Second Law of Thermodynamics

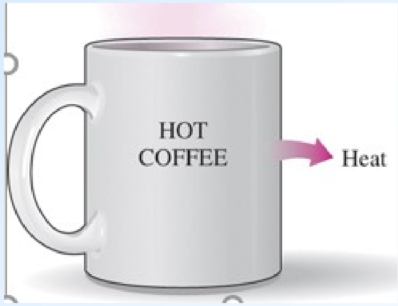
Energy has both quantity and quality. The first law is about quantity; the second low is about quality.

The first law says quantity is always conserved. The second law says the quality, i.e., the capability to do work, is reduced in all real processes.

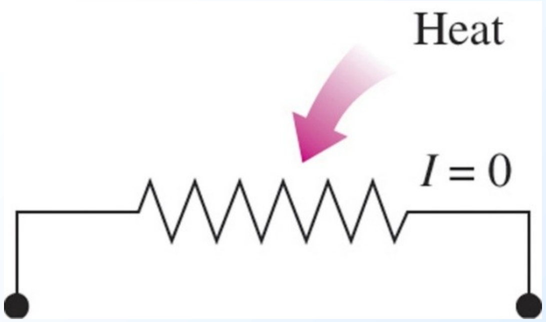
Do two bricks at a medium temperature ever go to a state where one is hot and the other is cold? The first law is not against that. The second law says it is impossible. Left to itself, energy always flows from high potential to low potential, and not the other way around.

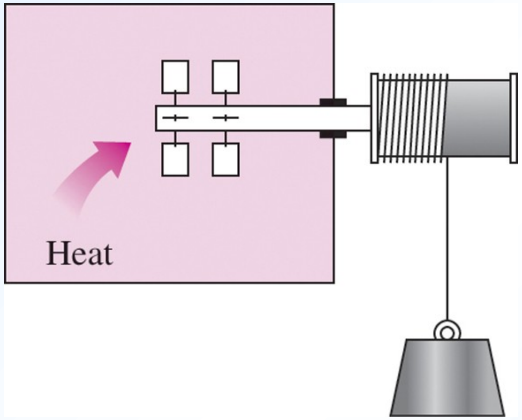
Like the first law, the second law is a generalization from an enormous amount of observations. There is no rigorous mathematical proof.

These processes cannot occur even though they are not in violation of the first law.



A cup of hot coffee does not get hotter in a cooler room.





Transferring heat to a paddle wheel will not cause it to rotate.

Transferring heat to a wire will not generate electricity.

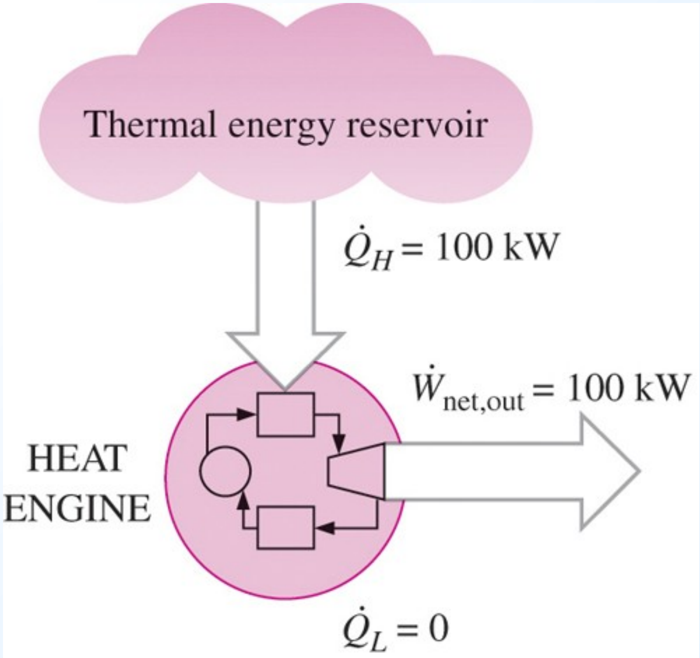
Second-Law Statements:

There are several ways in which the second law of thermodynamics can be stated. Listed below are three that are often encountered.

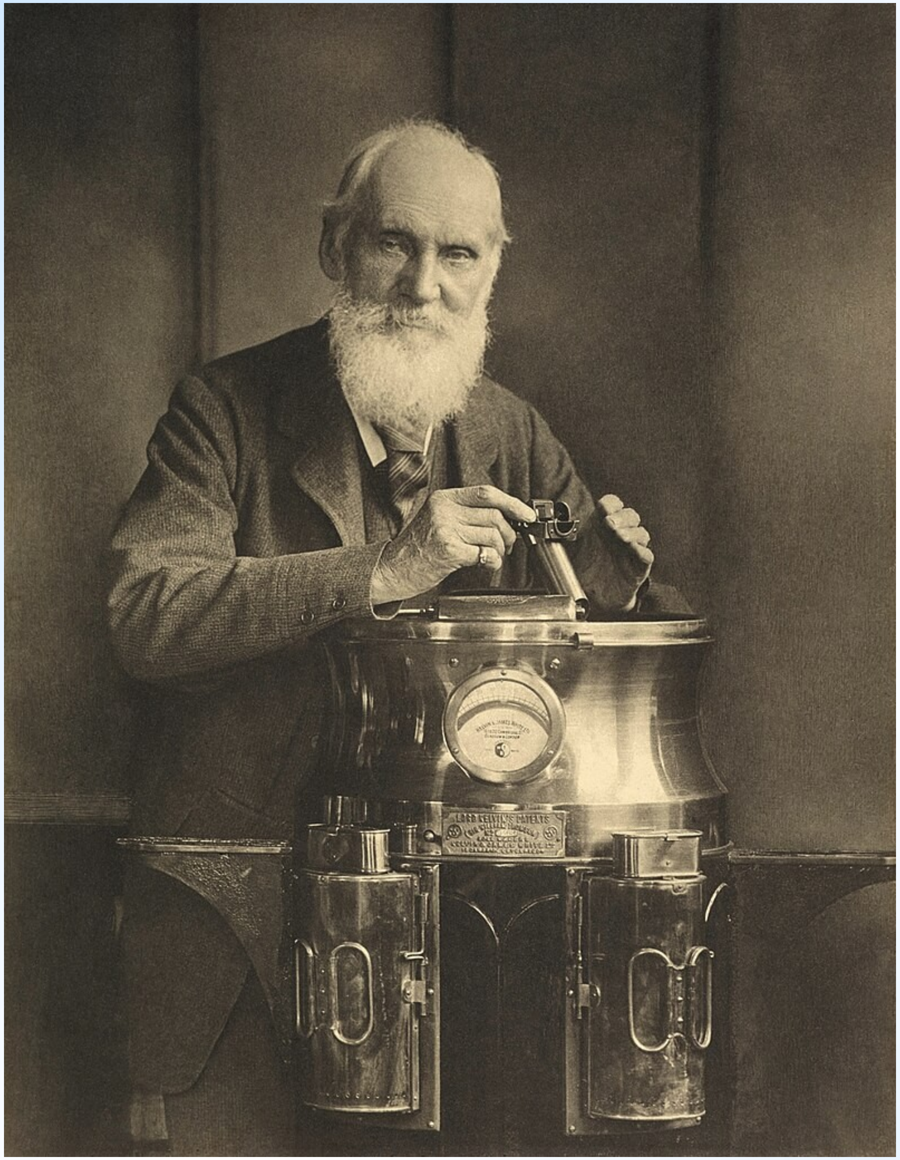
Altough the three may not appear to have much connection with each other, they are equivalent.

Kelvin-Planck statement:

No process is possible whose **sole** result is the absorption of heat from a reservoir and the conversion of this heat into work.



This is not possible



William Thomson, 1st Baron Kelvin

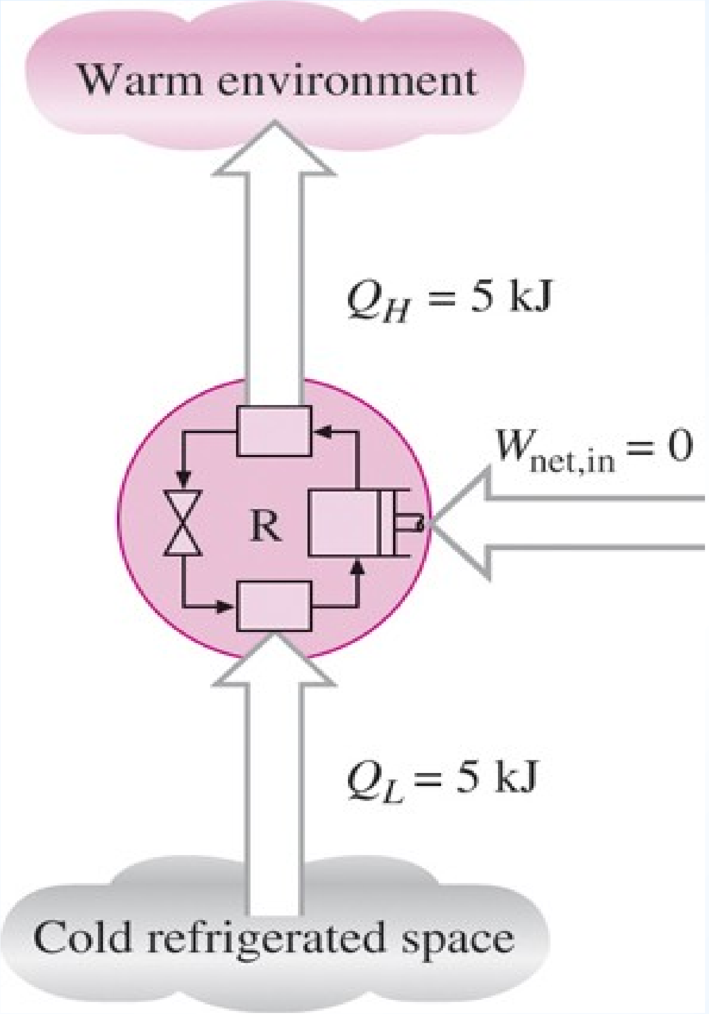
British Mathematician

1824 - 1907

Max Karl Ernst Ludwig Planck

German Physicist

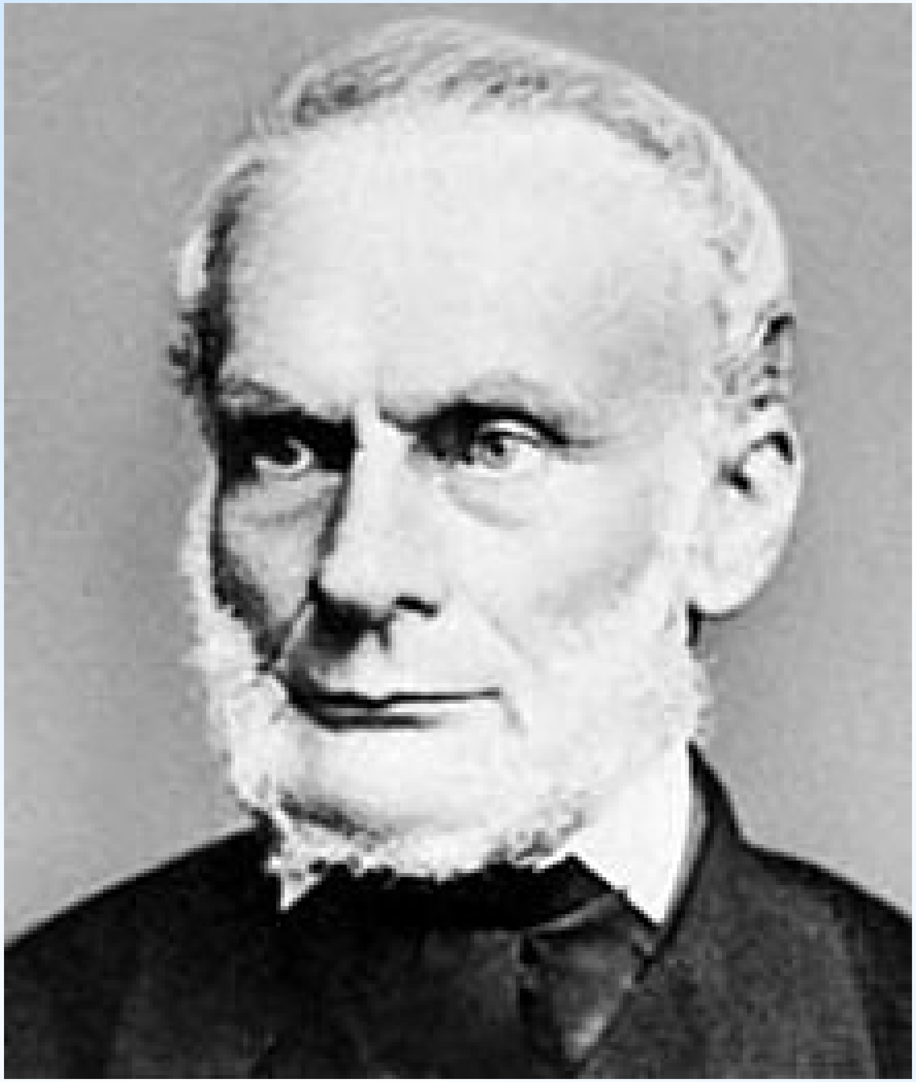
1858 - 1947



Clausius statement:

No process is possible whose **sole** result is the transfer of heat from a cooler to a hotter body.

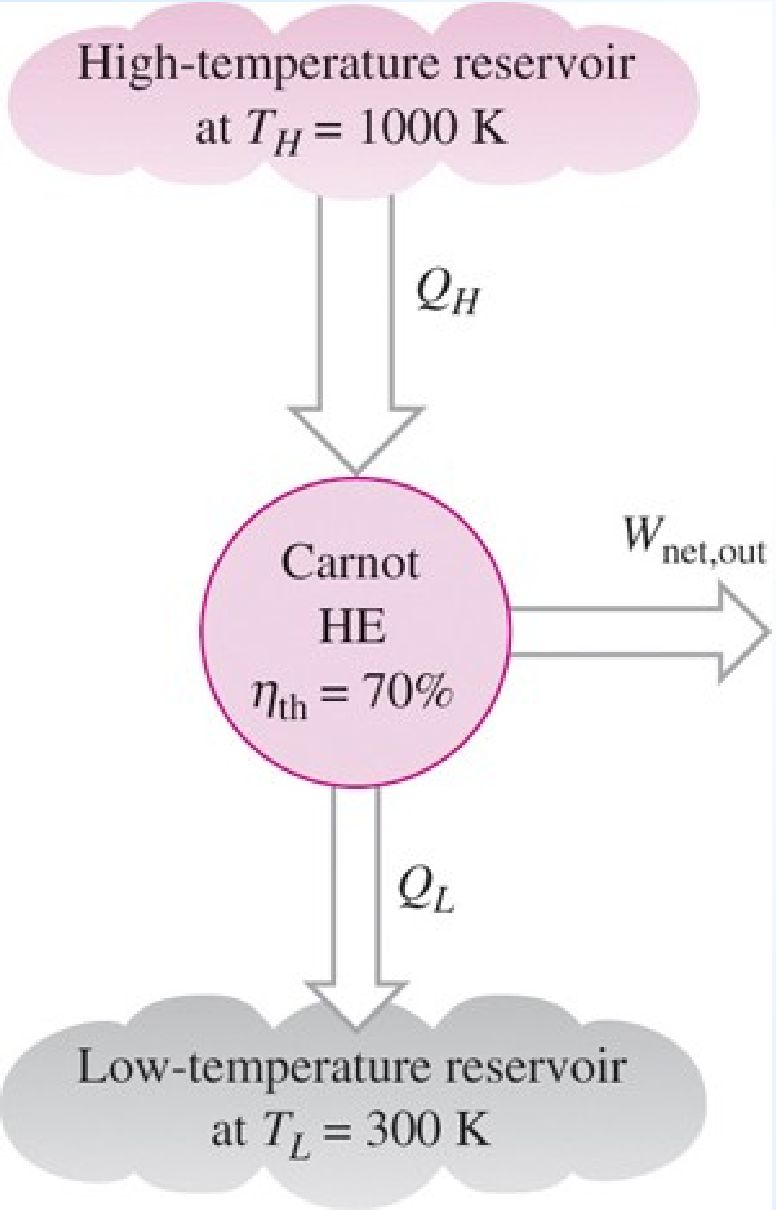
$T_1 > T_2$



Rudolf Julius Emanuel Clausius

German Physicist

1822 - 1888

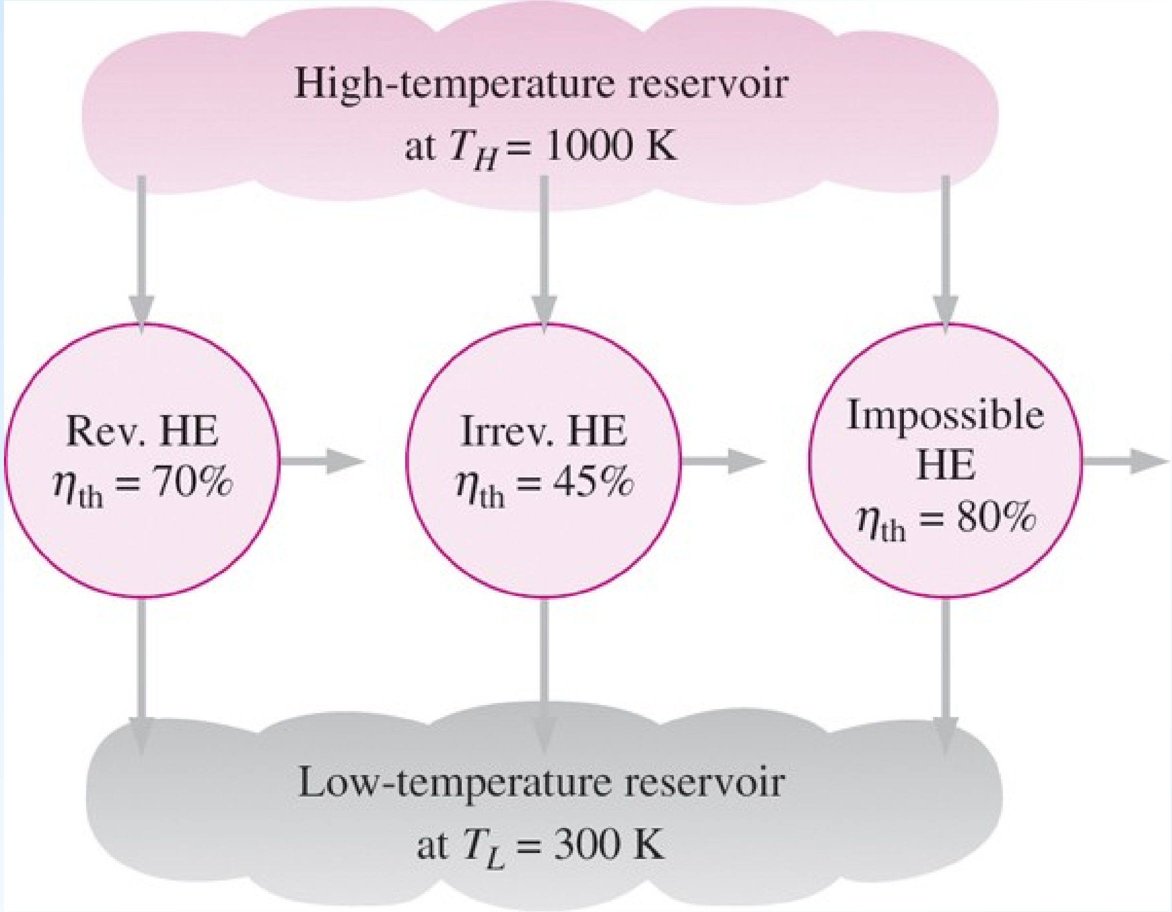


Any Heat Engine

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

Carnot Heat Engine

$$\eta_{th} = \eta_C = 1 - \frac{T_L}{T_H}$$



No heat engine can have a higher efficiency than a reversible heat engine operating between the same high- and low-temperature reservoirs.

$$\eta_{th} \begin{cases} < \eta_{th,rev} & \text{Irreversible} \\ = \eta_{th,rev} & \text{Reversible} \\ > \eta_{th,rev} & \text{Impossible} \end{cases}$$

Third statement of the Second Law:

There exists for every system in equilibrium a property called **entropy, S,** which is a thermodynamic property of a system.

For a reversible process, changes in this property are given by

$$dS = \frac{\partial Q_{reversible}}{T}$$

Entropy is a scientific concept as well as a measurable physical property of a system that is most commonly associated with a state of **disorder, randomness, or uncertainty**.

The term and the concept are used in diverse fields, from classical thermodynamics, where it was first recognized, to the microscopic description of nature in statistical physics, and to the principles of information theory. It has found far-ranging applications in chemistry and physics, in biological systems and their relation to life, in cosmology, economics, sociology, weather science, climate change, and information systems including the transmission of information in telecommunication.

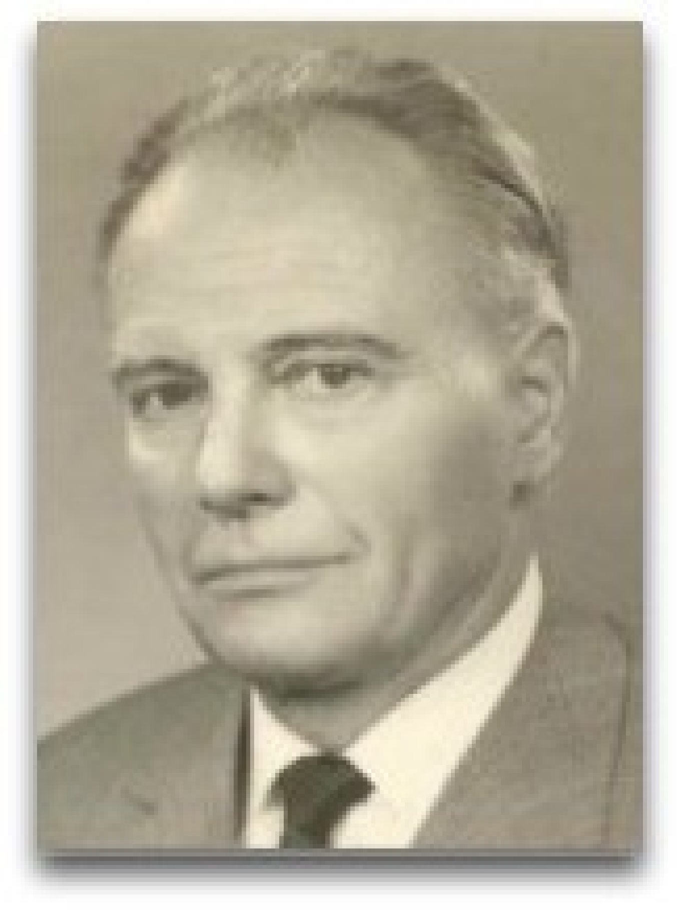
In 1865, the German physicist Clasius coined the term «entropy», replacing the root of the Greek word ἔργον ('ergon', 'work') by that of τροπή ('tropy', 'transformation').

Available Work (Availability – Exergy) & Second-Law Analysis

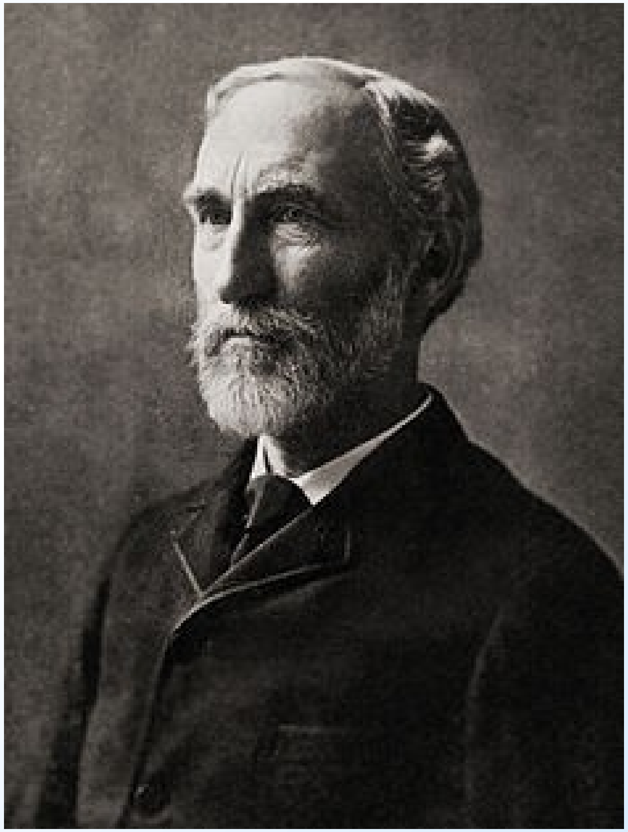
Available work or **exergy** is the portion of the total energy of a system that is available for conversion to useful work; in particular, the quantity of work that can be performed by a fluid relative to a reference condition, usually the surrounding ambient condition.

The term "exergy" was coined in 1956 by Zoran Rant (1904 – 1972) by using the Greek "*ex*" and "*ergon*" meaning "from work", but the concept was developed by J. Willard Gibbs in 1873.

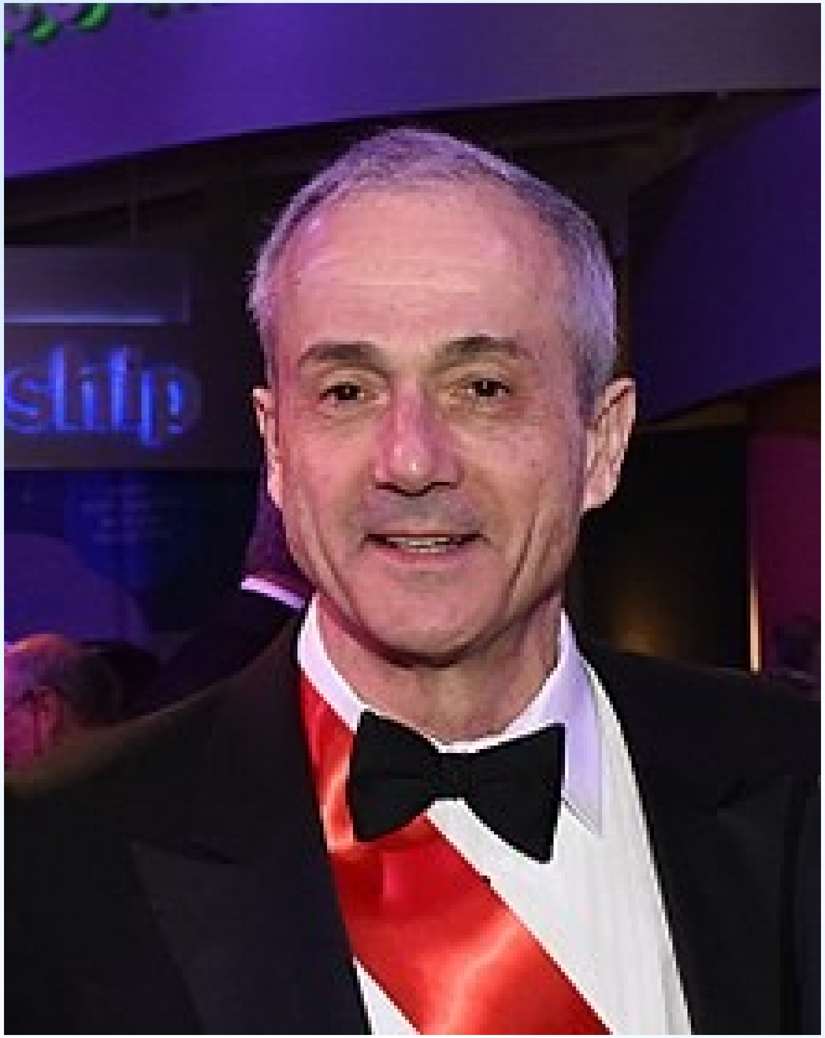
See OdtuClass for the journal article on exergy by Adrian Bejan.



Zoran Rant
Slovene mechanical engineer
1904 - 1972



Josiah Willard Gibbs
US physicist and chemist
1839 - 1903



Adrian Bejan

US Scientist

1948 -

Exergy is a combination property of a system and its environment because it depends on the state of both the system and its environment.

The exergy of a system in equilibrium with the environment is zero.

Exergy is neither a thermodynamic property of matter nor a thermodynamic potential of a system.

Exergy and energy both have units of Joules.

Exergy or second-law analysis of a system helps us to do the following:

- Determine the type, location, and magnitude of energy losses; and
- Find means to reduce losses to make the system more efficient.

Exergy

Exergy (available work or availability) is the theoretical limit for the work potential that can be obtained from a source or a system at a given state when interacting with a reference (environment) at a constant condition.

A system is said to be in the dead state when it is in thermodynamic equilibrium with its environment. Unless otherwise stated, assume the dead state to be:

$$P_0 = 1 \text{ atm} \quad \text{and} \quad T_0 = 25\ ^\circ\text{C}$$

A system delivers the maximum possible work as it undergoes a reversible process from the specified initial state to the state of its environment (dead state). This represents the *useful work potential*, or *exergy*, or *availability*.

Exergy is a property of the system-environment combination (not the system alone).

Irreversibilities:

- Friction
- Unstrained expansion – work required to restore state
- Heat transfer through ΔT (as opposed to dT) – cooling required to restore state which rejects heat to surroundings
- Mixing of two substances – work required to separate
- Joule heating (I & R)
- Combustion

External vs Internal Irreversibilities:

- Internal → No irreversibility occurs within boundary during process
- External → No irreversibility occurs outside boundary during process

Exergy of Kinetic Energy

Kinetic energy can be converted to work entirely; thus:

$$x_{KE} = KE = \frac{\forall^2}{2} \text{ kJ/kg}$$

where $\forall$ is the velocity of the system relative to the environment.

Exergy of Potential Energy

Potential energy is also a form of mechanical energy and can be converted to work entirely; thus:

$$x_{PE} = PE = g\,z \text{ kJ/kg}$$

Some Definitions

Surroundings Work: is the work done by or against the surroundings during a process. This work cannot be recovered and utilized. For a cylinder-piston assembly, one can write:

$$W_{surr} = P_0 \left(\forall_2 - \forall_1\right)$$

The difference between the actual work *W* and the surroundings work W_{surr} is called the useful work, W_u:

$$W_u = W_{act} - W_{surr} = W_{act} - P_0 \left(\forall_2 - \forall_1\right)$$

Reversible Work: W_{rev} is the maximum (ideal) amount of useful work that can be produced (or the min work needs to be supplied) as the system undergoes a process between the initial and final states. **When the final state is the dead state, the reversible work equals exergy (available work or availability).**

Irreversibility: “I” is equal to the exergy destroyed; thus for a reversible process, the irreversibility or exergy destruction is zero.

$$I = W_{rev,out} - W_{u,out} \quad \text{or} \quad I = W_{u,in} - W_{rev,in}$$

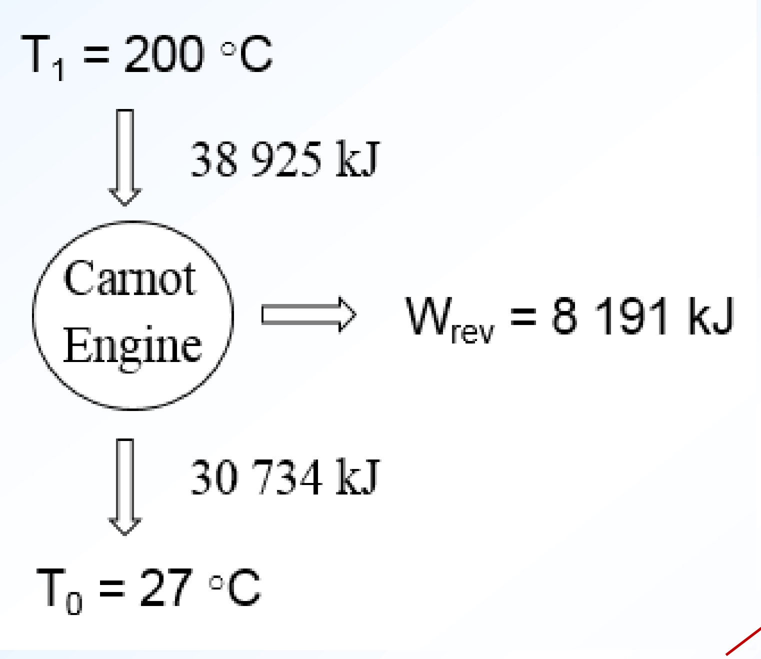
Example 1

A 500-kg iron block is initially at 200 °C and is allowed to cool to 27 °C by transferring heat to the surrounding air at 27 °C.

Determine the reversible work and the irreversibility for this process.

Assumptions:

1) the kinetic potential energies are negligible.
2) the process involves no work interactions.



$$W_{rev} = \int_{T_1}^{T_0} \underbrace{\left(1 - \frac{T_0}{T}\right)}_{\eta_{Carnot}} \delta Q_{in}$$

$$W_{rev} = \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right) \left(- m\, c_{av}\, dT\right)$$

$$\boxed{W_{rev} = m\, c_{av}\, (T_1 - T_0) - \underbrace{m\, c_{av}\, T_0 \ln\left(\frac{T_1}{T_0}\right)}_{T_0\,\Delta S}}$$

$$= 8191 \text{ kJ}$$

Same as Exergy, X, if T_0 is the dead state

$$\left.\boxed{\Delta S = m\, c_{av} \ln\left(\frac{T_1}{T_0}\right)}\quad\right\}$$ For solids and liquids

The Second Law Efficiency

The second law efficiency, η_{II}, is the ratio of actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions:

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}$$

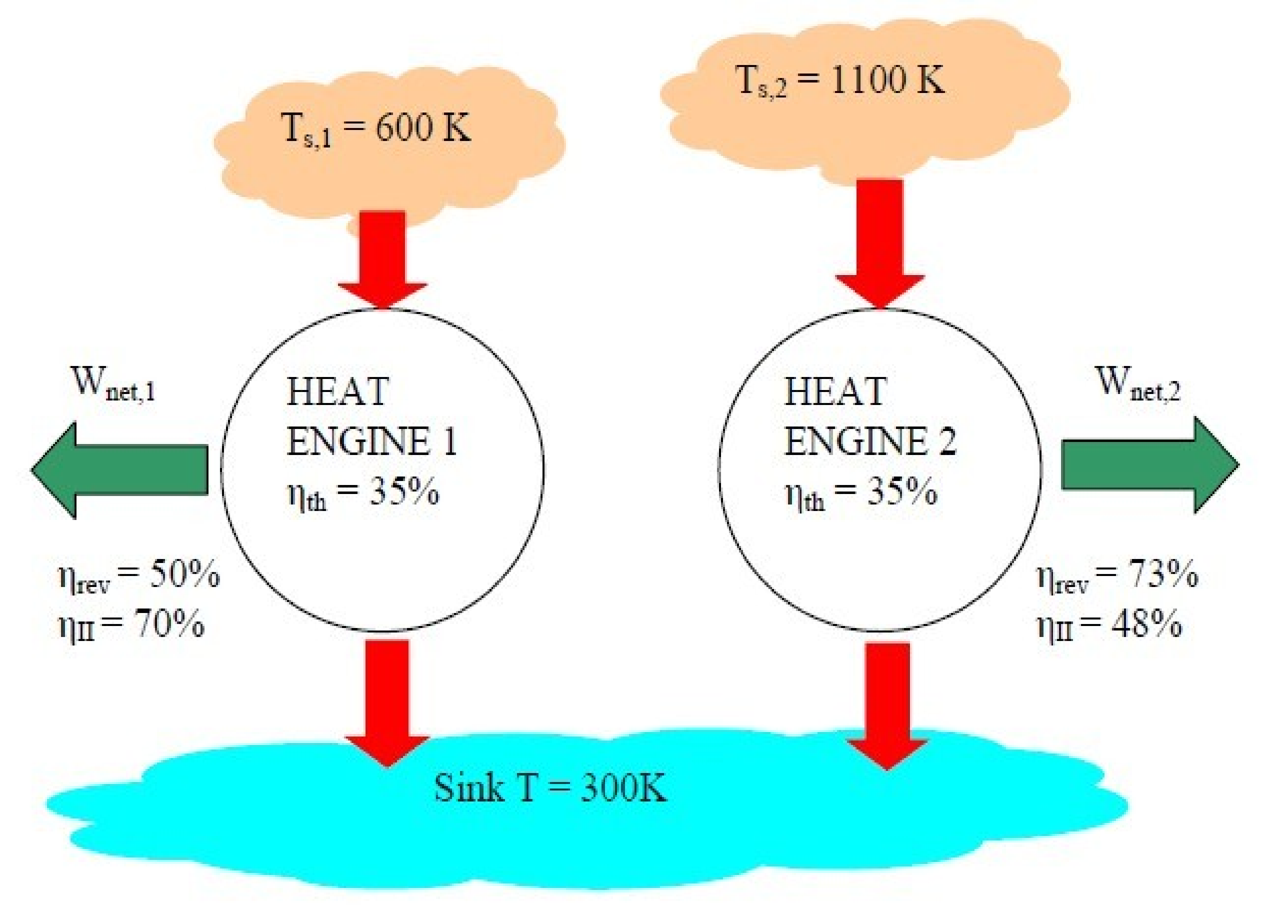
$$\eta_{II} = \frac{\eta_{th}}{\eta_{th,C}}$$ heat engines

$$\eta_{II} = \frac{W_u}{W_{rev}}$$ work-producing devices

$$\eta_{II} = \frac{W_{rev}}{W_u}$$ work-consuming devices

$$\eta_{II} = \frac{COP}{COP_{rev}}$$ refrigerators and heat pumps

The second law efficiency serves as a measure of approximation to reversible operation; thus its value should range from 0 to 1.



$$\eta_{rev} = 1 - \frac{300}{600} = 0.50$$

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th,rev}} = \frac{0.35}{0.50} = 0.70$$

$$\eta_{rev} = 1 - \frac{300}{1100} = 0.73$$

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th,rev}} = \frac{0.35}{0.73} = 0.48$$

Example 2

A power plant receives Q_{H1} = 25 kW at T_{H1} = 825 °C. The plant also receives Q_{H2} = 50 kW at T_{H2} = 240 °C. The plant rejects heat to the environment at T_0 = 20 °C and produces power W_{out} = 12 kW. Determine

a) The first-law (thermal) efficiency; and

b) The second-law efficiency of the plant.

$$\eta_{th} = \frac{\dot{W}_{out}}{\dot{Q}_{H1} + \dot{Q}_{H2}} = \frac{12}{25 + 50} = 0.16$$

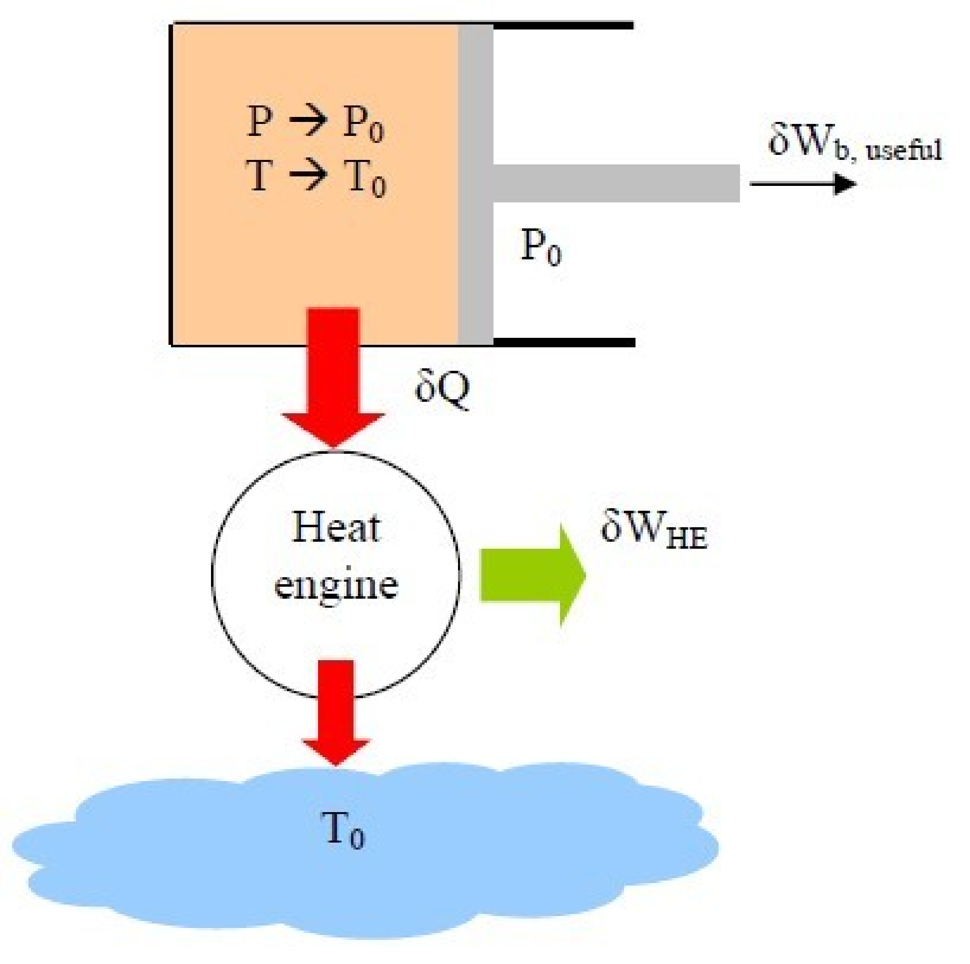
$$\eta_{II} = \frac{\dot{W}_{out}}{\dot{W}_{rev}}$$

$$\eta_{II} = \frac{\dot{W}_{out}}{\dot{X}_{H1} + \dot{X}_{H2}}$$

$$\dot{X}_{Q_{H1}} = \dot{Q}_{H1}\left(1 - \frac{T_0}{T_{H1}}\right) \quad , \quad \dot{X}_{Q_{H2}} = \dot{Q}_{H2}\left(1 - \frac{T_0}{T_{H2}}\right)$$

$$\eta_{II} = 0.302$$

Exergy of a fixed mass



Consider a stationary cylinder-piston assembly that contains a fluid of mass *m* at *T, P, U,* and *S*. The system is allowed to undergo a differential change.

The energy balance for the system during this differential process can be expressed as:

$$-\delta Q - \delta W = dU$$

$$\delta W = -dU - \delta Q$$

The system involves some boundary work: $\delta W = - dU - \delta Q$

$$\delta W = P\, d\forall = \left(P - P_0\right) d\forall + P_0\, d\forall = \delta W_{b,useful} + P_0\, d\forall$$

For a system, which is at temperature *T*, to have a reversible heat transfer with the surroundings at T_0, heat transfer must occur through a reversible heat engine ($\eta_{th} = 1 - T_L/T_H$). For the reversible heat engine, one can write:

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right) \delta Q = \delta Q - \delta Q\, \frac{T_0}{T}$$

For a reversible process we have: $dS = \dfrac{\delta Q}{T}$ thus

$$\delta W_{HE} = \delta Q - \left(- T_0\, dS\right) \qquad \delta Q = \delta W_{HE} - T_0\, dS$$

$$\delta Q = P_0\, d\forall - T_0\, dS$$

Using the above equations one can find

$$\delta W_{total,useful} = \delta W_{HE} + \delta W_{b,useful} = - dU - P_0 \, d\forall + T_0 \, dS$$

Integrating from initial state to final state gives

$$W_{total,useful} = (U - U_0) + P_0 \, (\forall - \forall_0) - T_0 \, (S - S_0)$$

where $W_{total,useful}$ is the total useful work delivered as the system undergoes a reversible process from the given state to the dead state which is the exergy of the system.

Total exergy for a closed system including the potential and kinetic energies can be written as:

$$\boxed{X = (U - U_0) + P_0 \, (\forall - \forall_0) - T_0 \, (S - S_0) + m \frac{V^2}{2} + m \, g \, z}$$

Exergy of a Flow System

A flowing fluid has flow energy; that is the energy needed to maintain flow in a pipe or line;

$$W_{flow} = P \forall$$

The flow work is the boundary work done by a fluid on the fluid downstream. The exergy associated with flow energy can be written as

$$X_{flow} = (P - P_0) \forall$$

Total exergy for an open system including the potential and kinetic energies can therefore be written as:

$$\boxed{X = (H - H_0) - T_0 (S - S_0) + m \frac{V^2}{2} + m g z}$$

Exergy by Heat Transfer

Heat is a form of disorganized energy, thus only a portion of it can be converted to work. Heat transfer *Q* at a location at thermodynamic temperature *T* is accompanied by exergy transfer:

$$\boxed{X_{heat} = \left(1 - \frac{T_0}{T}\right) Q}$$

Environment

Exergy Transfer by Work

Environment

$$X_{work} = \begin{cases} W - W_{surr} = W - P_0\left(\forall_2 - \forall_1\right) & \text{for boundary work} \\ W & \text{for other forms of work} \end{cases}$$

Therefore the exergy transfer with work such as shaft work and electrical work is equal to the work *W*.

Note that the work done or against atmosphere is not available for any useful purpose, and should be excluded from available work.

Example 3

A coal-fired furnace is used in a power plant. It delivers 5000 kW at 1000 K. The environment is at 300 K. What is the exergy of the added heat? You can use two steps to solve this problem.

(a) Determine the maximum percentage of the heat that can be converted to work.

(b) Using your answer from the first part, determine the maximum work possible.

$$\eta_C = 1 - \frac{T_0}{T} = 1 - \frac{300\ \text{K}}{1000\ \text{K}} = 0.7 \quad \text{or} \quad 70\ \%$$

$$W_{rev,u} = \text{Exergy} = (5000)\,(0.7) = 3500\ \text{kW}$$ Maximum possible ideal useful work

Example 4

A freezer is maintained at - 6 °C by removing heat from it at a rate of 75 kJ/min. The power input to the freezer is 0.70 hp (1 hp = 735 W), and the surrounding air is at 25 °C. Determine

a) The reversible power;

b) The irreversibility; and

c) The second-law efficiency of this freezer.

T_L = - 6 °C = 267 K $\quad$ T_H = 25 °C = 298 K

Heat removal rate: $\dot{Q}_{in}$ = - 75 kJ/min = - 1.25 kW

Input power: $\dot{W}_{u,in}$ = (0.7 hp) (735 W/hp) = 514.5 W

Reversible power:

$$\dot{W}_{rev,out} = (\eta_{th,rev})\,(\dot{Q}_{in}) = \left(1 - \frac{T_L}{T_H}\right)(\dot{Q}_{in}) = \left(1 - \frac{267}{298}\right)(-\,1.25) = -\,130\ \text{W}$$

$$\dot{W}_{rev,in} = 130\ \text{W}$$

It means that, if the process is reversible, we need to put in only 130 W. For the real process, which is irreversible, the same job is done with 514.5 W instead of only 130 W.

Irreversibility: $I = \dot{W}_{u,in} - \dot{W}_{rev,in} = 514.5 - 130\ \text{W} = 384.5\ \text{W}$

Note that irreversibility for a real process is always positive. It is zero when the process is reversible.

Second-law efficiency of the freezer:

$$\eta_{II} = \frac{\dot{W}_{rev,in}}{\dot{W}_{u,in}} = \frac{130}{514.5} = 0.253 \quad \text{or} \quad 25.3\ \%$$

The Decrease of Exergy Principle

Entropy and exergy for an **isolated** and closed system (no transfer of mass and energy) can be related through an energy balance and entropy balance as follows:

Energy balance: $E_{in} - E_{out} = \Delta E \quad \rightarrow \quad 0 = E_2 - E_1$

Entropy balance: $S_{in} - S_{out} + S_{gen} = \Delta S_{system} \quad \rightarrow \quad S_{gen} = S_2 - S_1$

Multiplying the second equation by T_0 and subtracting it from the first equation

$$- T_0 \, S_{gen} = E_2 - E_1 - T_0 \, (S_2 - S_1)$$

For a closed system we know that

$$X_2 - X_1 = \underbrace{\left(E_2 - E_1\right) + P_0 \, (\forall_2 - \forall_1)}_{= 0 \text{ for an isolated system}} - T_0 \, \underbrace{(S_2 - S_1)}_{= S_{gen}}$$

Combine the last two equations: $-T_0 S_{gen} = X_2 - X_1 \leq 0$

Since T_0 is a positive value: $\Delta X_{isolated} = (X_2 - X_1)_{isolated} \leq 0$

The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant.

Irreversibilities such as friction, mixing, chemical reaction, heat transfer, unrestrained expansion always generate entropy (or destroy exergy).

$$\boxed{X_{destroyed} = T_0 S_{gen} \geq 0}$$

Exergy destruction cannot be negative. The decrease of exergy principle can also be expressed as:

$$\boxed{X_{destroyed} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}}$$

Exergy Balance: Closed System

$$X_{in} - X_{out} - \underbrace{X_{destroyed}}_{= T_0 \, S_{gen}} = \Delta X_{system}$$

For a closed system that does not involve any mass flow, the exergy balance can be written as:

$$\boxed{\sum_k \left(1 - \frac{T_0}{T_k}\right) Q_k - \left[W - P_0 \left(\forall_2 - \forall_1\right)\right] - T_0 \, S_{gen} = X_2 - X_1}$$

or, in the rate form:

$$\boxed{\sum_k \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left[\dot{W} - P_0 \frac{d\forall_{system}}{dt}\right] - T_0 \, \dot{S}_{gen} = \frac{dX_{sytem}}{dt}}$$

Exergy Balance: Control Volumes

$$X_{heat} - X_{work} - X_{mass,in} - X_{mass,out} - X_{destroyed} = (X_2 - X_1)_{CV}$$

Or

$$\boxed{\sum_k \left(1 - \frac{T_0}{T_k}\right) Q_k - \left[W - P_0(\forall_2 - \forall_1)\right] + \sum_{fluid,in} X - \sum_{fluid,out} X - X_{destroyed} = (X_2 - X_1)_{CV}}$$

In the rate form:

$$\sum_k \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left[\dot{W} - P_0 \frac{d\forall_{CV}}{dt}\right] + \sum_{fluid,in} \dot{m}x - \sum_{fluid,out} \dot{m}x - \dot{X}_{destroyed} = \frac{dX_{CV}}{dt}$$

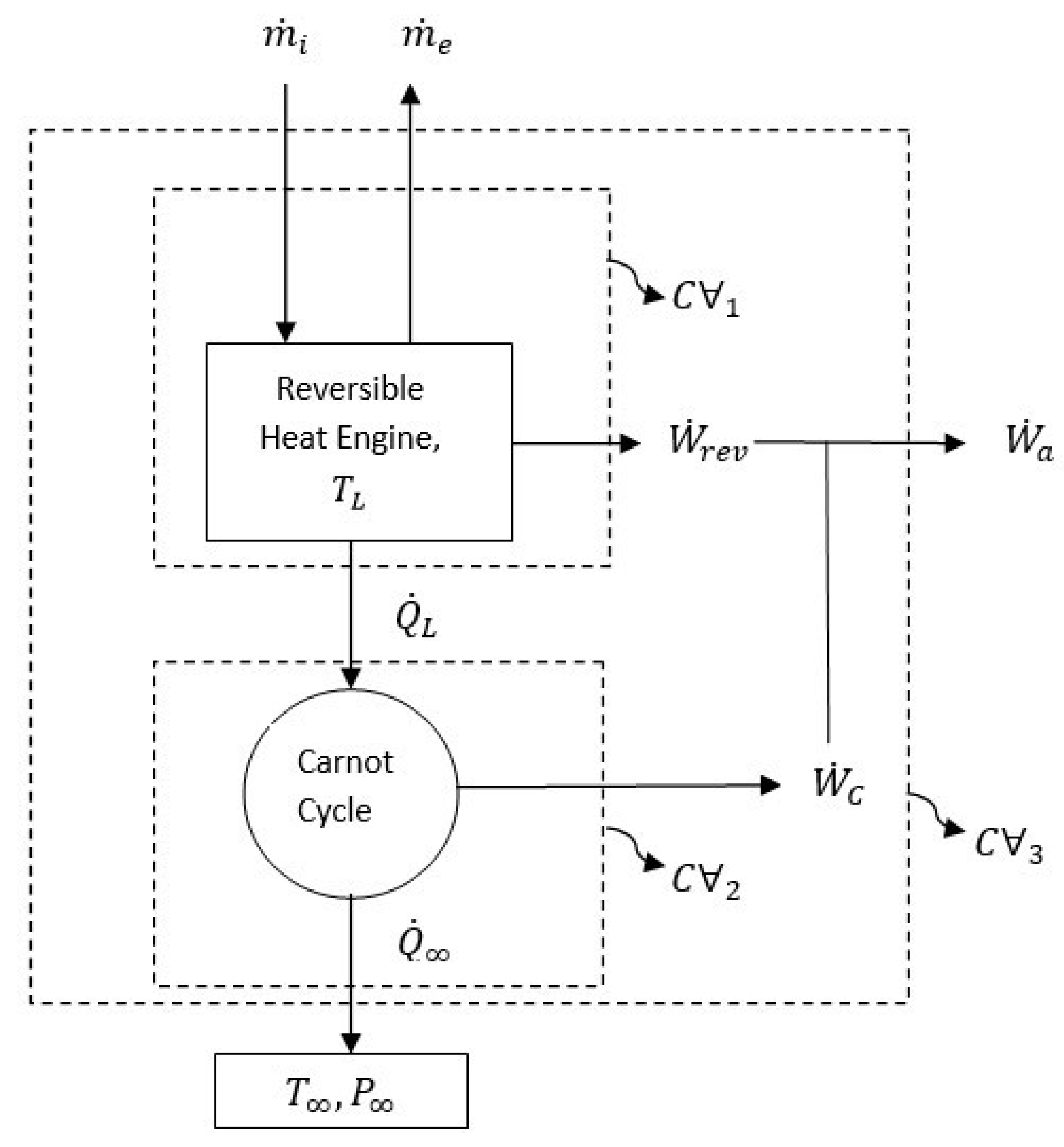
where the initial and final states of the control volume are specified , the exergy change of the control volume is:

$$X_2 - X_1 = m_2\, x_2 - x_1\, m_1$$

Steady-flow devices such as: turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts do not experience any changes in their mass, energy, entropy, and exergy content as well as their volumes. Therefore:

$$\frac{d\forall_{CV}}{dt} = 0 \quad \text{and} \quad \frac{dX_{CV}}{dt} = 0$$

$$\boxed{\sum_k \left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k - \dot{W} + \sum_{\text{fluid,in}} \dot{m}x - \sum_{\text{fluid,out}} \dot{m}x - \dot{X}_{\text{destroyed}} = 0}$$



$$\dot{W}_a = \dot{W}_{rev} + \dot{W}_C$$

Available work depends upon the initial and final thermodynamic states and the degree of non-equilibrium with the surroundings; NOT path dependent.

CV 1 Steady; Uniform flow; Potential and kinetic energy changes are negligible

1st Law: $\dot{Q} - \dot{W} = \dot{m}\left(h_e - h_i\right)$

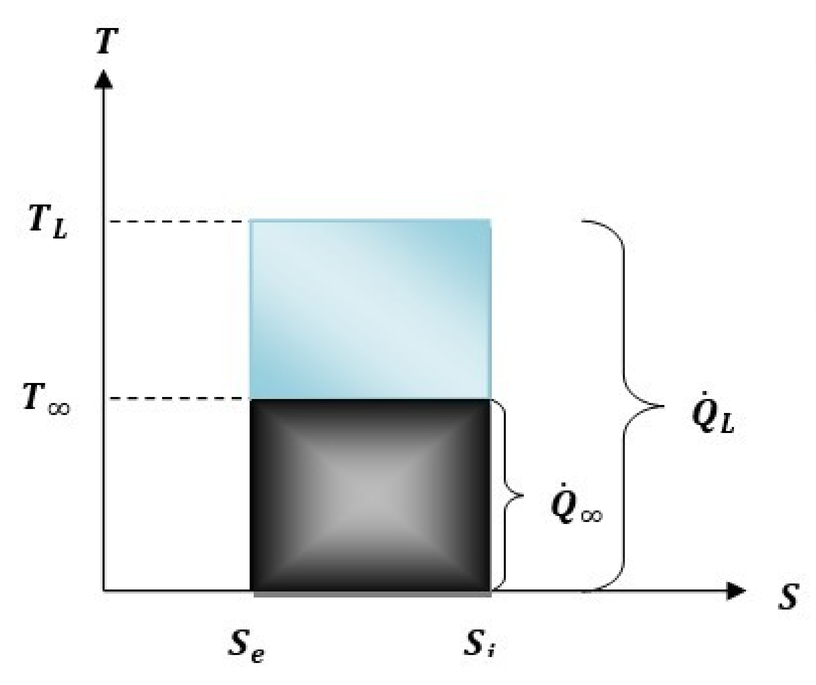
$$\dot{W}_{rev} = \dot{m}\left(h_e - h_i\right) - \dot{Q}_L$$

→ Reversible heat transfer out of CV_1

CV 2 $\dot{W}_C = \dot{Q}_L - \dot{Q}_\infty$ Cycle

Add two works:

$$\begin{aligned}\dot{W}_a &= \dot{W}_{rev} + \dot{W}_C \\ &= \left[\dot{m}\left(h_e - h_i\right) - \dot{Q}_L\right] + \left[\dot{Q}_L - \dot{Q}_\infty\right] \\ &= \dot{m}\left(h_e - h_i\right) - \dot{Q}_\infty\end{aligned}$$



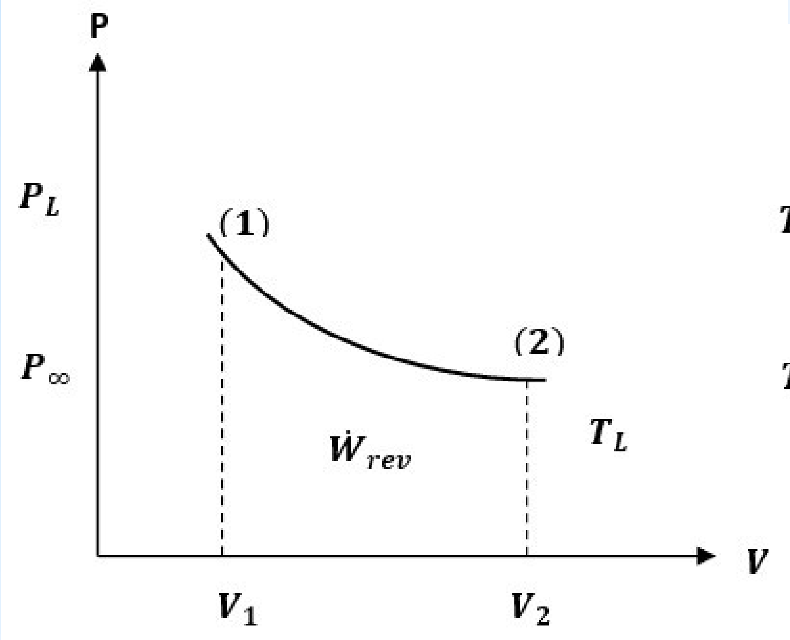
$$\dot{Q}_L = T_L\,(s_i - s_e)\,\dot{m}$$

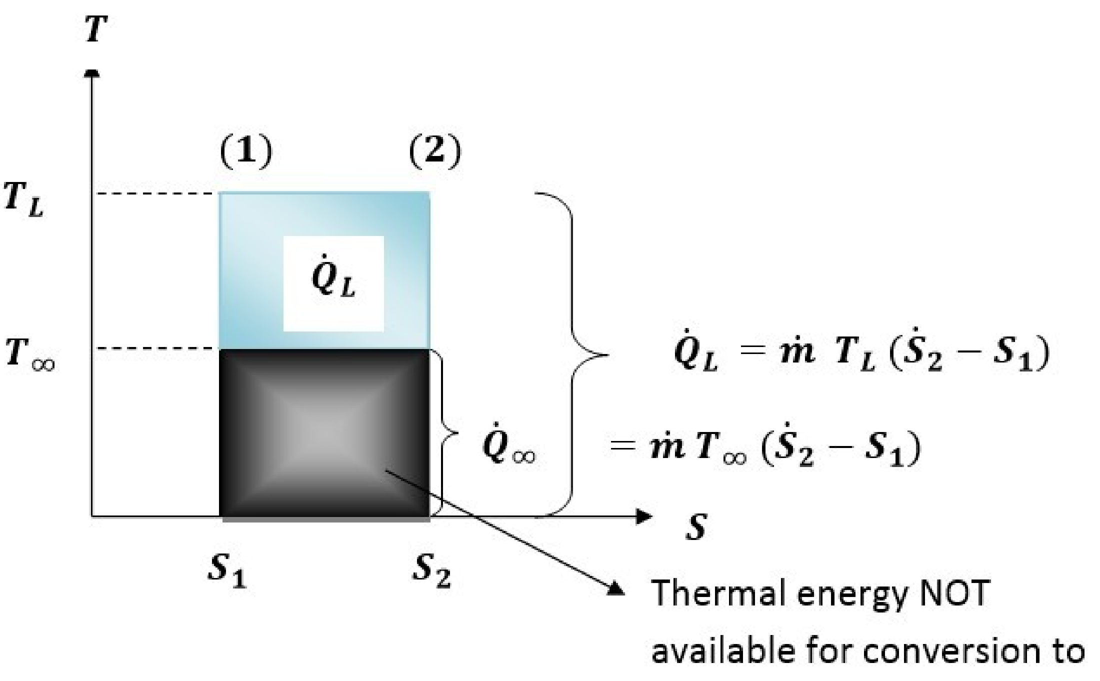
$$\dot{Q}_\infty = T_\infty\,(s_i - s_e)\,\dot{m}$$

$$\boxed{\dot{W}_a = \dot{m}\,(h_e - h_i) - \dot{Q}_\infty}$$

$$\dot{W}_a = \dot{m}\left[(h_e - h_i) - T_\infty\,(s_i - s_e)\right]$$ Available work in kW

$$w_a = (h_e - h_i) - T_\infty\,(s_i - s_e)$$ Available work in kJ/kg





Thermal energy NOT available for conversion to

Available work $\dot{W}_a = \dot{m}\left[(h_e - h_i) - T_\infty\ (s_i - s_e)\right]$

Lost work $\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}_{actual}$

The difference is due to internal irreversibilities, such as friction, etc.

Losses due to internal irreversibility: $\dot{W}_{lost} = \dot{W}_{rev} - \dot{W}_{actual}$

Losses due to external irreversibility: $\dot{W}_{Carnot} = \dot{Q}_{L} - \dot{Q}_{\infty}$

$$\left.\begin{array}{rl} \text{Available work:} & \delta W_a = \delta W_{rev} - \delta W_c \\ \text{Lost work:} & \delta W_{lost} = \delta W_{rev} - \delta W_{actual} \end{array}\right\} \quad \delta W_a = \delta W_{actual} + \left(\delta W_{lost} + \delta W_C\right)$$

$$\delta W_{lost} + \delta W_C = \left(T\,dS - \delta Q_{actual}\right) - \left(\delta Q_L\big|_{rev} - \delta Q_\infty\big|_{rev}\right)$$

$$= \left(T\,dS - \delta Q_{actual}\right) - \left(T\,dS - T_\infty\,ds\right) = T_\infty\,dS - \delta Q_{actual}$$

Solving for dS: $$dS = \frac{\left(\delta W_{lost} + \delta W_c\right)}{T_\infty} - \frac{\delta Q_{actual}}{T_\infty}$$

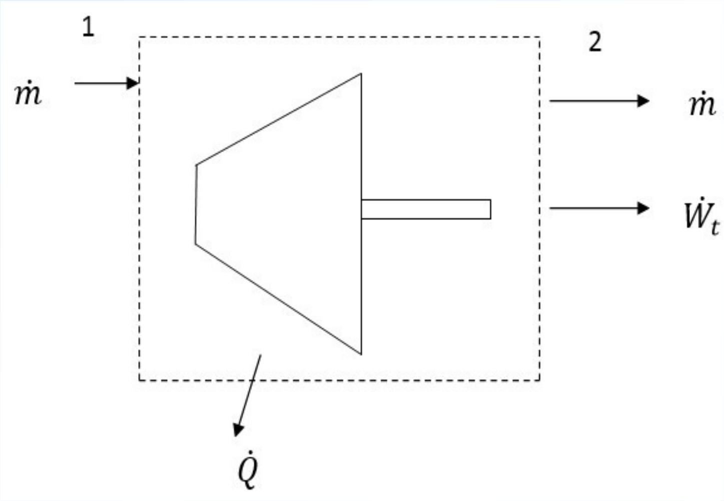
In rate form: $$\frac{dS}{dt} = S_{gen} = \frac{1}{T_\infty}\left[\delta\left(\dot{W}_{lost} + \dot{W}_c\right) - \delta\dot{Q}_{actual}\right]$$

Entropy generation;

Production

Example 5

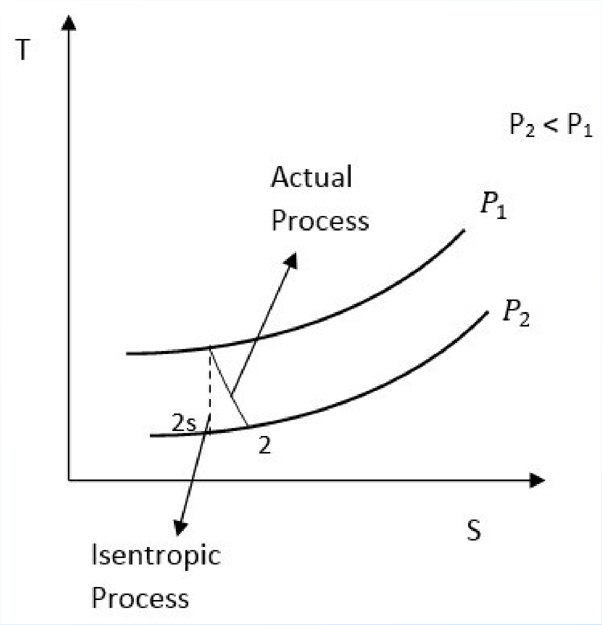
Air enters a turbine at 150 psia (1 MPa) and 2000 F (1100 °C) and expands to 14.7 psia (100 kPa) and 1000 F (540 °C). The heat loss from the turbine is approximately 2 % of the turbine power. The ambient temperature is 77 F (25 °C). Calculate the actual work and the available work for this turbine process



State 1: 150 psia
2000 °F

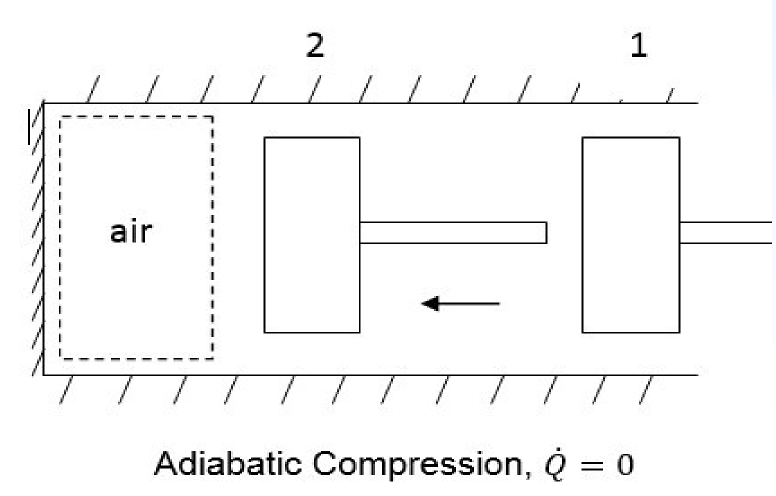
State 2: 14.7 psia
1000 °F

State ∞: 14.7 psia
77 °F



Answer: $W_T = 253.3$ Btu/lbm, $W_a = 258.2$ Btu/lbm

Example 6



Adiabatic Compression, $\dot{Q} = 0$

Air is compressed adiabatically in a piston cylinder process. For initial conditions of 20 psia and 100 °F and final conditions of 50 psia and 297.5 °F, calculate the actual work and available work in the process. The ambient temperature is 77 °F.

Solution:

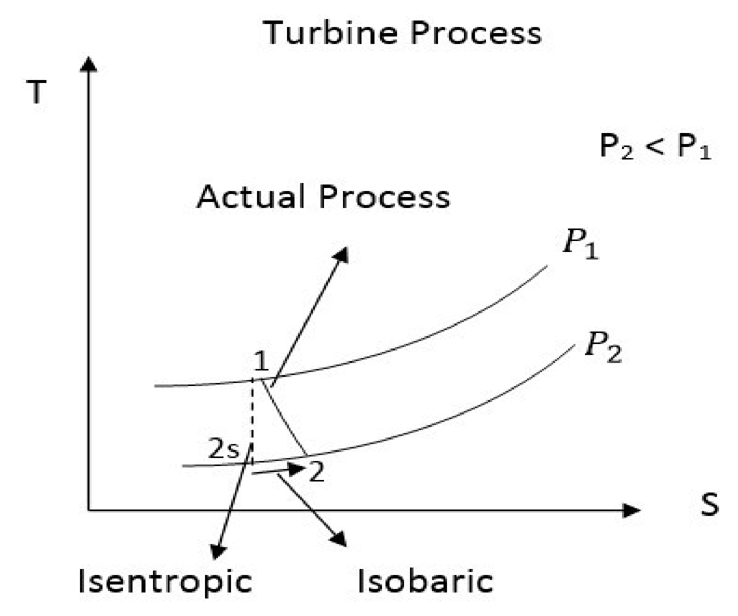
$$- w_P = u_2 - u_1 = c_v \, (T_2 - T_1) = 33.6 \text{ Btu/lbm} \quad \Rightarrow \quad w_P = - 33.6 \text{ Btu/lbm}$$

$$w_a = (u_1 - u_2) - T_\infty \, (s_1 - s_2)$$

$$= c_v \, (T_1 - T_2) - T_\infty \left[c_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right) \right] = - 28.3 \text{ Btu/lbm}$$

Example 7

For the turbine operation in the previous example, it is assumed that the air first expands isentropically to 14.7 psia and then changes to the final state by a reversible isobaric process. Calculate for this entire turbine process (a) the reversible work, (b) the Carnot work, (c) the lost work, and (d) the unavailable work.



Actual expansion: 1 → 2

Idealized expansion: 1 → 2s, 2s → 2

Assume air is an ideal gas, c_p = 0.24 Btu/lb$_m$.R

Ambient temperature is 77 °F

(a) Reversible work

$$w_{rev} = {}_1w_{2s} + \cancelto{= 0}{{}_{2s}w_2} = h_1 - h_{2s} = c_p\,(T_1 - T_{2s})$$

Isentropic expansion of an ideal gas: $T_{2s} = T_1 \left(\frac{P_{2s}}{P_1}\right)^{\frac{k-1}{k}}$

$$T_{2s} = T_1 \left(\frac{P_{2s}}{P_1}\right)^{\frac{k-1}{k}} = (2460\text{ R}) \left(\frac{14.7}{150}\right)^{\frac{1.4-1}{1.4}} = 1265.9\text{ R} = 805.9\text{ F}$$

$$w_{rev} = c_p\,(T_1 - T_{2s}) = \left(0.24\,\frac{\text{Btu}}{\text{lb}_m.\text{R}}\right)(2460 - 1265.9)\text{ R} = 286.6\text{ Btu/lb}_m$$

(b) Carnot work is the sum of two Carnot works

$$w_c = \underset{= 0}{\cancel{{}_1w_{c,2s}}} + {}_{c,2s}w_{c,2} = -\int_{2s}^{2}\left(1 - \frac{T_\infty}{T}\right)\underset{\partial q = c_p\, dT}{\cancel{\partial q}}$$

$$w_c = -\int_{2s}^{2}\left(1 - \frac{T_\infty}{T}\right) c_p\, dT = c_p\left[-(T_2 - T_1) + T_\infty \ln\left(\frac{T_2}{T_{2s}}\right)\right]$$

$$= \left(0.24\ \frac{\text{Btu}}{\text{lb}_m.\text{R}}\right)\left[-(1460 - 1265.9) + (77 + 460)\ln\left(\frac{1460}{1265.9}\right)\right]$$

$$= -\ 28.2\ \text{Btu/lb}_m$$

(c) The lost work due to internal irreversibilities is the difference between the reversible work and the actual work

$$w_{lost} = w_{rev} + w_{actual} = 286.6 - 235.3 = 51.3\ \text{Btu/lb}_m$$

(d) The unavailable work is the sum of the lost work plus the Carnot work

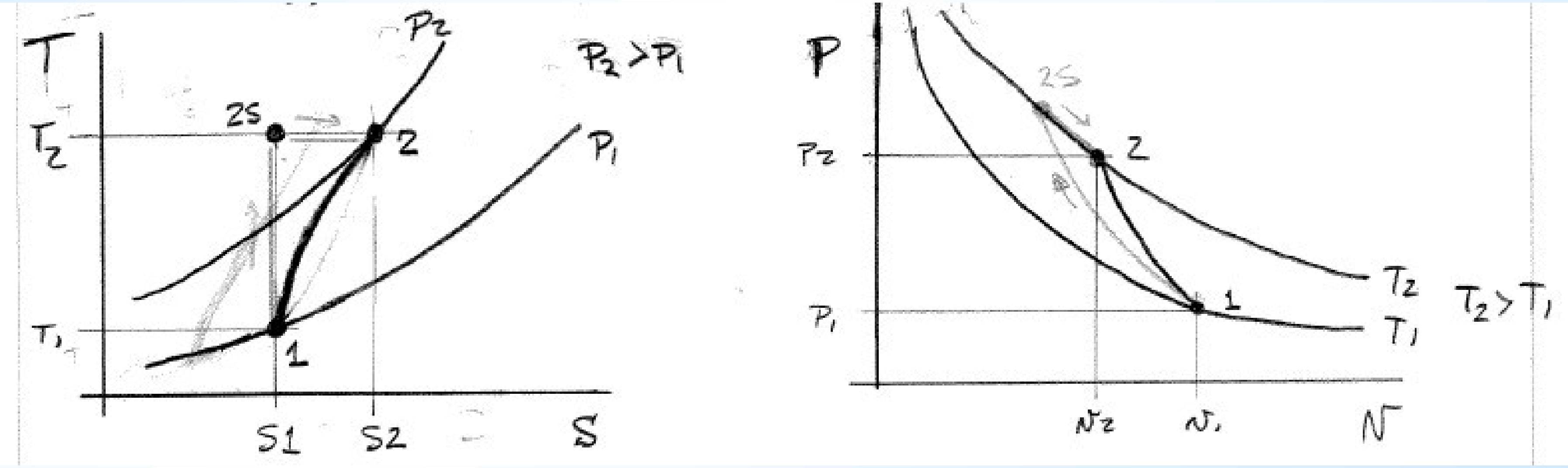
$$w_{unavailable} = w_{c} - w_{actual} = 51.3 - 28.2 = 22.9\ \text{Btu/lb}_m$$

Note that the values depend upon choice of process idealizations.

Unavailable work is unchanged for specified initial and final states.

Example 8

For the compressor specified in the previous example, it is assumed that the air is first compressed isentropically, then expanded reversibly and isothermally to the final state. For this entire process calculate (a) the reversible work, (b) the lost work, (c) the Carnot work, and (d) the unavailable work.



Availability

Available work: $w_a = (h_i - h_e) - T_\infty (s_i - s_e)$

$$= \left[(h_i - h_\infty) - T_\infty (h_i - h_\infty)\right] - \left[(h_e - h_\infty) - T_\infty (s_i - s_\infty)\right]$$

$= a_i - a_e$ availabilities

Availability (Exergy): measure of departure of state from the state of the environment or surroundings.

Flow process: $a_f = (h - h_\infty) - T_\infty (s - s_\infty)$

Non-flow process: $a_{nf} = (u - u_\infty) - P_\infty (v - v_\infty) - T_\infty (s - s_\infty)$

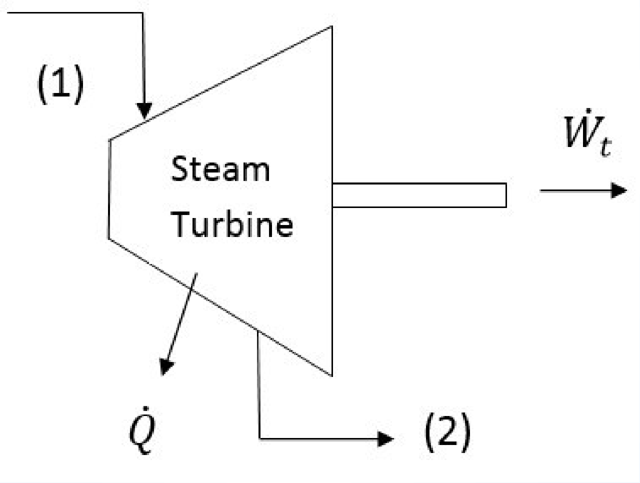
$= a_f + v (P_\infty - P)$ form of Gibb's relation

Example 9

Steam enters a well-insulated turbine at 800 °C and 10 MPa at a flow rate of 2.5 kg/s. The steam exits at 50 kPa. The isentropic efficiency of the turbine is 0.9332. The surroundings are at 25 °C and 1 atm.

(a) Determine the rate at which availability enters the turbine; and

(b) Determine the rate of availability destruction.



(1) $T_1 = 800$ °C

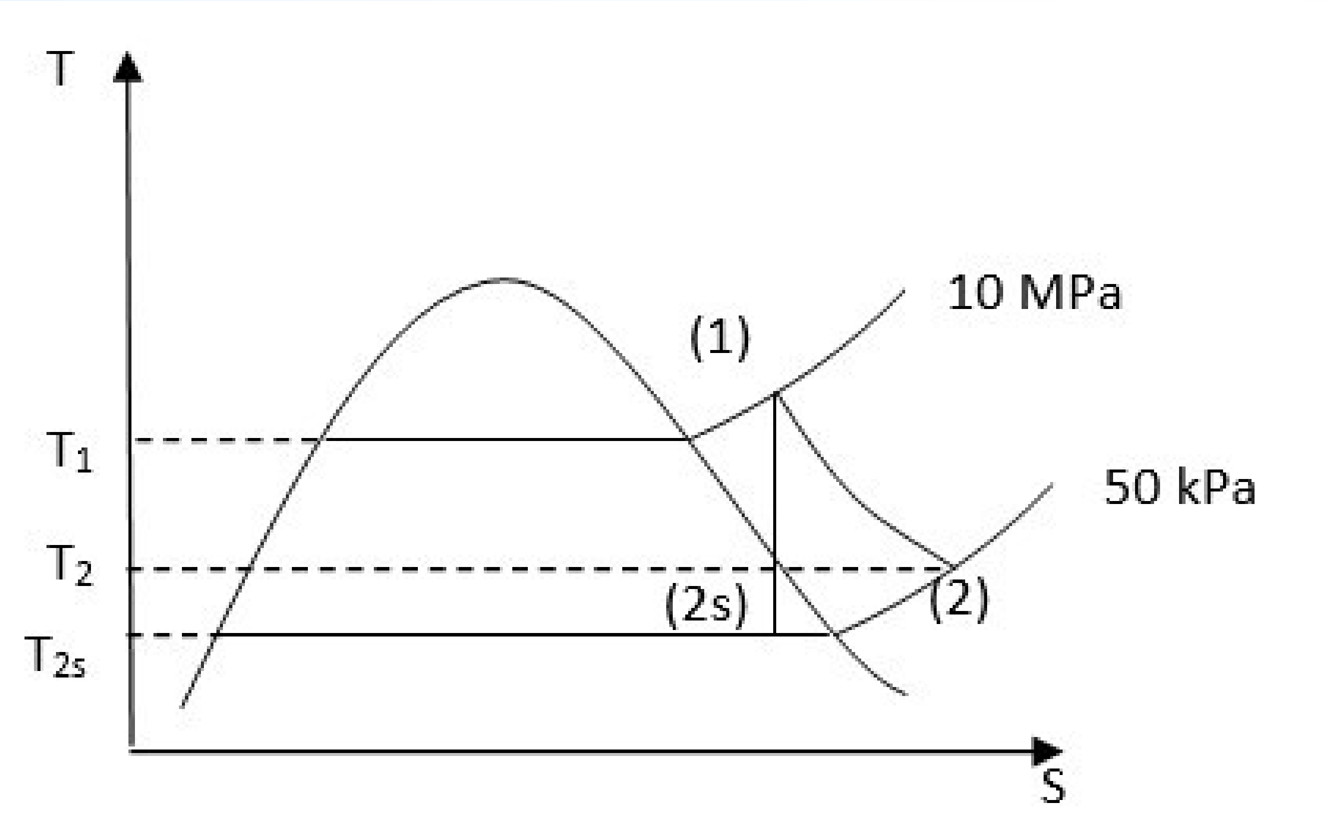
$P_1 = 10$ MPa

(2) $P_2 = 50$ kPa

(∞) $T = 25$ °C

$P = 1$ atm

- Steady state
- Steady flow
- Adiabatic
- Negligible changes in KE and PE


T
T1
T2
T2s
(1)
10 MPa
50 kPa
(2s)
(2)
S

Use steam tables for states 1 and ∞:

State 1: $T_1 = 800\ °C$

$P_1 = 10\ MPa$

$h_1 = 4114.8\ kJ/kg$

$s_1 = 7.4077\ kJ/kg.K$

State ∞ : $T_\infty = 25\ °C = 298\ K$

$P_\infty = 1\ atm$

$h_\infty = 104.92\ kJ/kg$

$s_\infty = 0.3672\ kJ/kg.K$

Availabilities:

$$a_1 = \left(h_1 - h_\infty\right) - T_\infty\ \left(s_1 - s_\infty\right)$$

$$= \left(4114.8 - 104.92\right) - (298)\ \left(7.4077 - 0.3672\right) = 1911.8\ kJ/kg$$

$$\dot{A}_1 = \dot{m}\, a_1 = (2.5\ kg/s)\ (1911.8\ kJ/kg) = 4779.5\ kW$$

$$\dot{A}_2 = \dot{m} a_2 = \dot{m}\left[\left(h_2 - h_\infty\right) - T_\infty \left(s_2 - s_\infty\right)\right]$$

Need to determine state 2

Use the definition of isentropic turbine efficiency:

$$\eta_T = 0.9332 = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad => \quad h_2 = h_1 - \eta_T \left(h_1 - h_{2s}\right)$$

h_{2s} can be calculated if $h_{2s,f}$, $h_{2s,g}$, and x are known.

Quality, x, found from $s_{2s} = s_1$; T_{2s} and P_{2s} are known.

$$x = \frac{s_{2s} - s_{2s,f}}{s_{2s,g} - s_{2s,f}} = \frac{s_1 - s_{2s,f}}{s_{2s,g} - s_{2s,f}} = \frac{7.4077 - 1.091}{7.5939 - 1.091} = 0.9714$$

$$h_{2s} = (1 - x)\, h_{2s,f} + x\, h_{2s,g} = (1 - 0.9714)\,(340.49) + (0.9714)\,(2645.9)$$

$$= 2579.9 \text{ kJ/kg}$$

$$h_2 = h_1 - \eta_T\, (h_1 - h_{2s}) = 4114.8 - (0.9332)\,(4114.8 - 2579.9)$$

$$= 2682.4 \text{ kJ/kg}$$

Knowing h_2 and P_2 => $s_2 = 7.6952$ kJ/kg.K

$$a_2 = \left(h_2 - h_\infty\right) - T_\infty \left(s_2 - s_\infty\right)$$

$$= \left(2682.4 - 104.92\right) - (298)\,\left(7.6952 - 0.3672\right) = 393.74 \text{ kJ/kg}$$

$$\dot{A}_2 = \dot{m}\, a_2 = (2.5 \text{ kg/s})\,(393.74 \text{ kJ/kg}) = 934.3 \text{ kW}$$

Difference in availability:

$$\Delta\dot{A} = \dot{A}_1 - \dot{A}_2 = 4779.5 - 984.3 = 3795.2 \text{ kW}$$

Turbine work:

$$\dot{W}_T = \dot{m}(h_1 - h_2) = (2.5)(4114.8 - 2682.4) = 3581 \text{ kW}$$

Availability destroyed by internal irreversibilities:

$$\dot{A}_{destroyed} = \Delta\dot{A} - \dot{W}_T = 3795.2 - 3581 = 214.2 \text{ kW}$$

Alternative calculation:

$$\dot{A}_{destroyed} = T_\infty \dot{S}_{gen} = \dot{m} T_\infty (s_2 - s_1)$$

$$= (2.5)(298)(7.6952 - 7.4077) = 214.2 \text{ kW}$$

Example 10

Consider a steam throttling process through an expansion valve. Steam enters the valve at 1900 psia and 1000 °F and leaves at 1800 psia. Calculate the flow availability per unit mass of steam at the inlet and exit.

Solution: Use steam tables

$a_1 = (h_1 - h_\infty) - T_\infty (s_1 - s_\infty) = 637.8$ Btu/lbm Availability at the inlet

$a_2 = (h_2 - h_\infty) - T_\infty (s_2 - s_\infty) = 635$ Btu/lbm Availability at the exit

Note that the availability (exergy) decreases during the throttling process even though no work or heat is extracted from the flow. The throttling process is internally irreversible.

CONCEPTS and DEFINITIONS

Second Law Entropy, S Carnot Engine, Carnot Efficiency, η_C

Exergy, Availability, Available work ⇨ X or A

Actual work (W_a), Useful work (W_u), Reversible (Carnot) work, (W_{rev})

Irreversibility, $I = W_u - W_{rev}$ Second law efficiency, $\eta_{II} = W_u / W_{rev}$

$$W_{rev} = m\, c_{av}\, (T_1 - T_2) - m\, c_{av}\, T_0 \ln\left(\frac{T_1}{T_2}\right)$$

For solids and liquids

$$\Delta s = c_{av} \ln\left(\frac{T_1}{T_2}\right)$$

For solids and liquids

$$\Delta s = c_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right)$$

For ideal gasses