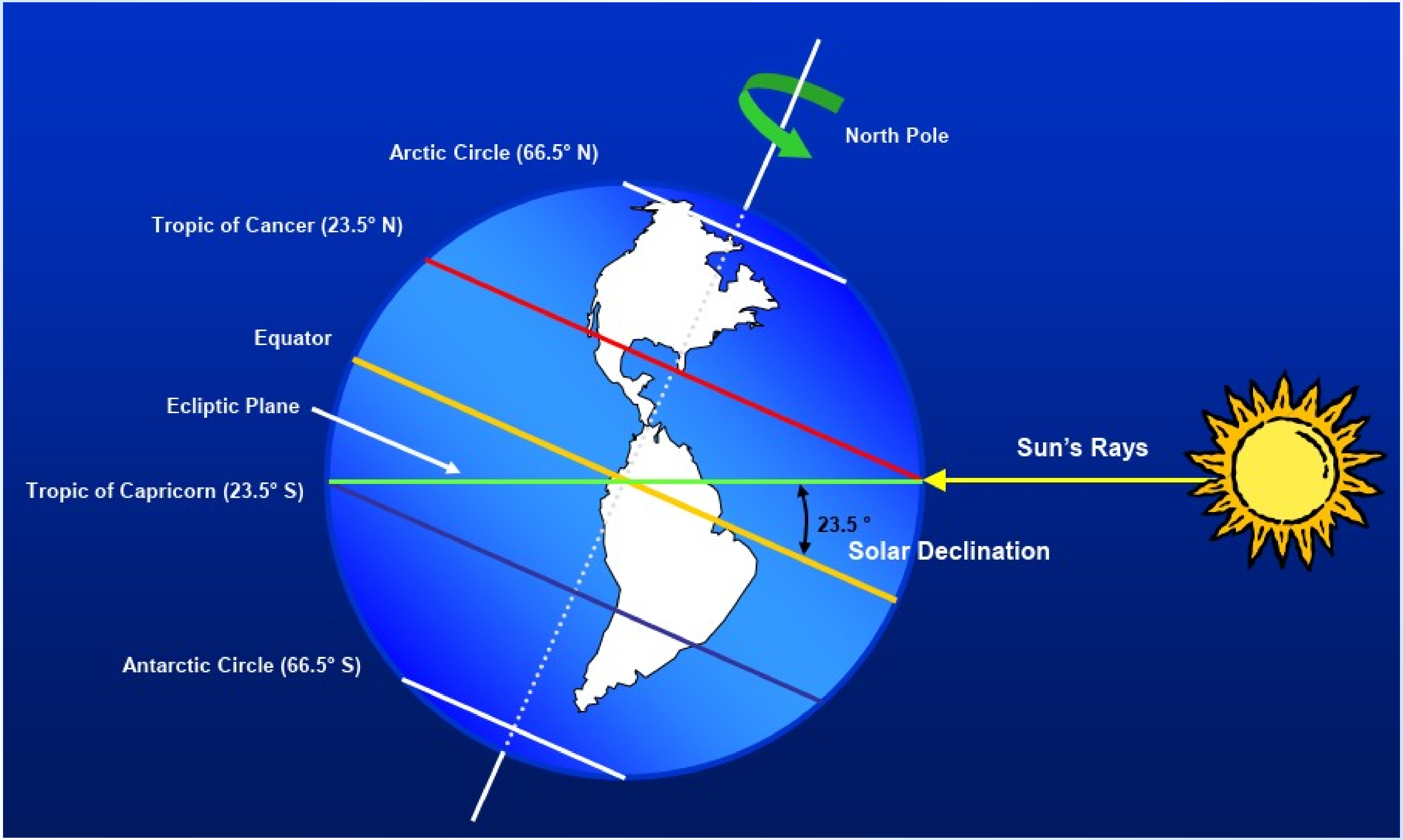
Declination angle

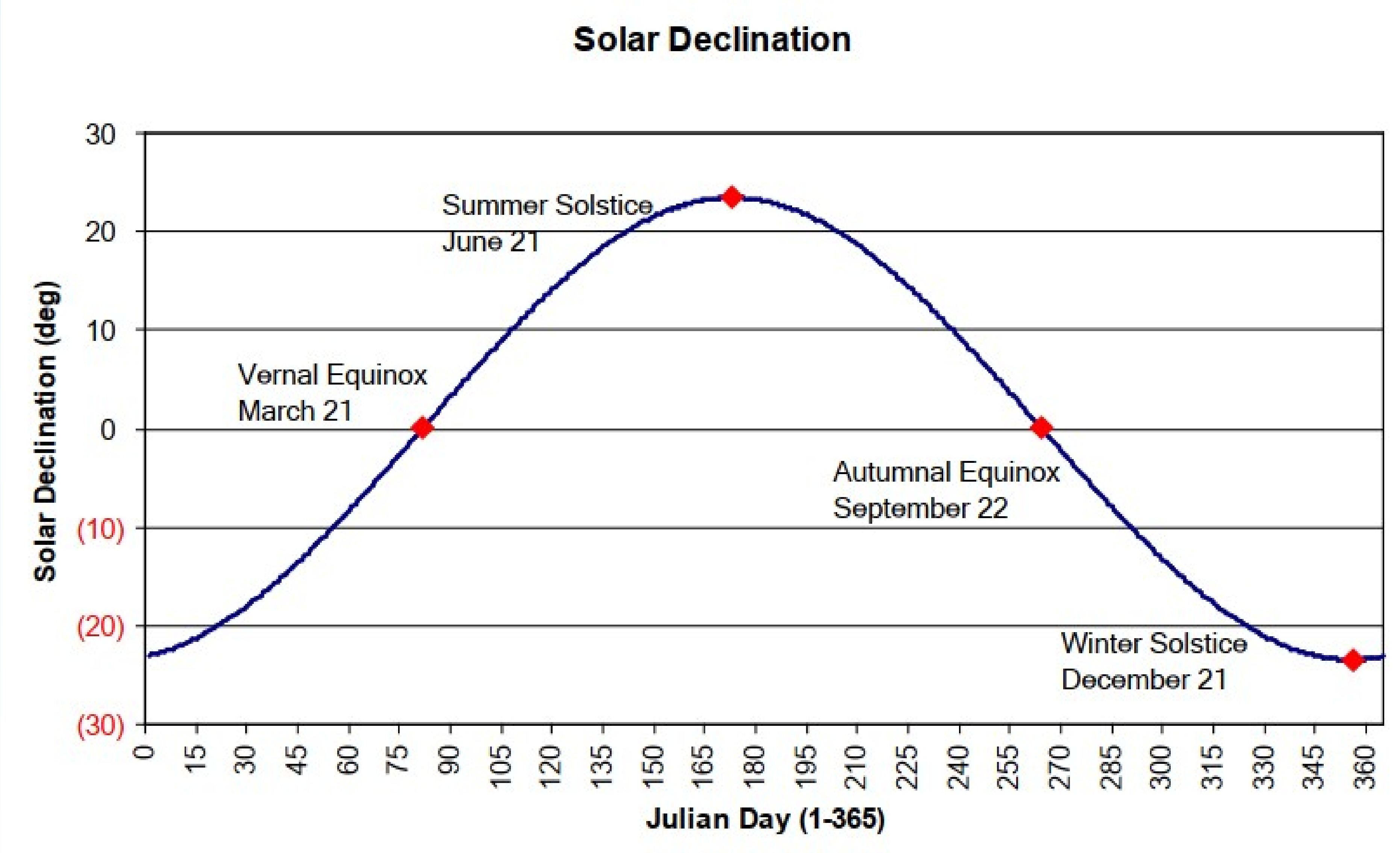
$$\delta = (23.45^\circ)\ \sin\left(\frac{360}{365}(284+n)\right) \quad , \quad 1 \le n \le 365$$



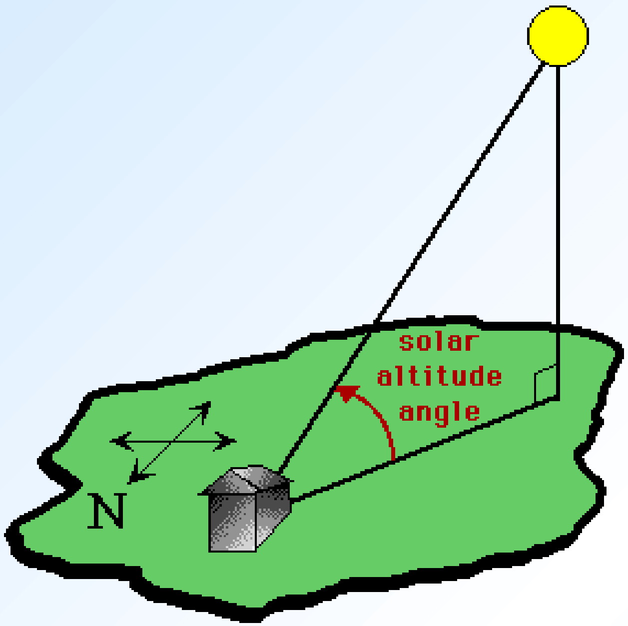
Declination (°)	*Approximate dates*
+23.45	June 22
+20	May 21, July 24
+15	May 1, Aug 12
+10	Apr 16, Aug 28
+5	Apr 3, Sept 10
0	Mar 21, Sept 23
−5	Mar 8, Oct 6
−10	Feb 23, Oct 20
−15	Feb 9, Nov 3
−20	Jan 21, Nov 22
−23.45	Dec 22

Declination angle $\delta = (23.45^\circ)\ \sin\left(\frac{360}{365}(284 + n)\right)$, $1 \le n \le 365$




Solar Declination
30
20
10
0
(10)
(20)
(30)
Solar Declination (deg)
Summer Solstice
June 21
Vernal Equinox
March 21
Autumnal Equinox
September 22
Winter Solstice
December 21
0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360
Julian Day (1-365)

Altitude Angle, β_1



Ground (Horizontal surface)

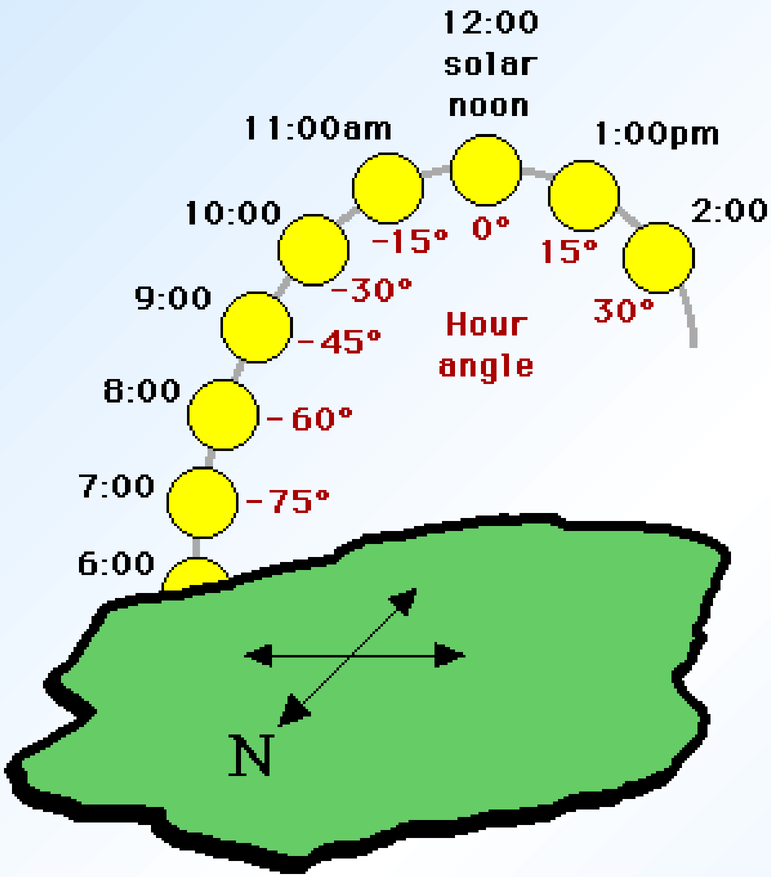
$$\sin(\beta_1) = \cos(L)\cos(\delta)\cos(H) + \sin(L)\sin(\delta)$$

L = Latitude Angle

δ = Declination Angle

H = Hour Angle

Hour Angle, H



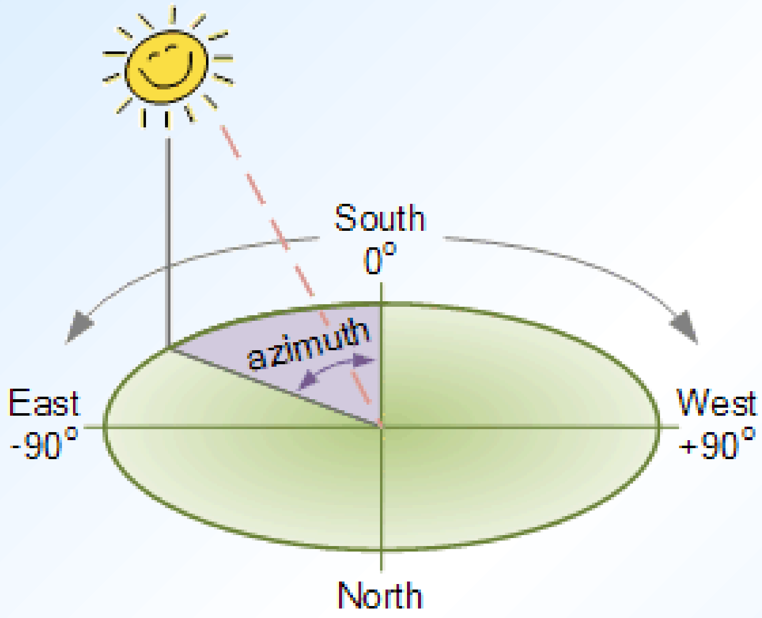
H is measured from local solar time (AST) noon

Maximum at sunset (+ angle)

Minimum at sunrise (- angle)

Solar Azimuth Angle, α_1

$$\cos(\alpha_1) = \frac{\sin(\delta)\cos(L) - \cos(\delta)\sin(L)\cos(H)}{\cos(\beta_1)}$$

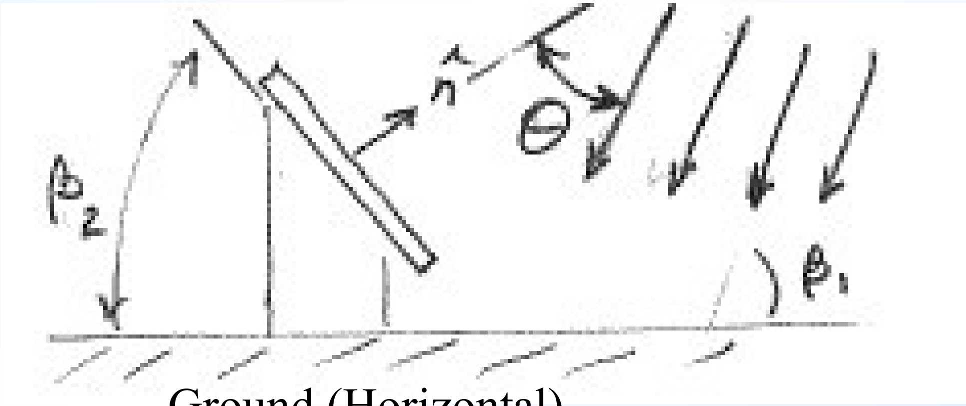


Or

$$\sin(\alpha_1) = \frac{\cos(\delta)\sin(H)}{\cos(\beta_1)}$$

Watch signs (quadrants) when solving for α_1

Tilt Angle of the Surface, β_2



Ground (Horizontal)

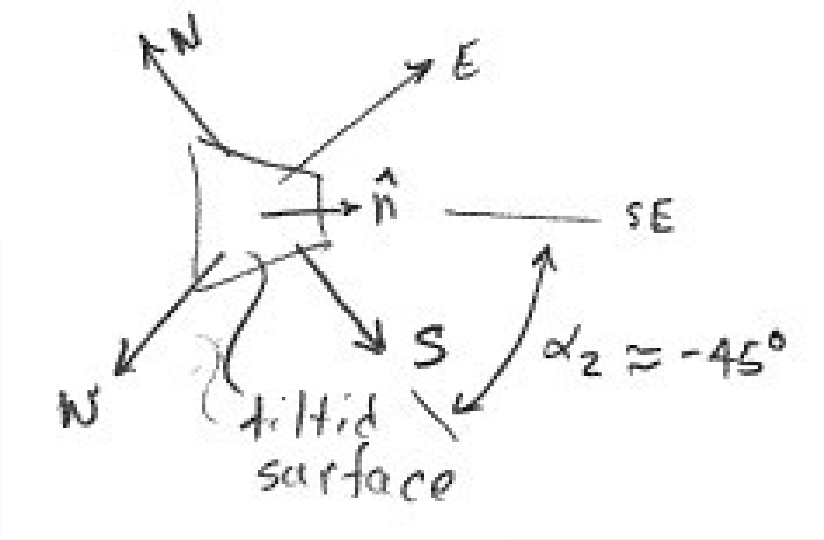
Horizontal surface: $\beta_2 = 0°$

$\theta = 90 - \beta_1$

Vertical surface: $\beta_2 = 90°$

$\theta = \beta_1$

Azimuth Angle of Surface Normal, α_2



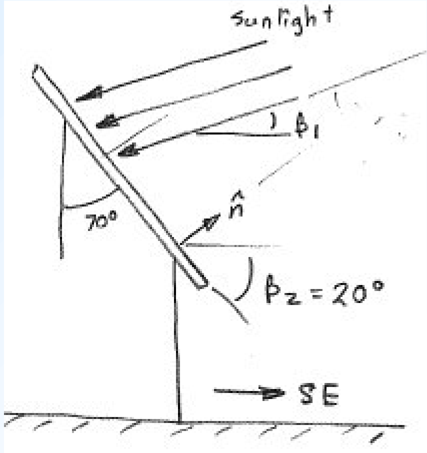
Measured from due South

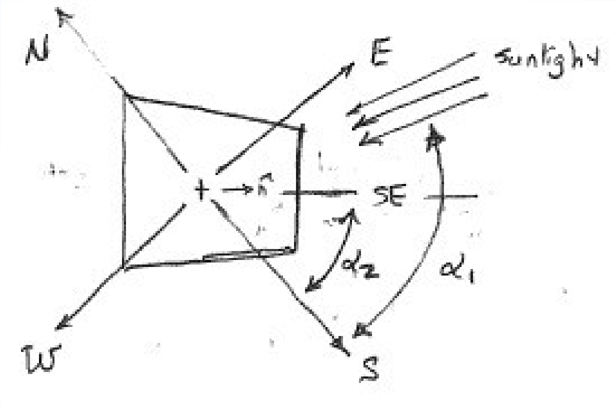
Example 2

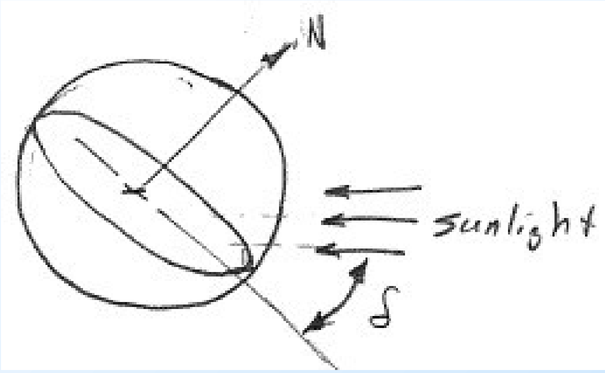
An old galvanized roof (ε = α = 0.9), inclined at 70° from vertical. 38° N latitude, 67° W longitude, 9 AM (local daylight savings time in effect) on July 7.

a) Calculate the incidence angle, θ

b) Determine the absorbed beam and diffuse-scattered solar insolation

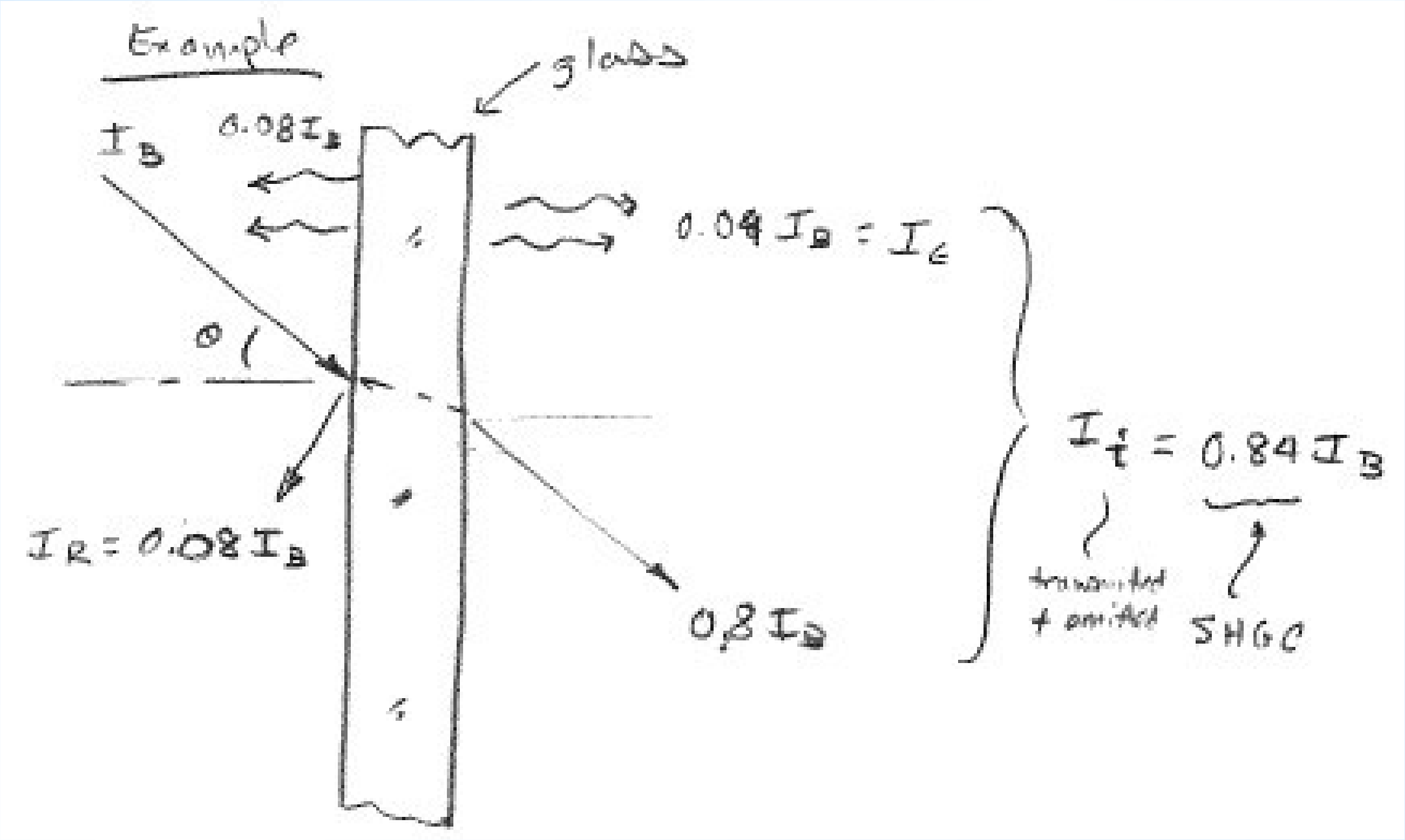




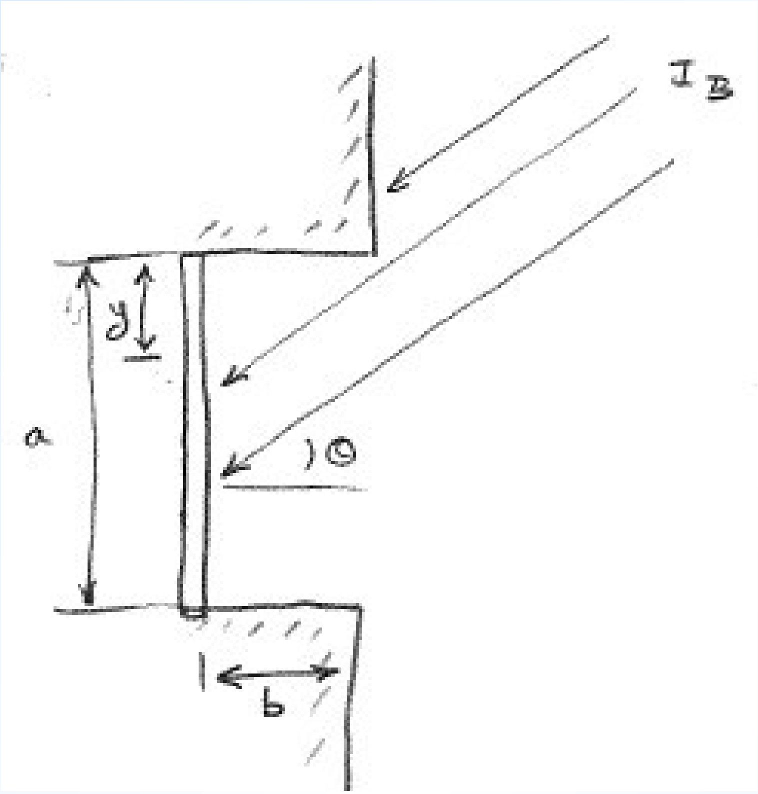


$$\cos\theta = (\sin\beta_1)\left(\cos\beta_2\right) + (\cos\beta_1)\left(\sin\beta_2\right)\left(\cos\delta\right)$$

Fenestrations (Openings in a wall)



Shading



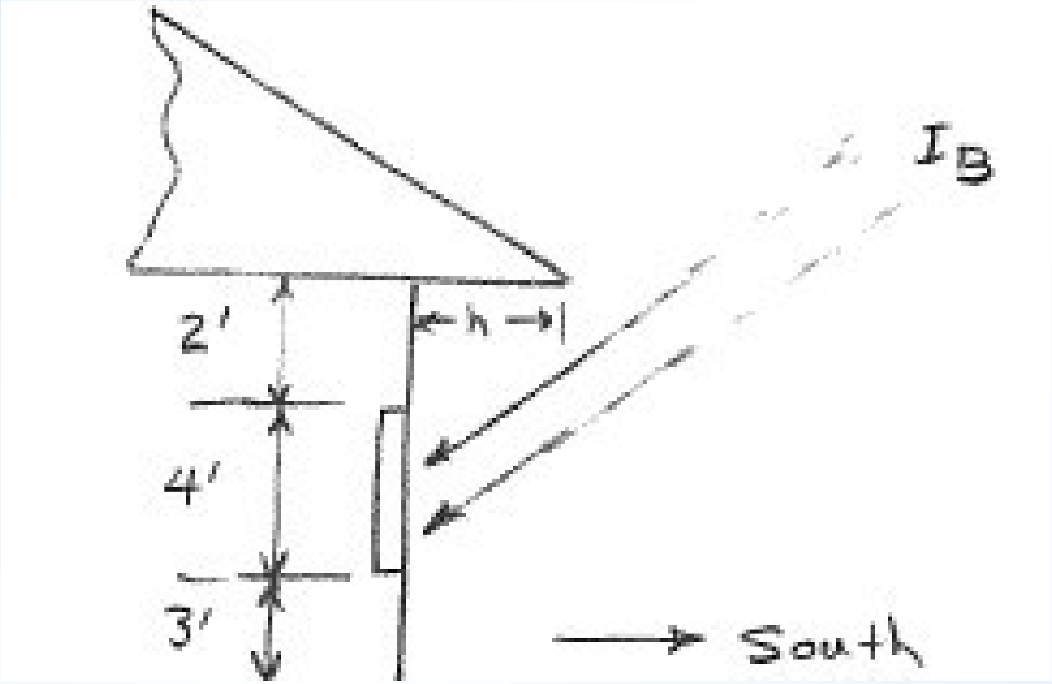
$$y = b\,\frac{\tan\beta_1}{\cos(\alpha_1 - \alpha_2)}$$

Vertical portion shaded

$$x = b\tan(\alpha_1 - \alpha_2)$$

Horizontal portion shaded

Example 3



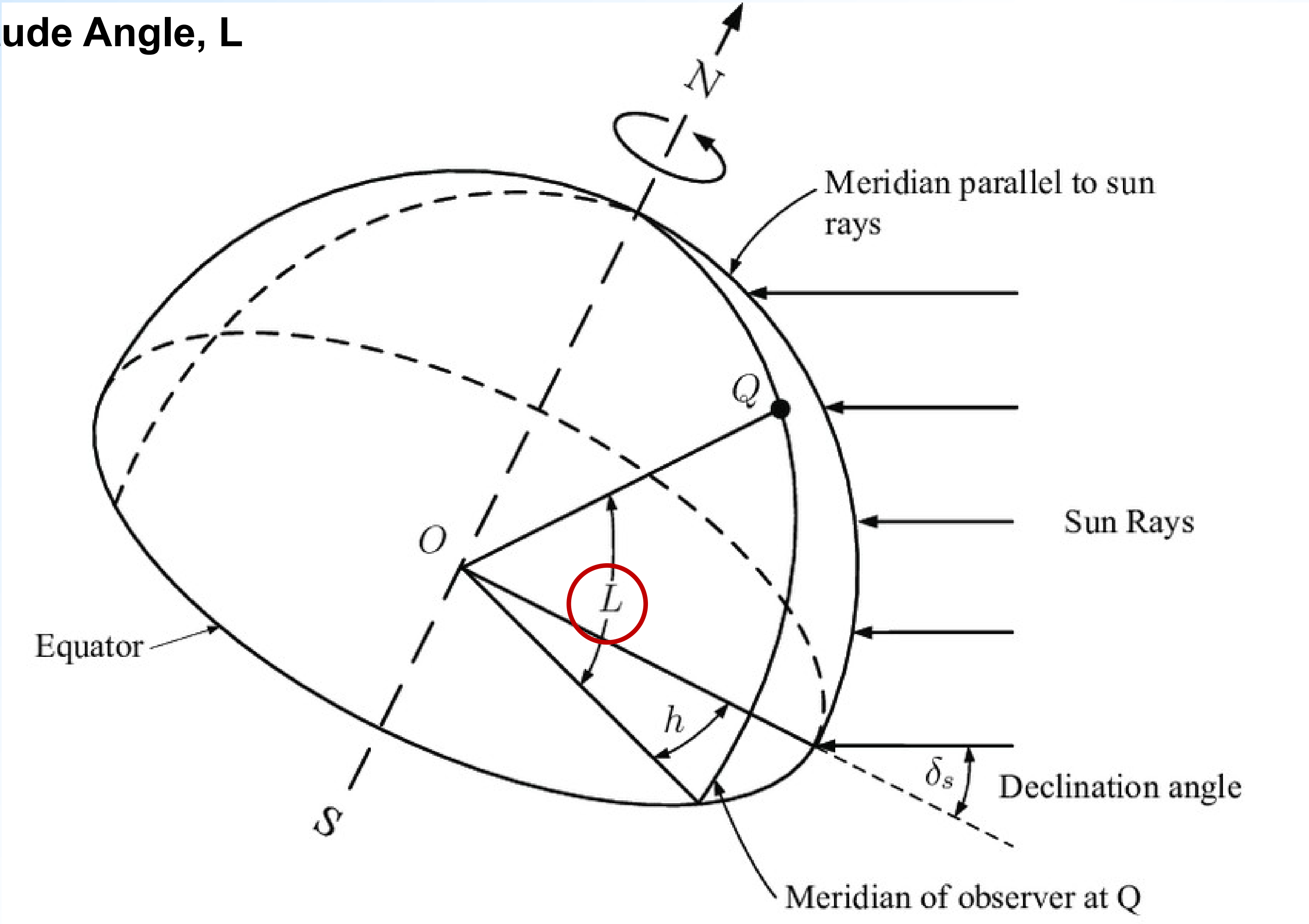
800 m above sea level

What should the overhang be (h = ?) so that the south-facing window is shaded at solar noon on June 21st. The house is located in Ankara at an elevation of 800 m above sea level.

SOLAR ANGLES

Latitude Angle, L - Location on Earth

Latitude Angle, L



SOLAR ANGLES

Latitude Angle, L - Location on Earth

Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

Tilt Angle of the Surface, β_2 or α

Ground (Horizontal)

Horizontal surface: $\beta_2 = 0°$

Vertical surface: $\beta_2 = 90°$

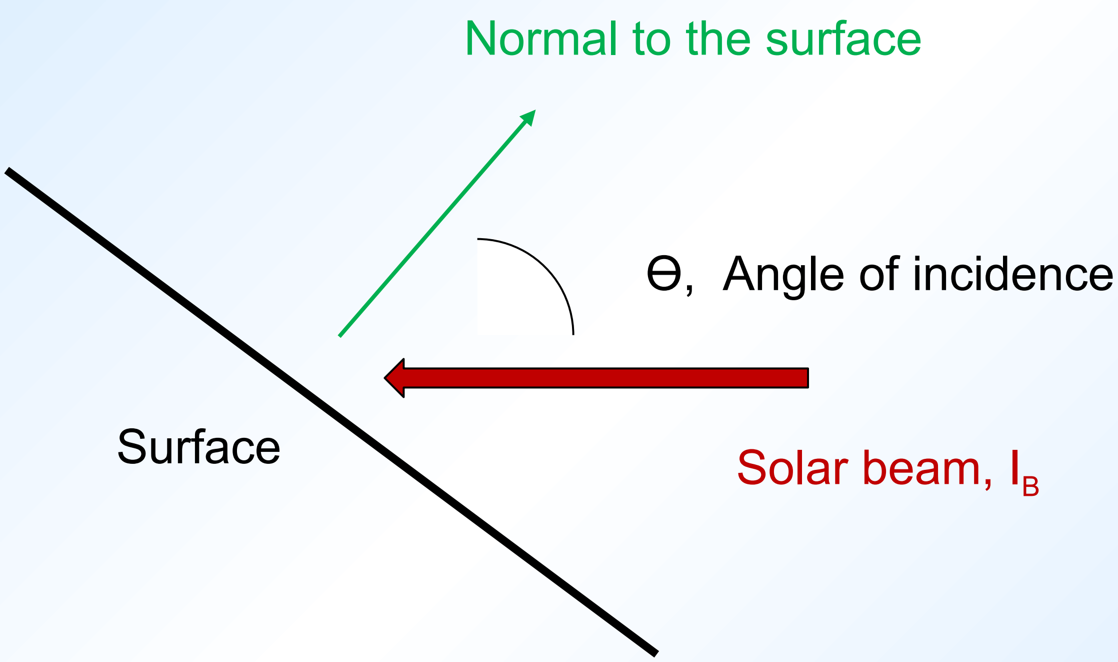
Θ: Angle of Incidence

SOLAR ANGLES

Latitude Angle, L - Location on Earth

Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

Angle of Incidence, θ



$$I_{BN} = I_B \cos(\theta)$$

SOLAR ANGLES

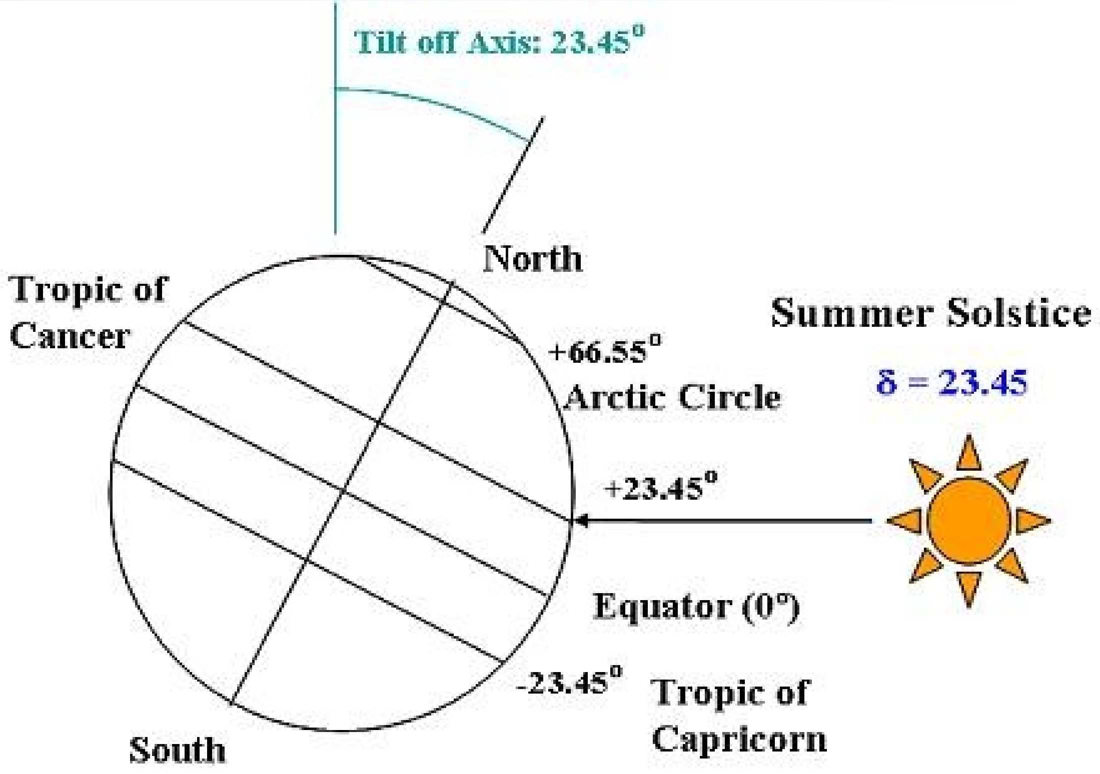
Latitude Angle, L - Location on Earth

Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

Angle of Incidence, θ

Declination angle, δ - Day of the year

Declination Angle $\delta = (23.45^\circ)\ \sin\left(\frac{360}{365}(284 + n)\right) \quad , \quad 1 \le n \le 365$



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−5	Mar 8, Oct 6
−10	Feb 23, Oct 20
−15	Feb 9, Nov 3
−20	Jan 21, Nov 22
−23.45	Dec 22

SOLAR ANGLES

Latitude Angle, L - Location on Earth

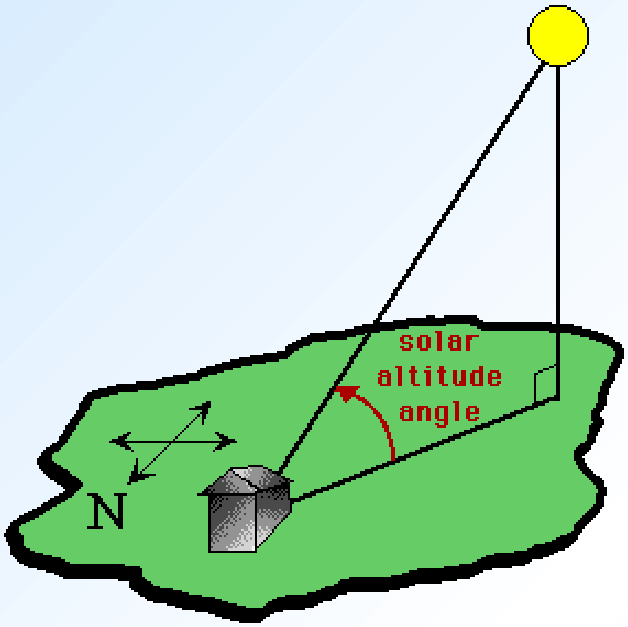
Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

Angle of Incidence, θ

Declination angle, δ - Day of the year

Altitude angle, β_1 - Time of the day

Altitude Angle, β_1



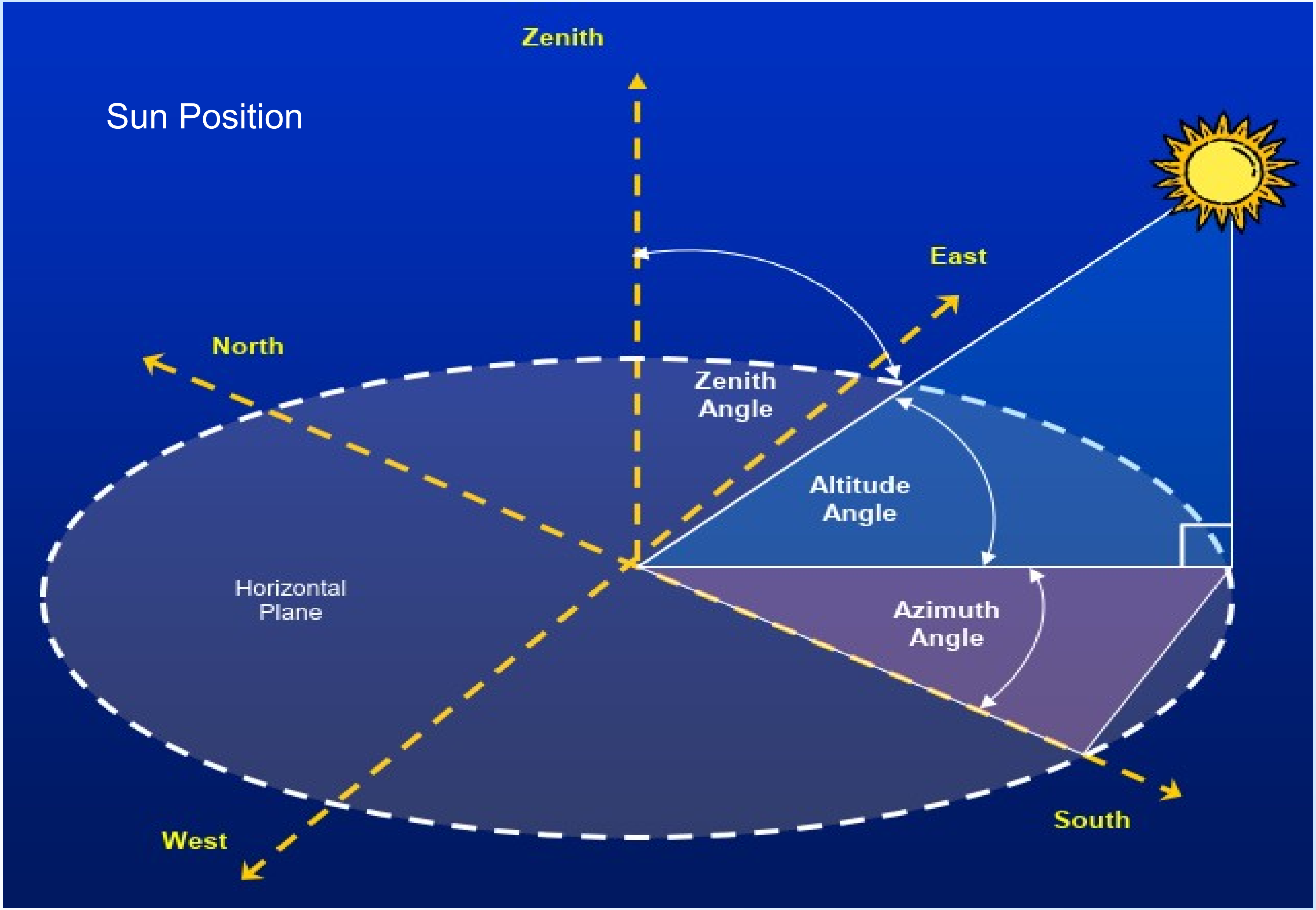
Ground (Horizontal surface)

$$\sin(\beta_1) = \cos(L)\cos(\delta)\cos(H) + \sin(L)\sin(\delta)$$

L = Latitude Angle

δ = Declination Angle

H = Hour Angle


Zenith
Sun Position
East
North
Zenith
Angle
Altitude
Angle
Horizontal
Plane
Azimuth
Angle
South
West

SOLAR ANGLES

Latitude Angle, L - Location on Earth

Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

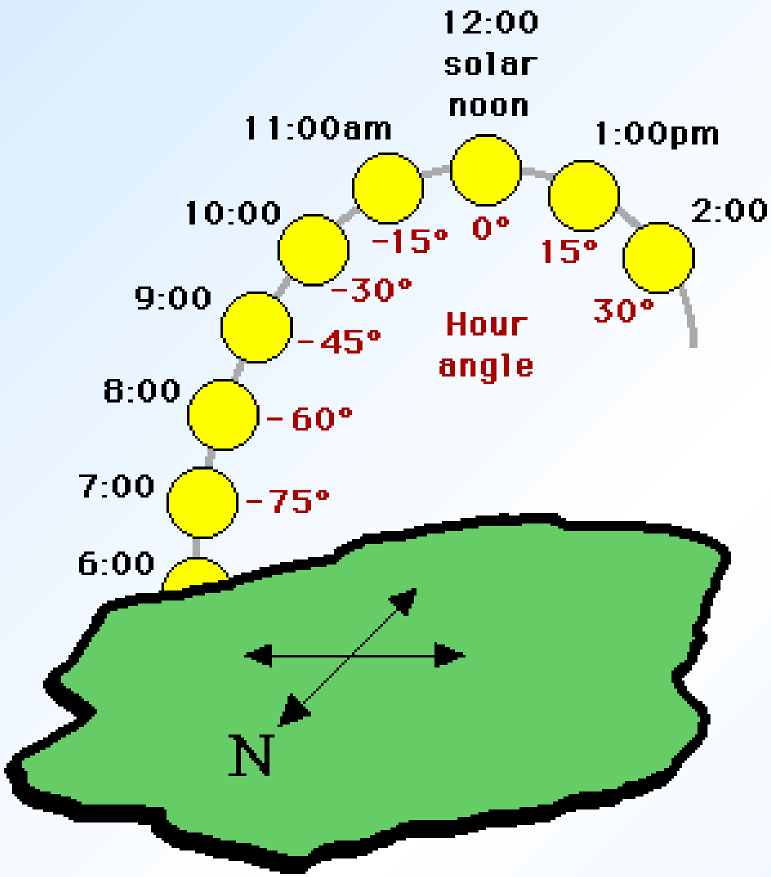
Angle of Incidence, θ

Declination angle, δ - Day of the year

Altitude angle, β_1 - Time of the day

Hour angle H - Time of the day (sunrise, sunset)

Hour Angle, H



H is measured from local solar time (AST) noon

Maximum at sunset (+ angle)

Minimum at sunrise (- angle)

SOLAR ANGLES

Latitude Angle, L - Location on Earth

Tilt Angle (tilted due South), β_2 or α - Orientation of the surface of interest

Angle of Incidence, θ

Declination angle, δ - Day of the year

Altitude angle, β_1 - Time of the day

Hour angle H - Time of the day (sunrise, sunset)

Local Solar Time (LST)

$$\text{LST} = \begin{pmatrix}\text{local}\\ \text{standard}\\ \text{time}\end{pmatrix} + (4)\left[\begin{pmatrix}\text{local}\\ \text{longitude}\end{pmatrix} - \begin{pmatrix}\text{standard}\\ \text{meridian}\\ \text{longitude}\end{pmatrix}\right] + \text{EoT}$$

$$\cos\theta = (\sin\beta_1)(\cos\beta_2) + (\cos\beta_1)(\sin\beta_2)(\cos\delta)$$

$$\begin{aligned} I_{total} &= I_{BN} + I_{DS} + I_R \\ &= I_B\cos\theta + I_B C_s F_{ss} + \rho_g (I_B + I_B C_s F_{ss}) F_{wg} \\ &= I_B (\cos\theta + C_s F_{ss} + \rho_g F_{wg} + C_s \rho_g F_{ss} F_{wg}) \end{aligned}$$

$$I_{total} = I_B \left\{\cos\theta + \frac{1}{2}\left[C_s(1 + \cos\beta_2) + \rho_g(1 - \cos\beta_2)\right] + \frac{1}{4} C_s \rho_g \sin^2\beta_2\right\}$$

Example 4

Consider a city with latitude angle 42.3° North (Boston) in mid February (day number: N = 45). The ground reflectance is 0.7 (snow more than 2.5 cm deep). Solar data for February are H = 8.61 MJ/m^2 and K_T = 0.435. H is daily integrated average insolation on a horizontal surface, and K_T is monthly clearness index which is an average value over a month. These parameters are derived from analysis of local insolation data over time using national data Web sites.

Calculate the expected daily insolation to fall on a south-facing solar collector panel having a tilt angle of 45°. Assume isotropic sky conditions.

Turkiye: 36 - 42 North Latitude

26 - 44 East Longtitude

Solution

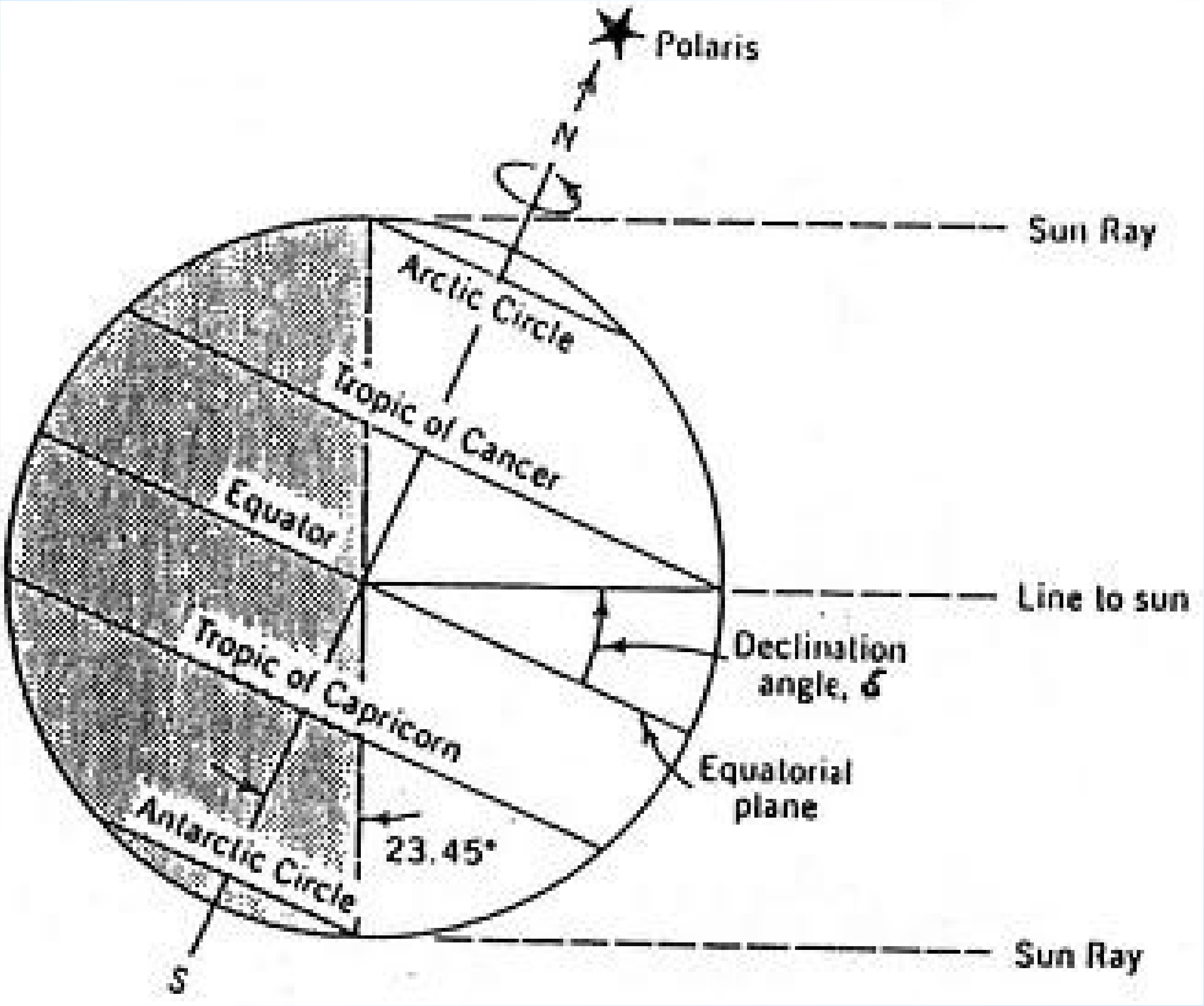
Declination angle: $\delta = 23.45 \sin\left(\dfrac{360\,(284 + 45)}{365}\right) = -13.6 \text{ deg} = -0.237 \text{ rad}$

Solar declination angle is between the solar rays and the equator plane of the earth, measured from the plane to the rays, positive to the north. N is the day number.

Sunset hour angle (for a south-facing or horizontal surface):

$w_s = \cos^{-1}\left(-\tan(L)\tan(\delta)\right)$ for a month between fall and spring equinoxes.

$w_s' = \text{Min}\left[w_s\ ,\ \cos^{-1}\left(-\tan(L - \beta)\tan(\delta)\right)\right]$ for a month between spring and fall equinoxes.


Polaris
N
Sun Ray
Arctic Circle
Tropic of Cancer
Equator
Line to sun
Declination angle, δ
Tropic of Capricorn
Equatorial plane
Antarctic Circle
23.45°
Sun Ray
S

$$w_s = \cos^{-1}\left(-\tan(L)\tan(\delta)\right) = 77.3 \text{ deg}$$

This is equivalent to 77.3/15 = 5 h and 9 min after solar noon.

$$w_s = w_s'$$

L is the local latitude, angle north and south of the equator, positive if north.

The ratio of average direct beam insolation on a tilted surface to that on a horizontal surface:

$$R_{b,\beta} = \frac{\cos(L-\beta)\cos(\delta)\sin(w_s') + w_s'\sin(L-\beta)\sin(\delta)}{\cos(L)\cos(\delta)\sin(w_s) + w_s\sin(L)\sin(\delta)} = 1.973$$

The ratio of average total insolation on a tilted surface to that on a horizontal surface:

$$R = \left(1 - \frac{H_d}{H}\right) R_{b,\beta} + \left(\frac{H_d}{H}\right)\left(\frac{1 + \cos(\beta)}{2}\right) + \frac{\rho\,(1 - \cos(\beta))}{2}$$

Direct | Diffuse | Ground Reflected

Diffuse component of the daily insolation for isentropic sky conditions:

$$\frac{H_d}{H} = 1.39 - 4.03\,K_T + 5.53\,K_T^2 - 3.11\,K_T^3$$

H is the daily insolation, H_d is its diffuse component, and K_T is the clearness index

R = 1.598 => 59.8 % more total insolation than on the horizontal surface

The average daily total insolation on the tilted surface: $H_T = H\ R$

$$H_T = (8.61)\ (1.598) = 13.5\ MJ/m^2$$

Components of H_T : Direct: (8.61) (1.130) = 9.73 MJ/m^2

Diffuse: (8.61) (0.365) = 3.14 MJ/m_2

Ground reflected: (8.61) (0.103) = 0.89 MJ/m^2

See OdtuClass:

E-Book: «Physics of Solar Energy» by J. Julina Chen

DETERMINATION OF THE DISTRIBUTION OF GLOBAL SOLAR RADIATION FOR TURKEY USING MSG SATELLITE DATA (In Turkish)

TÜBA-ENERJİ DEPOLAMA TEKNOLOJİLERİ RAPORU, 2020

TÜBA-GÜNEŞ ENERJİSİ TEKNOLOJİLERİ RAPORU, 2018

MEASUREMENT OF SOLAR RADIATION IN ANKARA, TURKEY, 2012