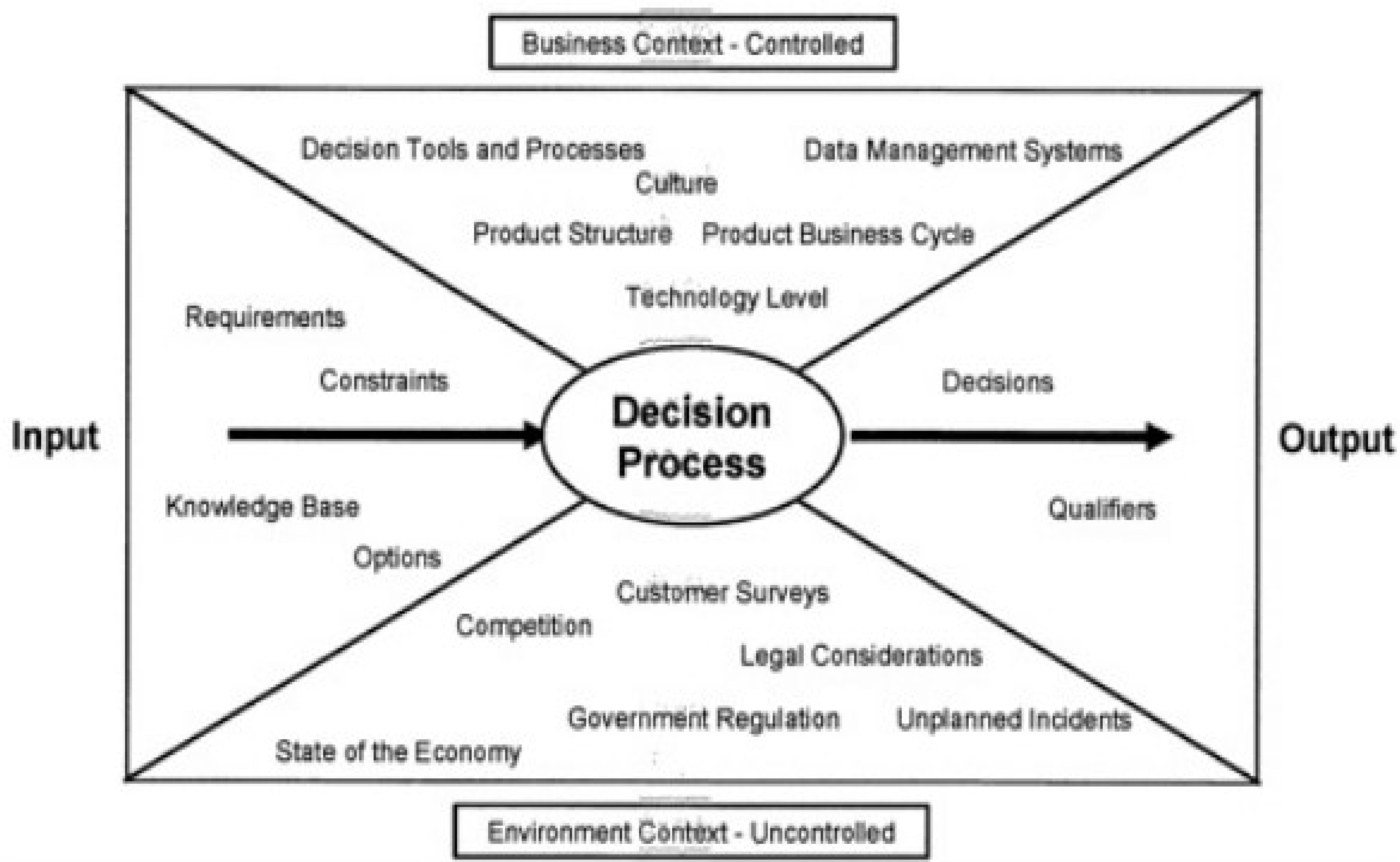

Business Context - Controlled
Decision Tools and Processes
Data Management Systems
Culture
Product Structure
Product Business Cycle
Technology Level
Requirements
Constraints
Decisions
Input
Decision Process
Output
Knowledge Base
Qualifiers
Options
Customer Surveys
Competition
Legal Considerations
Government Regulation
Unplanned Incidents
State of the Economy
Environment Context - Uncontrolled

Energy Economics

Energy economics is a specialized field used to make decisions on

- energy purchases;
- selection of competing energy generation technologies; and
- financing of energy technologies.

A thorough study of this subject is beyond the scope of this course, but every engineer should have a basic understanding of energy economics in order to bridge the gap between engineering decision analysis and economic decision analysis.

See "odtuclass" for an e-book and some articles on engineering economy (Electricity Cost and Tidal Power).

Energy Costs

Energy costs can generally be divided into two categories both of which are called by many different names:

1. Capital, startup, investment, initial, …, costs.
2. Recurring, ongoing, operational, operations and maintenance (O&E), …, costs.

Examples of **capital costs** include cash or borrowed money used for construction of facilities, equipment purchase, and/or equipment installation.

Capital costs are usually one-time expenses incurred at the beginning of a project. An increase in the capacity factor decreases the capital cost per energy output.

Examples of **recurring costs** are numerous; including

- Salaries;
- Taxes;
- Annuity payments (series of payments at regular intervals for mortgage, insurance, etc.);
- Maintenance costs for scheduled or unscheduled shutdowns;
- Loan payments; and
- Fuel costs.

These costs may be uniform payments in time (regular) or sporadic (irregular).

These may be not only costs but also recurring gains.

The difference between the two categories has to do with time.

Capital costs are always in **today's value** of money whereas recurring costs occur at some **future value** of money. This poses difficulties when comparing competing energy systems.

For example, an economic comparison of a conventional gas-fired heater to a solar home hot-water heater requires evaluation of money at different times. The conventional hot water heater has a very low capital cost, but has recurring future fuel costs. The solar hot water heater has a relatively high capital cost, but has minimal recurring costs.

Energy economic analysis attempts to **convert all monetary costs and gains to a common basis** to enable quantitative comparisons of competing technologies.

Time Value of Money

The premise of time value of money is that a TL received today is perceived to be worth more than a TL received in the future. There are several factors that drive this perception:

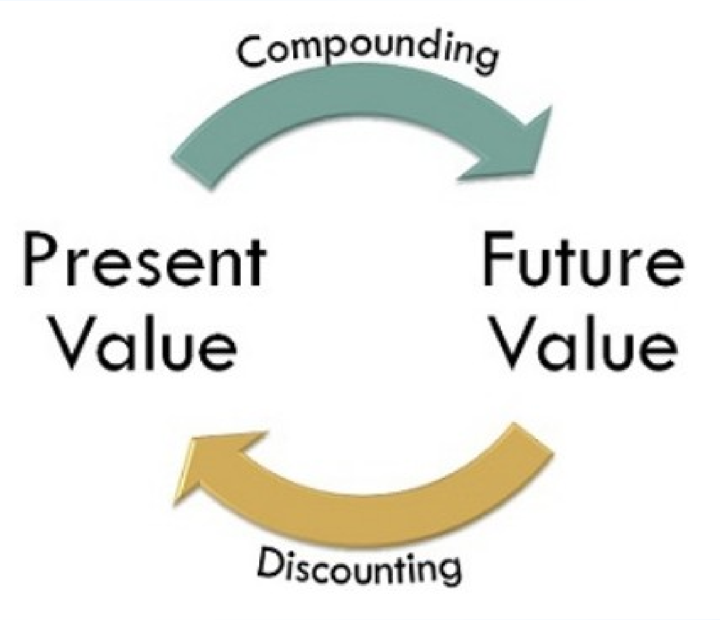
- time preference;
- Interest rate;
- Inflation;
- Escalation of costs (fuel prices, etc.);
- Profit (MARR – Minimum Acceptable Rate of Return);
- Others.

Time preference of humans is the reason that present consumption (money) is preferred to future consumption. Time preference is naturally different for each person in a society. It is difficult to measure for individuals and even more difficult to aggregate for a society.

A second factor affecting the time value of money is the “rental rate", or **interest rate**, on funds. There is a future cost associated with the present value of the rented (borrowed) funds.

A third factor is the possibility of currency **inflation**, which reduces the future purchasing power of present funds.

There are two methods used for ascertaining the worth of money at different points of time, namely, **compounding and discounting**.



Compounding method is used to know the future value of present money. Conversely, **discounting** is a way to compute the present value of future money.

Discount Rate

The value of money has historically declined with time. The effect of a TL purchasing less today than 20 years ago is known as **inflation**. There are many factors that can cause inflation, not the least of which is the perception that "*a TL in hand today is more valuable than the one to be received at some time in the future*".

In order to compare an energy system's benefits and costs that occur at different points in time, all the monetary factors should be converted to a common time basis. This conversion is known as **discounting**.

Discount Rate is the fractional decline in the value of money used for comparing present and future costs on a common basis.

The discount rate may also be thought of as the rental rate on funds needed for the investment that could be undertaken. This investment may be for an energy system, or a potential alternative investment used as a comparison.

The value used for a discount rate is industry specific, but there are some common means for determining the discount rate. These are:

- Rate higher than a state bond (from the central bank of Turkey);
- Anticipated rate of inflation;
- Rate of interest that balances costs and benefits (savings and/or revenues);
- Rate of financing available; and
- Rate of return on an alternative investment with a similar risk.

The choice of discount rate depends on the specific scenario being analyzed. Low discount rates tend to be favorable towards projects with long-deferred benefits. High discount rates tend to be favorable towards projects with quick paybacks. The discount rate should reflect the "**opportunity cost**" of the capital to be invested.

METHODOLOGIES OF ECONOMIC ANALYSIS

The type of analysis chosen has much to do with type of energy project being considered. For instance, a short-lived project may not be affected by the future value of money, but a project which is expected to take decades, such as a power plant, will certainly be affected by future costs.

The cost effectiveness of a short-lived project may be accomplished using a **simple payback method**.

The long-lived project may be better assessed through some form of a **life cycle analysis** (LCA).

Simple Payback Method

determines the time period to recover capital costs. Typical considerations are accumulation of savings, no future value of money, no interest on debt, and no comparison to fuel costs. The Simple Payback Method penalizes projects with long life potentials in part because any savings beyond the payback period are ignored. There is no accounting for inflation or for escalation of future savings in fuel costs that historically have increased at a rate faster than that of inflation.

Life Cycle Analysis, also known as **Engineering Economic Analysis**, considers the total cost over an anticipated useful life, where useful life is the lesser of lifetime or obsolescence. Analysis may include:

Capital costs

Operating costs

Maintenance costs and contracts

Interest on investment

Fuel cost

Salaries

Insurance

Salvage value

Taxes

Life Cycle Analysis (LCA)

may account for all costs including indirect costs paid by society but not reflected as cash flow. An example would be health and environmental costs associated with pollution due to electric power generation from coal; a cost not directly paid by the power generating utility.

The difficulty with life cycle analysis is that many of the costs are in the future and can only be estimated with some uncertainty. New technologies may also result in unanticipated obsolescence that, in hindsight, will turn a “cost effective' decision into an investment loss. For the purposes here, Life Cycle Analysis is a method that encompasses several variations.

All of the economic evaluation analysis methods are attempting to do two things:

- The first is to manipulate costs and savings in time to some common basis.
- The second is to assess these costs against some comparative objective; i.e.,

i. Which energy system has the lowest total expense;

ii. Which system has maximized return on investment; or

iii. Which system will maximize savings in energy costs.

- **Life-Cycle Cost Method (LCC):** all future costs are brought to present values for a comparison to a base case. The base case may be a conventional energy system, design variations in alternative energy systems, or the alternative of not making the investment. LCC is commonly used to determine the `cost-minimizing' option that will achieve a common objective.

- **Levelized Cost of Energy (LCOE):** seeks to convert all costs (capital and recurring) to a value per energy unit that must be collected (or saved) to ensure expenses are met and reasonable pros collected. Future revenues are discounted at a rate that equals the rate of return that might be gained on an investment of similar risk; often called the "opportunity cost of capital". LCOE is often used to compare competing energy producing technologies.

Net Present Value (NPV): (also known as Net Benefits, Net Present Worth, Net Savings Methods) determines the difference between benefits and expenses with everything discounted to present value. NPV is used for determining long-term profitability.

Benefit-to-Cost Ratio (BCR): (also known as Savings-to-Investment Ratio) is similar to NPV, but utilizes a ratio instead of a difference. Benefits usually implies savings in energy cost. What to include in the numerator (benefits) and denominator (costs) varies and care should be taken when assessing a reported benefit-to-cost ratio. This method is often used when setting priorities amongst competing projects with a limited budget. Projects with the largest ratio get the highest priority.

Overall Rate-of-Return (ORR): determines the discount rate for which savings in energy costs are equal to total expenditures. This is equivalent to determining the discount rate that results in a zero NPV. This method enables cash flow to be expressed in terms of the future value at the end of the analysis period. Previous methods require specification of a discount rate; this method solves for the discount rate.

Discounted Payback Method (DPM): determines the time period required to offset the initial investment (capital cost) by energy savings or benefits. Unlike the simple payback method, the time value of money is considered. DPM is often used when the useful life of the project or technology is not known.

The various forms of analysis and vocabulary are generally built using two simple arithmetic concepts:

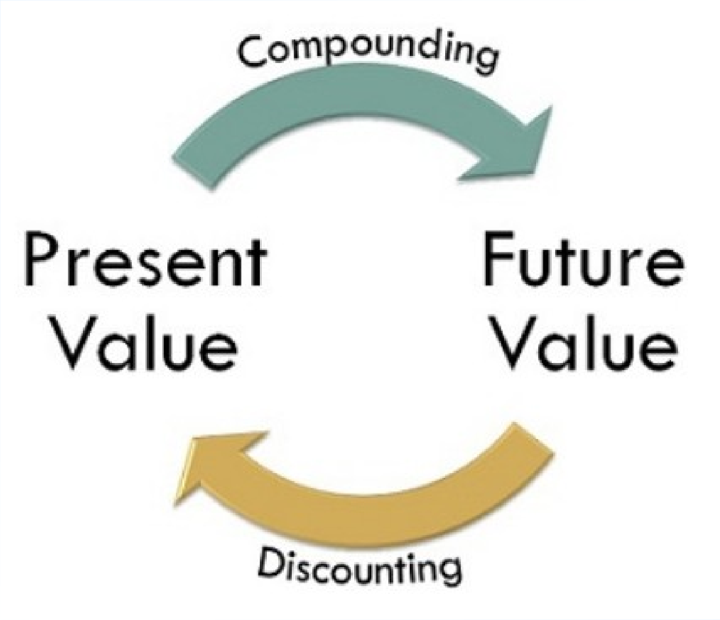
i. Compounding; and

ii. Uniform series.

Using these two arithmetic concepts, four methods for determining equivalent values of money will be examined;

- **Future value**;
- **Present value**;
- **Levelized cost (value)**; and
- **Capitalized cost (value).**

There are two methods used for ascertaining the worth of money at different points of time, namely, **compounding and discounting**.



Compounding method is used to know the future value of present money. Conversely, **discounting** is a way to compute the present value of future money.

Discounting Tool – Future Value

A basic measure of the time value of money is the future value obtainable through **compounding**. Compounding is a fractional growth rate based on finite time intervals; similar to exponential growth which is fractional growth at infinitesimal time intervals.

Compounding Formula

Compounding can be described mathematically in terms of a growth rate, a time period over which compounding occurs, and the number of compounding periods being considered.

P: quantity which increases by a fractional rate at fixed time intervals

j: Percent growth rate, or increase in value in a given time interval

n: number of compounding time intervals

Compound Interest Formula: $P_n = P_0 (1 + j)^n$

The compounding formula can be derived by examining the increase in P after each time interval. At the end of the first time interval, the initial value, P_0, has increased by the fractional amount (j) P_0. Similarly, at the end of the second time interval, the starting value, P_1 has increased by a fractional amount.

at the end of period 1: $P_1 = P_0 + (j)\,P_0 = P_0\,(1 + j)$

end of period 2: $P_2 = P_1 + (j)\,P_1 = P_1\,(1 + j) = P_0\,(1 + j)^2$

period 3: $P_3 = P_2 + (j)\,P_2 = P_2\,(1 + j) = P_1\,(1 + j\,\tau)^2 = P_0\,(1 + j\,\tau)^3$

…

period n: $P_n = P_0\,(1 + j)^n$

Compound Interest Formula: $P_n = P_0\,(1 + j)^n$

What is the value of 1000 TL in the bank with 10 % annual interest rate (APR) in 5 years?

$$P_n = P_0\,(1 + j)^n = 1000\,(1 + 0.1)^5 = 1610.5 \text{ TL}$$

What is the relationship between annual percent rate (APR) and percent rate of other periods (weeks, months, etc.)?

$$j' = \left(1 + \frac{APR}{n}\right)^{n/n'} - 1$$

Where j' = effective APR; n' = 1 year; and n = given period.

Example 1

Determine the effective annual percent interest rate on a loan with 6 % annual interest rate compounded quarterly (every 4 months).

Percent interest rate compounded quarterly:

$$j = \frac{0.06}{3} = 0.02 \quad \text{per quarter}$$

Equivalent annual percent interest rate:

$$j' = (1 + 0.02)^{3/1} - 1 = 0.0612 \quad => \quad 6.1\ \% \quad \text{per year}$$

Example 2

Determine the effective annual percent interest rate (APR) on a loan with 17.23 % annual interest rate compounded daily.

Percent interest rate compounded daily:

$$j = \frac{0.1723}{365} = 0.000472 \quad \text{per day}$$

Equivalent annual percent interest rate:

$$j' = (1 + 0.000472)^{365/1} - 1 = 0.1879 \quad => \quad 18.79\ \%$$

Multiple Compound Rates

When dealing with energy, the escalation rate due to inflation is often insufficient to describe future increase in costs. Historically, fuel prices have increased at a rate higher than inflation so if recurring costs include fuel then two rates should be considered. In the simplest terms, the two factors are multiplied. If i is the percent rate of inflation and j is the percent rate of escalation, then from the compound interest formula,

$$\frac{P_n}{P_0} = (1 + i)^n (1 + j)^n = \left[(1 + i)\ (1 + j)\right]^n$$

The growth factor (right side of equation) may be substituted with by a Total Escalation Rate growth factor,

$P_n = P_0 (1 + TER)^n$, where TER = (1 + inflation rate) (1 + escalation rate) - 1.

Effective Interest Rate (Short-Term Interest Rate)

Many financial institutions calculate interest payments on an annual basis even though the time interval for compounding is less than a year. The annual percentage rate (APR) is not necessarily the annual percent growth rate because of the multiple compounding periods which occur per annum.

For example, an APR of 18 % compounded monthly is in actuality 1.5 % interest on a balance applied monthly with an effective annual interest rate of 19.56 %. The effective annual interest rate can be determined by equating compounding formulas for which the beginning and ending balances should be the same; i.e., P_n must be the same regardless of how growth rate and compounding period are calculated.

Uniform Series Formula (Equal Payment Annuity Formula)

The uniform series formula expresses the growth of a quantity due to a fractional increase plus a regular annuity or payment. This may be used to determine the amount of money that should be collected per time interval in order to recoup capital costs.

The symbols used are the same as for the compound interest formula with the addition of the fixed payment, S, per time interval.

- P: quantity which increases by a fractional rate at fixed time intervals
- S: payment or annuity per time interval
- j : percent growth rate, fractional increase in value
- n: number of time intervals

The uniform series formula can be derived in a similar manner as the compound interest formula; by examining the increase in P after each time interval. At the end of the first time interval, an initial value of zero will increase by S. At the end of the second time interval, the value, P_1, will increased by a fractional amount plus another S.

n	
0	$P_0 = 0$
1	$P_1 = S$ Note that the first payment occurred after the first period
2	$P_2 = P_1 + (j)\,P_1 + S = P_1\,(1 + j) + S = S\,(1 + j) + S$
3	$P_3 = P_2 + (j)\,P_2 + S = P_2\,(1 + j) + S = S\,(1 + j)^2 + S\,(1 + j) + S$
…	
n	$P_n = S\,[(1 + j)^{n-1} + (1 + j)^{n-2} + \ldots + (1 + j) + 1]$

To simplify this series, multiply P_n by (1 + j) and then subtract P_n.

The uniform series formula becomes:

$$P_n = S\left[\frac{(1+j)^n - 1}{j}\right]$$

Starting point for first payment

The derivation of the **Uniform Series Formula** was based on the first payment beginning at the end of the first time interval. For payments that start at the beginning of the first time interval, there is an extra compounding period,

$$P_n = S\left[\frac{(1+j)^{n+1} - 1}{j}\right]$$

Gains and Losses

Gains (compounding interest) and losses (inflation) can both be accounted for when determining the required uniform series payments. If d is the percent decrease, then:

Gain: $(j)\, P_n = S\,[(1 + j)^n - 1]$ Loss: $(d)\, P_m = S\,[(1 + d)^m - 1]$

Losses can be considered as a negative gain. If the compounding periods are the same, then n = m and the gains and losses may be added together. When j and d are in percent, Solve for P_n:

Gains and Losses: $$P_n = S\left[\frac{(1+j)^n - (1+d)^n}{(j - d)}\right]$$

Example 3. Effective Heating Costs - Alternative Investment

A solar-powered home heating system can be built for $8000 and will supply all of the heating requirements for 20 years. Assume that the salvage value of the solar heating system just compensates for the maintenance and operational costs over the 20 year period. If the interest on money is 8 %, compounded annually, what is the effective cost of heating the house? Another way to ask this question is “How much would you have to save per year to equal the future value of $8,000 invested for 20 years at 8 %?"

The discount rate is based on the lost future value of the $8000 at 8 % compounded annually. Therefore, the total capital cost is based on the building cost plus the lost future value of an alternative investment.

Example 3. Effective Heating Costs - Alternative Investment

Future value of the capital cost:

$$P_{20} = P_0 \, (1 + j)^n = 8000 \, (1 + 0.08)^{20} = 37\,287.66 \text{ TL}$$

Corresponding yearly heating expense:

$$P_{20} = S \left[\frac{(1 + j)^n - 1}{j} \right] \quad => \quad S = P_{20} \left[\frac{j}{(1 + j)^n - 1} \right]$$

$$S = (37287.66) \left[\frac{0.08}{(1 + 0.08)^{20} - 1} \right] = 814.8 \text{ TL} \quad \text{per year}$$

Hypothetical yearly expense.

Note that inflation and escalation costs are assumed to be zero.

Example 4. Effective Heating Costs – Minimum payment

A proposed solar-heating system for a home costs $6,000 and has a rated operational life of 20 years. Purchase and installation of the system is to be financed by a 60-month loan with an annual percentage rate of 7.2 %. The salvage value of the solar-heating system will essentially be zero. Determine the maximum effective (hyphotetical) yearly heating costs for the system to pay for itself if the annual savings could be invested at 6 % interest compounded annually.

Assumption: No inflation and no escalation of fuel cost.

Example 4. Effective Heating Costs – Minimum payment

Assumptions: No inflation and no escalation of fuel cost.

Monthly interest rate of the loan (capital cost): $j = \dfrac{0.072}{12} = 0.006$

Future value of the capital cost: $P_{60} = 6000\,(1 + 0.006)^{60} = 8\,590.7$ TL

Present worth of the capital cost with 6 % annual interest:

$$P_0 = \frac{P_{60}}{(1 + 0.06)^n} = \frac{8590.7}{(1 + 0.06)^5} = 6419.5 \text{ TL}$$

Future value of the capital cost in 20 years with 6 % interest:

$$P_{20} = (6419.5)\,(1 + 0.06)^{20} = 20\,588.2 \text{ TL}$$

Yearly cost: $P_n = S\left[\frac{(1 + j)^n - 1}{j}\right]$ => $S = P_{20}\left[\frac{0.06}{(1 + 0.06)^{20} - 1}\right]$

S = 559.7 TL per year

Example 5.a Effective Heating Costs – With and Without Inflation

What should be the annual savings in heating costs in order to “break even" after twenty years on an $8000 solar-powered residential heating system? Consider two cases, one without inflation and one with inflation.

Case 1 (future value without inflation)

- long-term investment at 8 % per year, compounded annually
- no operational or maintenance costs,
- no inflation
- no fuel cost escalation
- no salvage value or tax incentives
- savings re-invested at 6 % APR, compounded monthly

Capital investment in 20 years: $P_{20} = 8000\ (1 + 0.08)^{20} = \$37\ 288$

8000 invested with 6 % APR compunded montly:

$$j = \frac{0.06}{12} = 0.005 \text{ per month} \qquad j' = (1 + j)^{n/n'} - 1 = (1.005)^{240/20} - 1 = 0.0617$$

Yearly saving:

$$P_{20} = S\left[\frac{(1 + 0.0617)^{20} - 1}{0.0617}\right] = 37\ 288$$

$$S = \$\ 995 \text{ per year}$$

Example 5.b Effective Heating Costs – With and Without Inflation

What should be the annual savings in heating costs in order to “break even" after twenty years on an $8000 solar-powered residential heating system? Consider two cases, one without inflation and one with inflation.

Case 2 (future value without inflation)

- long-term investment at 8 % per year, compounded annually
- no operational or maintenance costs,
- Inflation rate: 4 %
- no fuel cost escalation
- no salvage value or tax incentives
- savings reinvested at 6 % APR, compounded monthly

Capital investment in 20 years: $P_{20} = 8000\,(1 + 0.08)^{20} = \$37\,288$

8000 invested with 6 % APR compunded montly:

$$j = \frac{0.06}{12} = 0.005 \text{ per month} \qquad j' = \left(1 + j\right)^{n/n'} - 1 = \left(1.005\right)^{240/20} - 1 = 0.0617$$

Yearly saving:

$$P_{20} = S\left[\frac{\left(1 + j\right)^{20} - \left(1 + d\right)^{20}}{j - d}\right] = S\left[\frac{\left(1 + 0.0617\right)^{20} - \left(1 + 0.04\right)^{20}}{0.0617 - 0.04}\right] = 37\,288$$

$S = \$\ 722$ per year

Relation to Discounting Factors

The discounting factors are typically tabulated on an annual basis. Instead of annualized discount factors, we are working with two basic growth rate expressions.

Compounding: $P_n = P_0 \, (1 + j)^n$ Uniform Series: $P_n = S \left[\frac{(1 + j)^n - 1}{j} \right]$

While there is no common language or nomenclature, a commonly used nomenclature is F for future value (P_n), P for present value (P_0), and A for uniform series payment (S); the annual growth rate is i (or j) % over m (or n) years.

Discounting Tool – Present Value

The present value methods are used to bring all future costs, which may occur in different years, back to today's value of money. In this way, the cost effectiveness of different energy technologies can be compared on an equal basis.

Compounding – Present Value

Compounding in terms of present value is the inverse of future value.

$$P_0 = P_n \frac{1}{(1+j)^n} = P_n (1+j)^{-n}$$

In other words, use this equation and solve for P_0.

Uniform Series – Present Value

In terms of future value, a uniform series annuity was derived as

$$P_n = S\left[\frac{(1+j)^n - 1}{j}\right]$$

The present value can be determined by substituting equation (1) into the uniform series formula:

$$P_0 = S\left[\frac{(1+j)^n - 1}{j\,(1+j)^n}\right]$$

Another way to look at this is that there are two equations (1 & 2) and two unknowns (P_0 and P_n).

Example 6. Home Mortgage Payments & Present Value

A good example of the need to calculate present value is with a home mortgage («ipotek» in Turkish). A lump sum is borrowed at a fixed annual interest rate and uniform series payments are made on the mortgage while interest is accruing. Consider a 30-year fixed-rate mortgage for 250,000 at 6 % per year. Find the yearly and monthly payments.

Example 6. Home Mortgage Payments & Present Value

Present value: P_0 = $ 250 000

Annual interest: j = 0.06 or 6 %

Future value in 30 years if there is no payment:

$$P_{30} = 250\,000\,(1 + 0.06)^{30} = \$\,1\,435\,873$$

With yearly payments:

$$P_{30} = S\left[\frac{(1+j)^{30} - 1}{j}\right] = S\left[\frac{(1+0.06)^{30} - 1}{0.06}\right] = \$1\,435\,873$$

S = $ 18 162 per year

Alternative calculation:

$$P_0 = S\left[\frac{(1+j)^{30} - 1}{j\,(1+j)^{30}}\right] = S\left[\frac{(1+0.06)^{30} - 1}{(0.06)\,(1+0.06)^{30}}\right] = \$250\,000$$

S = \$18 162 per year

Normally, banks require monthly payments

Monthly interest: $j = \dfrac{0.06}{12} = 0.005$ per month

360 months in 30 years $P_0 = S\left[\frac{(1+j)^{360} - 1}{j\,(1+j)^{360}}\right] = S\left[\frac{(1+0.005)^{360} - 1}{(0.005)\,(1+0.005)^{360}}\right] = 250\,000$

S = \$1500 per month

Example 7. Least Current Cash Option

You are in charge of purchasing several new fleet of vehicles. You are offered two payment options. Option A requires a $30,000 payment at the end of the year for four years. Option B requires a $39,000 payment at the end of the year for the next three years. Which is the least costly option if the long-term interest is 8 %, compounded annually. In other words, what is the minimum amount of cash that should be set aside now to make the annual payments?

The two options require uniform payments, but over different periods of time. In order to compare the two options, calculate how much money today (present value) would be required to make the payments if the lump sum was invested at 8 %, compounded annually.

Example 7. Least Current Cash - Option A

Cash outlay in present value = (4) (30 000) = \$120 000

Future value of \$30 000 annual payment in 4 years:

$$P_4 = S\left[\frac{(1+j)^4 - 1}{j}\right] = 30\,000\left[\frac{(1+0.08)^4 - 1}{0.08}\right] = \$135\,183$$

Present value: $$P_0 = \frac{P_4}{(1+j)^4} = \frac{135\,183}{(1+0.08)^4} = \$99\,364$$

You may combine these two equations

That means, if you invest \$99 364 with 8 % interest rate, then you can make \$30 000 yearly payments and you break even at the end of 4 years.

Example 7. Least Current Cash - Option B

Cash outlay in present value = (3) (39 000) = \$117 000

Future value of \$39 000 annual payment in 3 years:

$$P_3 = S\left[\frac{(1+j)^3 - 1}{j}\right] = 39\,000\left[\frac{(1+0.08)^3 - 1}{0.08}\right] = \$126\,610$$

Present value: $$P_0 = \frac{P_3}{(1+j)^3} = \frac{126\,610}{(1+0.08)^3} = \$100\,507$$

The option A is less costly (99 364 < 100 507) although the cash outlay is higher (120000 > 117000).

Example 8. Alternative Energy Payback – Present Value

A small municipality is considering installing a 1 MW_e wind turbine that will cost $6.5 million to install, and then generate a net annuity of $400 000 per year for twenty-five years, with an estimated salvage value of $1 million. The inflation rate is estimated to be 5 % per year.

a) Use a simple payback method to assess the economic viability.

b) Calculate the present value to assess the economic viability.

Based on the present value analysis, the project is not economically viable at 5% inflation. Economic viability will require a lower inflation rate, higher income, or tax incentives or subsidies.

Example 8. Alternative Energy Payback – Present Value

Future value of income: $P_{i,25} = S_i \left[\frac{(1+j)^n - 1}{j}\right] = 400\,000 \left[\frac{(1+0.05)^{25} - 1}{0.05}\right] = \$19\,091\,000$

Present value of income: $P_{i,0} = \frac{P_{i,n}}{(1+j)^n} = \frac{19\,091\,000}{(1+0.05)^{25}} = \$5\,640\,000$

Present value of salvage: $P_{s,0} = \frac{P_{s,n}}{(1+j)^n} = \frac{1\,000\,000}{(1+0.05)^{25}} = \$295\,303$

Present value of capital: $P_{c,0} = \$6\,500\,000$

Income – Expense at present values: $P_{i,0} + P_{s,0} - P_{c,0} = -\$565\,000$

It is not viable. The municipality will lose money.

Notation	Name	Find / Given	Formula
$P_n = P_0(P_n/P_0,j,n)$	Single payment Compound amount	P_n / P_0	$P_n = P_0 \, (1+j)^n$
$P_0 = P_n(P/F,j,n)$	Single payment Present worth	P_0 / P_n	$P_0 = \dfrac{P_n}{(1+j)^n}$
$P_n = S(P_n/S,j,n)$	Uniform series Compound amount	P_n / S	$P_n = S \dfrac{(1+j)^n - 1}{j}$
$S = P_n(S/P_n,j,n)$	Sinking fund	S / P_n	$S = P_n \dfrac{j}{(1+j)^n - 1}$
$P_0 = S(P_0/S,j,n)$	Uniform series Present worth	P_0 / S	$P_0 = S \dfrac{(1+j)^n - 1}{j\,(1+j)^n}$
$S = P_0(S/P_0,i,n)$	Capital recovery	S / P_0	$S = P_0 \dfrac{j\,(1+j)^n}{(1+j)^n - 1}$

Discounting Tool – Levelized Value

Levelized value is a technique used to convert a series of non-uniform payments into a uniform series payment per time per energy unit. In this way, the cost of a project relative to the energy produced can be examined through an equal payment or dividend per some time period; usually a year. This method is useful when comparing two different energy technologies; especially when comparing a fossil fuel technology with a renewable energy technology.

fossil fuel	renewable energy
relatively low capital	high capital cost
significant recurring costs in fuel; may escalate at different rate than inflation	low recurring costs

Levelization of Values

The levelization process is straightforward:

1. convert each value (future, present, series) into a present value;
2. sum all of the present values;
3. convert total equivalent present value into an equivalent uniform series of values over the anticipated life of the project; usually this is on an annual basis, but the time interval can be anything;
4. divide (equivalent cost/time interval) by the energy produced/consumed in that time interval.

These steps will use only the two basic equations derived, compounding and uniform series, but will require some manipulation of the equations.

Compounding: $P_n = P_0 \, (1 + j)^n$ Uniform Series: $P_n = S \left[\dfrac{(1 + j)^n - 1}{j} \right]$

Example 9. Levelizing a non-uniform Series

A series of payments will be made annually for ten years. The initial payment is \$20,000 and each year the payment increases by \$5,000. The interest rate is 10 %, compounded annually. Determine the equivalent uniform series value from the non-uniform series value.

Solution

Present value of non-uniform payments for 10 years:

$$P_0 = \prod_{n=1}^{10} \frac{P_{i,n}}{(1+j)^n} = \frac{20\,000}{(1+0.1)^1} + \frac{25\,000}{(1+0.1)^2} + \ldots + \frac{65\,000}{(1+0.1)^{10}} = \$237\,000$$

Convert this to uniform payments over 10 years:

$$S = P_0 \left[\frac{j\,(1+j)^n}{(1+j)^n - 1} \right] = 237\,000 \left[\frac{(0.1)\,(1+0.1)^{10}}{(1+0.1)^{10} - 1} \right] = \$38\,600 \text{ per year}$$

Example 10. Levelizing Cost of Electricity

An electric power plant that produces 2 billion kWh_e per year has a capital cost of $500 million and anticipated lifetime of 20 years. The salvage value is estimated to cover the cost of dismantling the plant. The capital cost of the plant is repaid at 7 % interest, compounded annually. The total annual operational cost of the plant is $25 million, and the annual return to investors is estimated at 10 % of the operating cost plus the capital repayment cost. Determine the levelized cost of electricity for this plant, in $/$kWh_e$.

The levelization will be done on an annual time increment. First, annualize the capital cost by taking the initial loan plus the interest and converting it into a uniform series of annual costs. Then add the other annualized cost; operational costs and annual dividends to investors. The levelized cost is the total annualized (uniform series) cost divided by the annual energy output.

Total capital cost: $P_{c,20} = P_{c,0}\ (1 + j)^n = 500\ 10^6\ (1 + 0.07)^{20} = \$1.935\ 10^9$

Uniform series payment:

$$S_c = P_{c,n} \left[\frac{j}{(1 + j)^n - 1} \right] = 1\ 935\ 00 \left[\frac{0.1}{(1 + 0.1)^{10} - 1} \right] = \$42.7\ 10^6 \text{ per year}$$

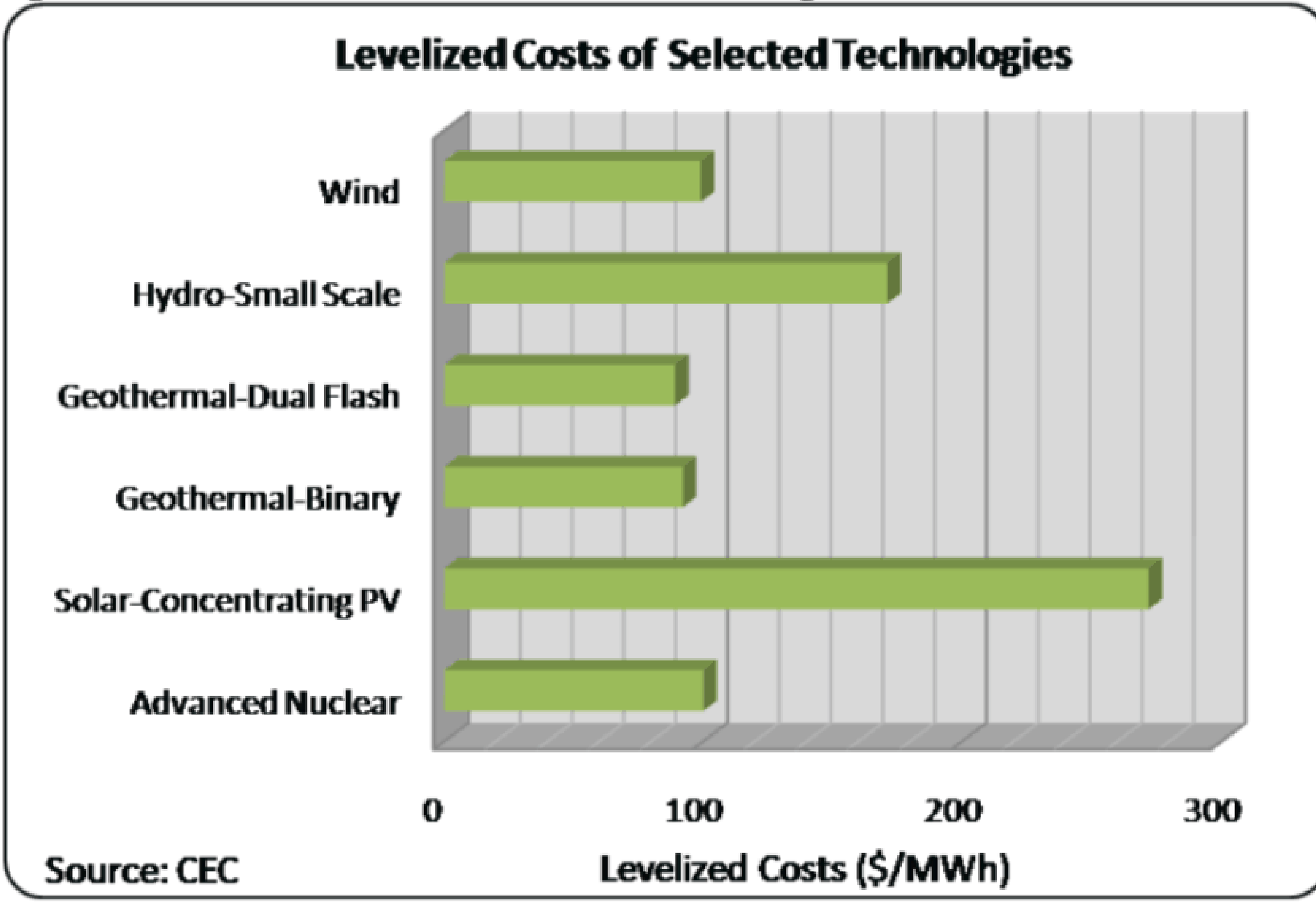
Annual operating cost: $S_0 = \$25\ 10^6$

Annual return to investors = $S_r = (0.10)\ (47.2 + 25)\ 10^6 = \$7.2\ 10^6$ per year

Total annual cost $S = (42 + 25 + 7.2)\ 10^6 = \$79.4\ 10^6$ per year

$$\text{Levelized cost} = \frac{79.4\ 10^6\ \$/\text{year}}{2\ 10^9\ \text{kWh/year}} \cong 0.04\ \$/\text{kWh}$$

Figure 27: Levelized Costs of Selected Technologies



Capitalized Cost

Capitalized cost, CC, refers to the present worth of cash flows which go on for an infinite period of time. Some public work projects like dams, bridges and parks fall into this category.

To compare two or more alternatives on the basis of capitalized cost, find CC for each alternative.

The alternative with the smallest capitalized cost should be selected.

$$P_0 = S\left[\frac{(1+j)^n - 1}{j\,(1+j)^n}\right]$$

If n is very large (goes to infinity)

$$P_0 \cong \frac{S}{j}$$

Example 11

1. A suspension bridge will cost $50 million to be built over 5 years with annual inspection and maintenance costs of $35,000. The building cost is spread out evenly over the 5 years. In addition, the concrete deck will have to be resurfaced every 10 years at a cost of $100,000. The cost of purchasing the right-of-way (on day 1) is expected to be $2 million for the suspension bridge.

2. A truss bridge and approach roads are expected to cost $30 million to be built over 2 years and annual maintenance costs of $20,000. The building cost is spread out evenly over the 5 years. The bridge will have to be painted every 3 years at a cost of $40,000. In addition, the bridge will have to be sandblasted every 10 years at a cost of $190,000. The cost of purchasing the right-of-way (on day 1) is expected to be $15 million for the truss bridge.

Compare the alternatives if the interest rate is 6 % per year.

$$CC_1 = 2\ 10^6 + 10\ 10^6 \left[\frac{(1.06)^5 - 1}{(1.06)^5\ 0.06}\right] + \frac{(35000/1.06)}{0.06} + \frac{\left(100000/(1.06)^{10}\right)}{0.06}$$

$$= 2\ 10^6 + 42\ 123\ 638 + 530\ 314.5 + 930\ 658 = 45\ 604\ 610$$

$$CC_2 = 15\ 10^6 + 15\ 10^6 \left[\frac{(1.06)^2 - 1}{(1.06)^2\ 0.06}\right] + \frac{(20000\ /\ 1.06)}{0.06} + \frac{\left(40000\ /\ (1.06)^3\right)}{0.06} + \frac{\left(190000\ /\ (1.06)^{10}\right)}{0.06}$$

$$= 15\ 10^6 + 27\ 500\ 890 + 314\ 465 + 559\ 746 + 1\ 768\ 250 = 45\ 143\ 352$$

The alternative 2 is more economical.