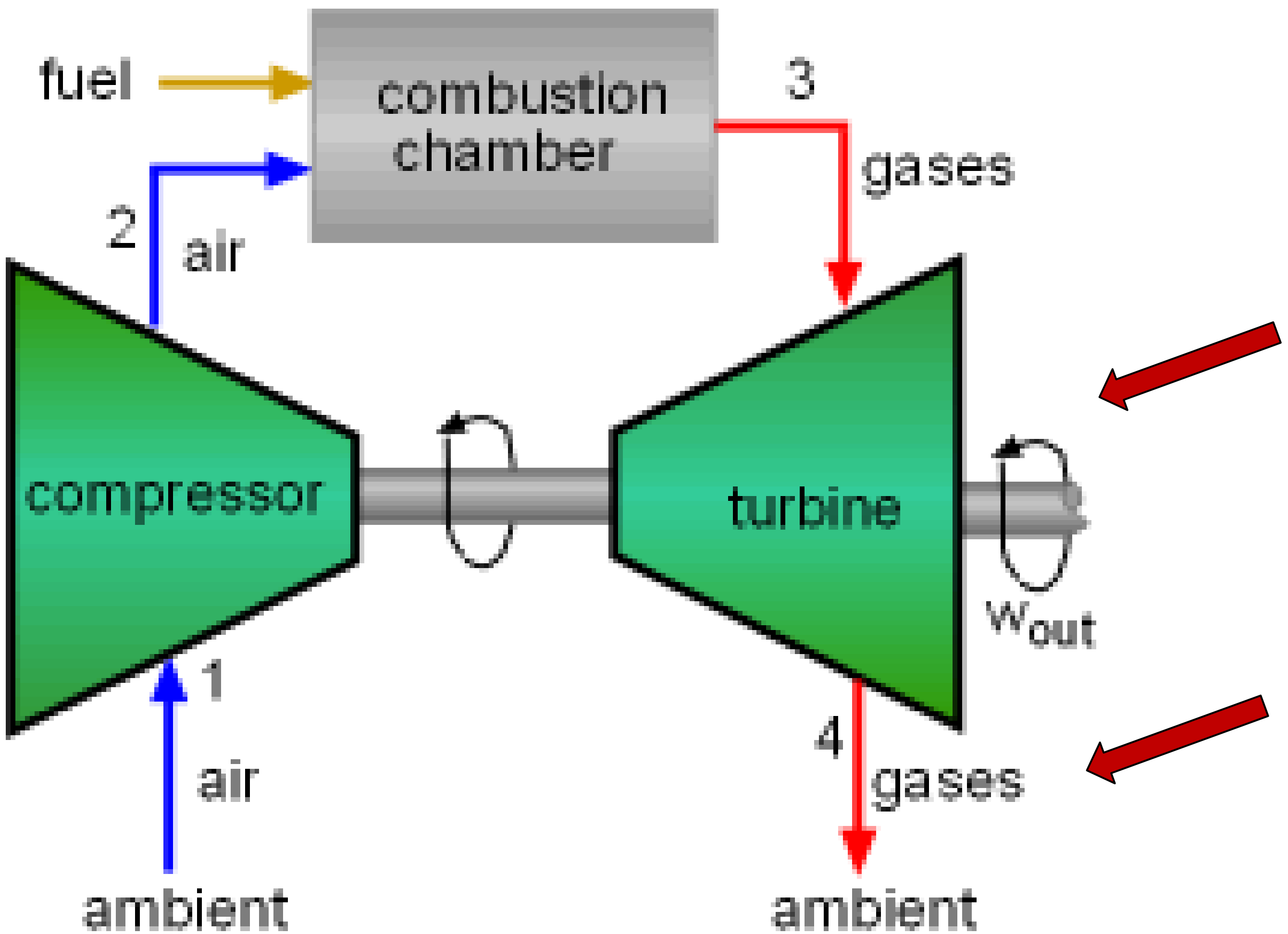
George Brayton

US Engineer

1830 -1892

Brayton (Gas) Cycle



The word ***engineer*** (Latin *ingeniator*) is derived from the Latin words *ingeniare* ("to contrive, devise") and *ingenium* ("cleverness«)

Engineers are professionals who invent, design, analyze, build and test machines, complex systems, structures, gadgets and materials to fulfill functional objectives and requirements while considering the limitations imposed by practicality, regulation, safety and cost.

The work of engineers forms the link between scientific discoveries and their subsequent applications to human and business needs and quality of life.

«Mühendis» - someone who knows (or works with) «hendese» (mathematics).

SOLUTION OF ENGINEERING PROBLEMS

Given	A real physical problem such as design and manufacture of a robot arm, heat exchanger, etc.
Determine	Length, area, volume, material, velocity, acceleration, forces, pressure, temperature, flow rates of mass and energy, etc.
Mathematical Formulation	Use simplifying assumptions => **Physical Modeling**
	Make force, momentum, mass, energy balances
	Obtain mathematical equations in the form of differential, integral, integro-differential, algebraic, set of algebraic, etc.
	Make further simplifications on the mathematical formulae => **Mathematical Modeling**

SOLUTION OF ENGINEERING PROBLEMS

Solve Mathematical Formulae	• Analytically (exactly) • **Numerically (approximately), using methods such as finite differences, finite elements, spectral methods, etc., using *computers and programming***

Report Errors and Interpret Results	With respect to the solution technique With respect to the simplifications in the mathematical formulae With respect to the simplifications in the physical model

Thermodynamic Relations

For an ideal gas & a polytropic process: $P\,\forall = m\,R\,T$ $\quad P\,\forall^n = \text{constant}$

$$\frac{P_1\,\forall_1}{T_1} = \frac{P_2\,\forall_2}{T_2} \quad \Rightarrow \quad \frac{\forall_1}{\forall_2} = \frac{P_2\,T_1}{P_1\,T_2}$$

$$P_1\,\forall_1^n = P_2\,\forall_2^n \quad \Rightarrow \quad \frac{\forall_1}{\forall_2} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\frac{P_2}{P_1}\,\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{-1/n}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{P_1}{P_2}\right)^{-1/n} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$$

For an ideal gas & isentropic process: $\Delta S = 0$

$$0 = S_2 - S_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

with enthapies: $0 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

→ Derived from 1st law with reversible work (P dv) and reversible heat (T ds)

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{c_v}\ln\left(\frac{v_2}{v_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{-\frac{R}{c_v}}$$

Recall:

$$R = c_p - c_v$$

$$k = \text{Isentropic Expansion Factor} = \frac{c_p}{c_v}$$

$$\frac{R}{c_v} = k - 1$$

Ideal Gas

Isentropic Process

$$\left.\frac{T_2}{T_1}\right|_{\Delta s=0} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

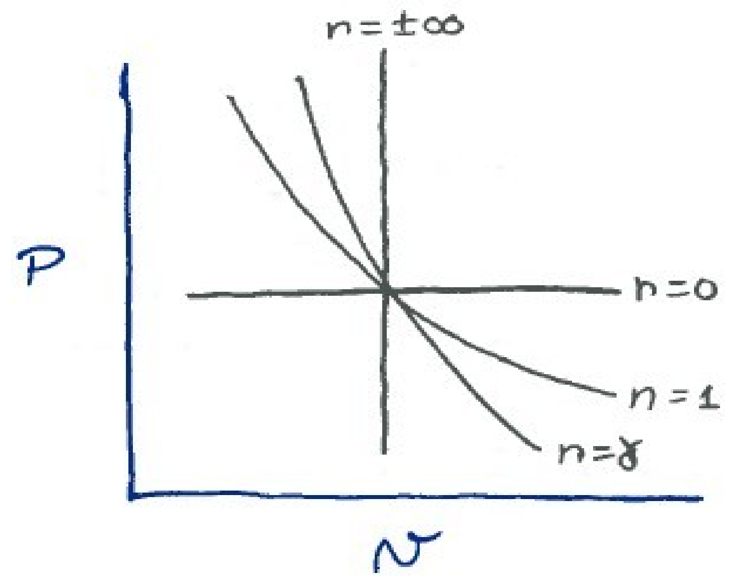
$$\left.\frac{P_2}{P_1}\right|_{\Delta s=0} = \left(\frac{v_1}{v_2}\right)^{k}$$

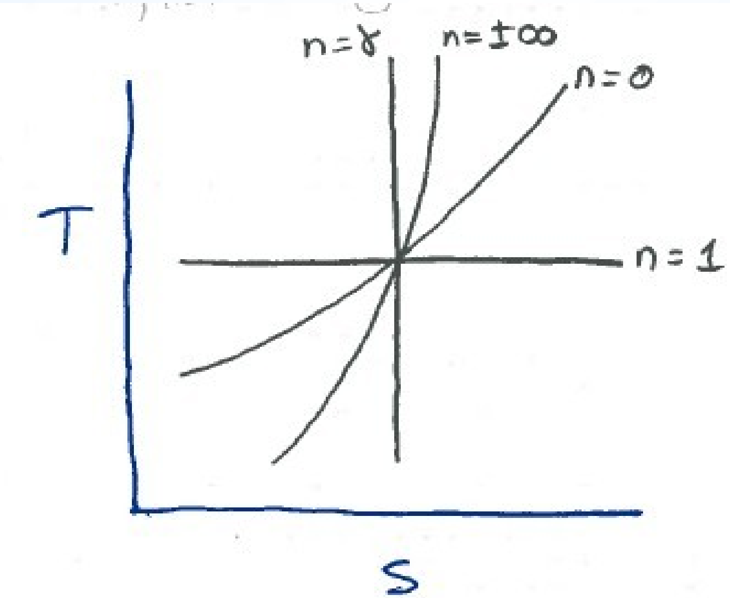
$$\left.\frac{T_2}{T_1}\right|_{\Delta s=0} = \left(\frac{P_2}{P_2}\right)^{\frac{k-1}{k}}$$

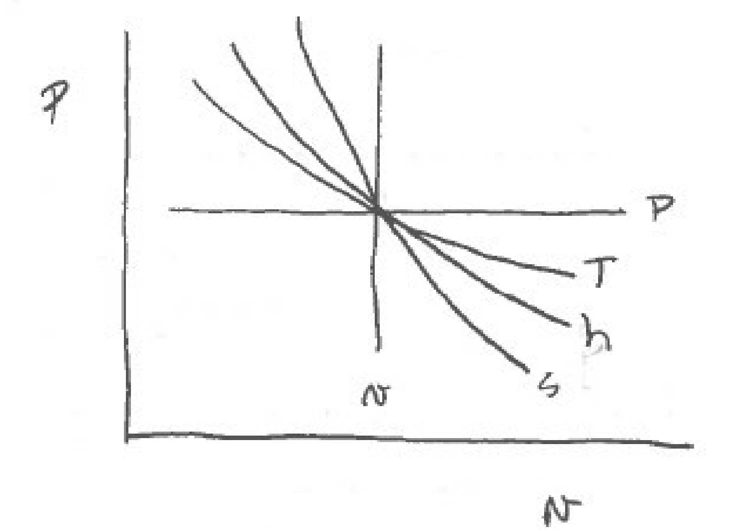
Polytropic Processes $P\,\forall^{n} = P_1\,\forall_1^{n} = P_2\,\forall_2^{n} = \text{constant}$

$$\delta Q = c\ m\ dT$$

Process	c	n
Constant Volume (Isochoric)	c_v	∞
Constant Pressure (Isobaric)	c_p	0
Constant Temperature (Isothermal)	∞	1
Adiabatic, Reversible (Isentropic)	0	$k = c_p / c_v$
Polytropic	$c_v \left(\frac{k - n}{1 - n} \right)$	0 to ∞


n=±∞
P
n=0
n=1
n=γ
v


n=γ
n=±∞
n=0
T
n=1
s


P
P
T
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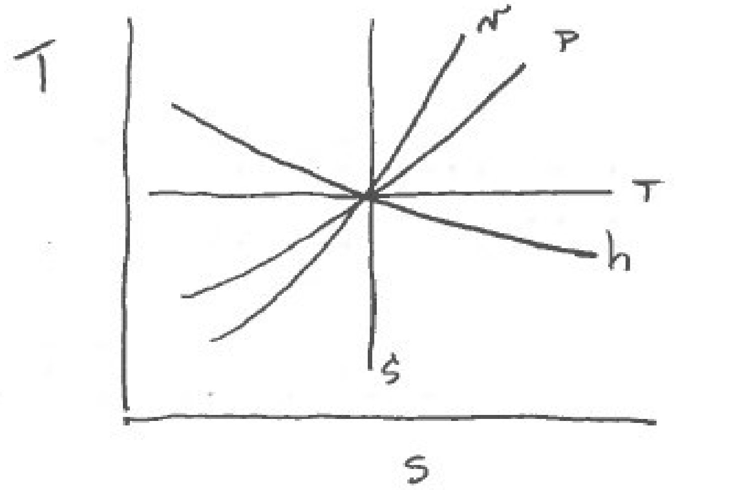
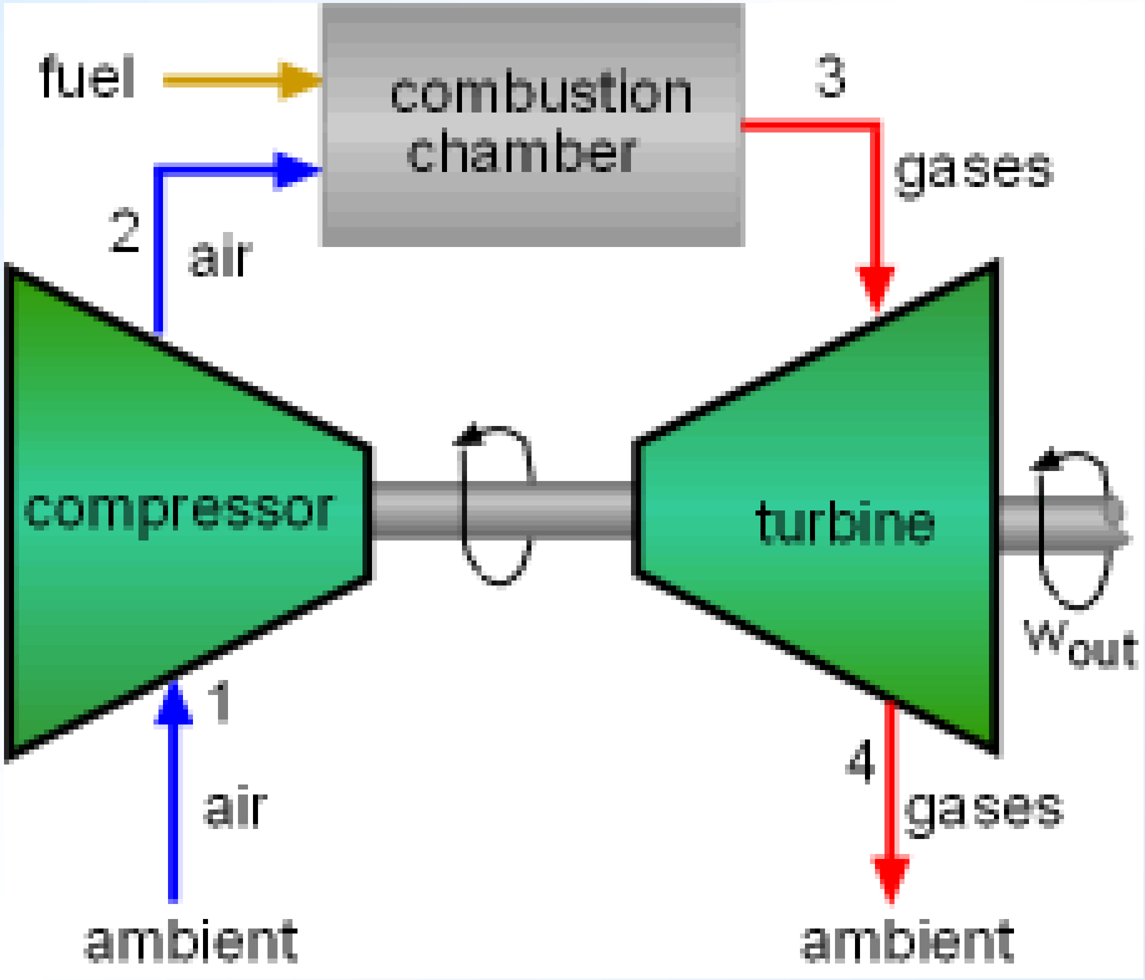
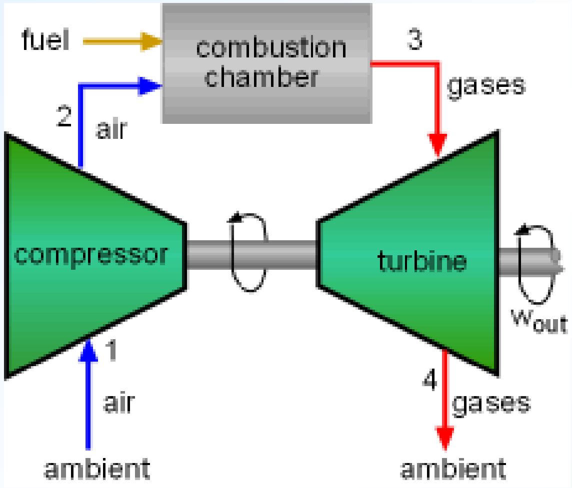

T
v
P
T
h
s
s

Open (to atmosphere) Ideal (isentropic exp/comp) Brayton Cycle



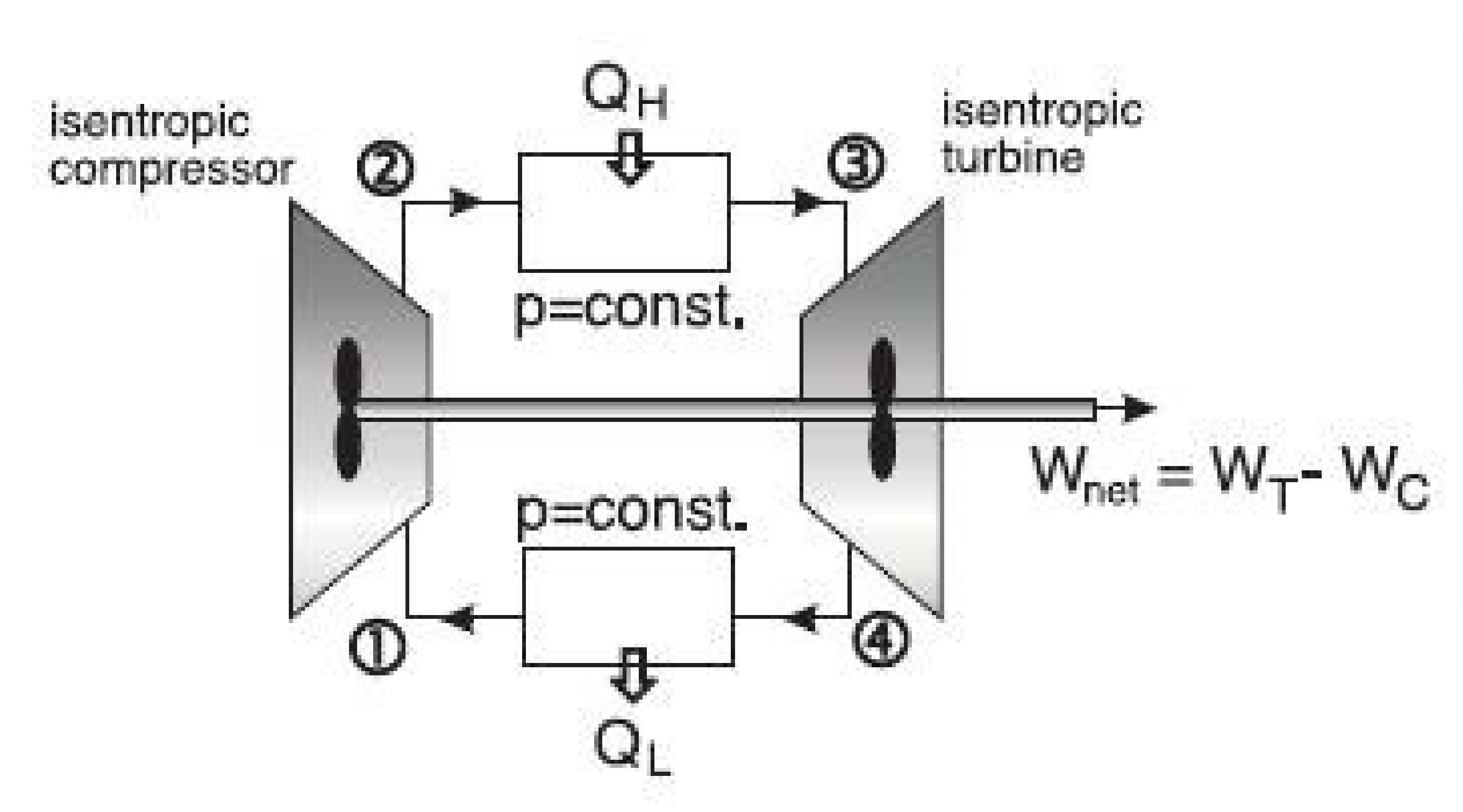
Ideal Brayton Cycle

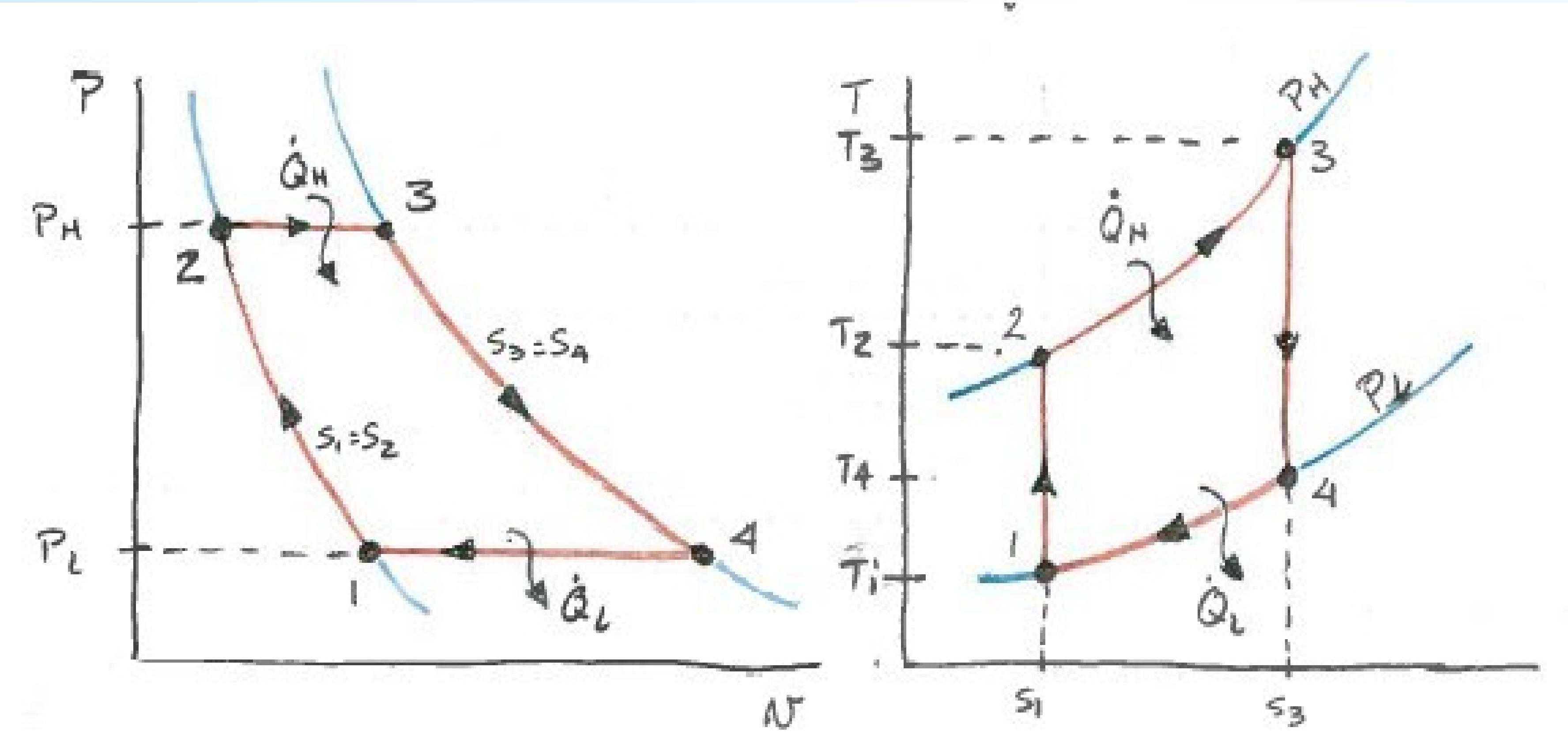


Air standard cycle:

- Fixed mass of air throughout; no inlet /exhaust process
- Air is an ideal gas
- Combustion process is replaced by heat transfer from an external source
- Cycle is completed by heat transfer to surroundings
- All processes are internally reversible
- Air has a constant specific heat

Indirect Closed Brayton Cycle





P
$\dot{Q}_H$
3
P_H
2
$s_3 = s_4$
$s_1 = s_2$
P_L
4
1
$\dot{Q}_L$
v
T
T_3
P_H
3
$\dot{Q}_H$
T_2
2
P_L
T_4
4
1
T_1
$\dot{Q}_L$
s_1
s_3

The Brayton cycle is widely used in **aircraft engines**, where it powers the propulsion system. Gas turbine engines, which operate on the Brayton cycle, provide the necessary thrust for aircraft to overcome drag and achieve flight. These engines are known for their high power-to-weight ratio, making them ideal for aviation applications.

In addition to aviation, the Brayton cycle is also employed in power (**electricity**) generation. Gas turbine power plants utilize the Brayton cycle to convert the chemical energy of fuel into mechanical work, which is then used to drive a generator and produce electricity. These power plants are known for their quick startup time, high efficiency, and flexibility in fuel selection.

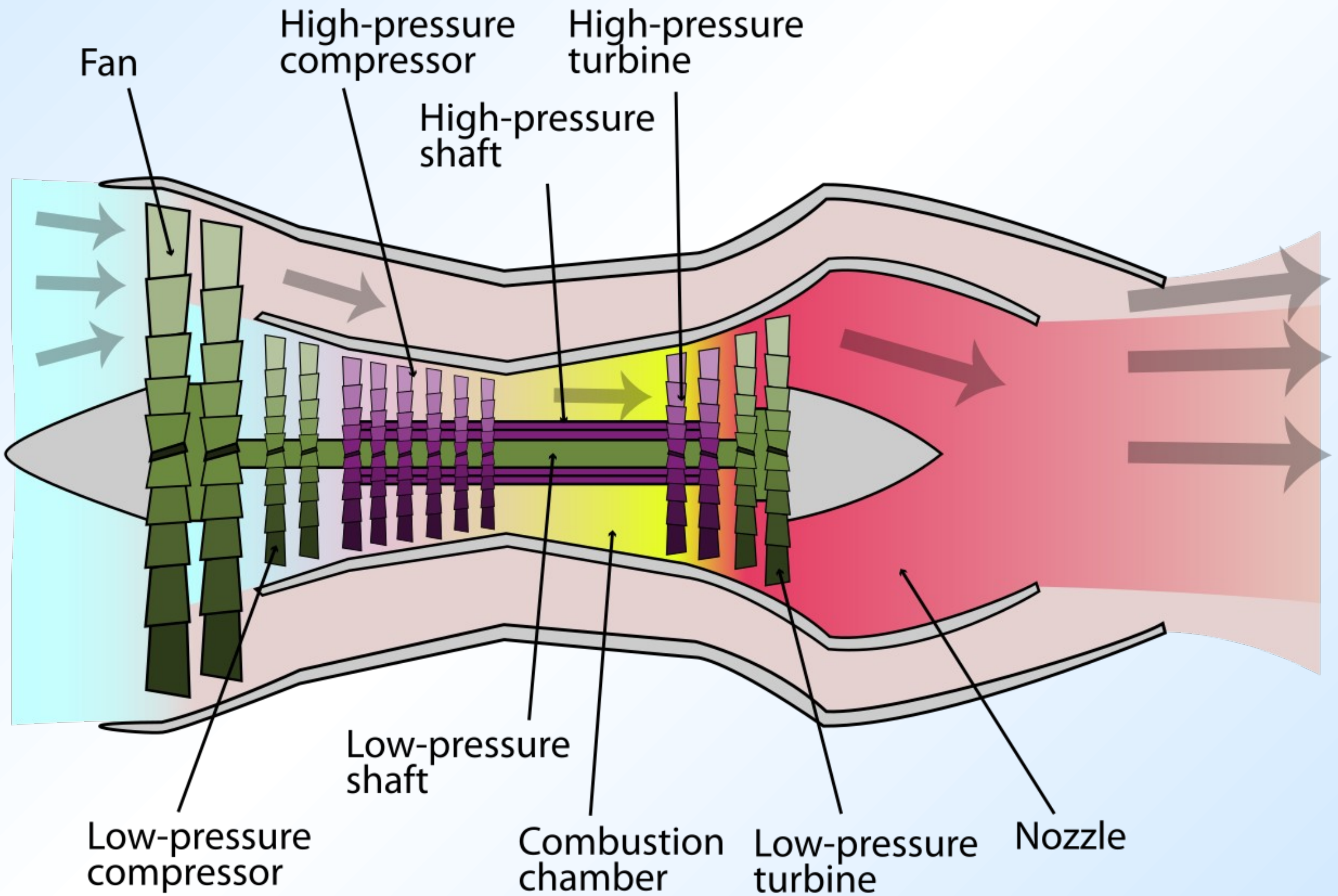
The Brayton cycle offers several **advantages**, including high power density, compact size, and quick startup time. Gas turbine engines based on the Brayton cycle can achieve high thermal efficiencies, especially in combined cycle configurations where waste heat from the turbine is recovered to generate additional power. However, the Brayton cycle has limitations in terms of lower thermal efficiency compared to the Rankine cycle and higher emissions due to the combustion of fossil fuels.

On the other hand, the Rankine cycle offers higher thermal efficiencies compared to the Brayton cycle, especially in large-scale power generation applications. The use of water as the working fluid allows for efficient heat transfer and the ability to utilize waste heat from other processes. However, the Rankine cycle has limitations in terms of slower startup time, larger footprint, and the need for a constant supply of water.

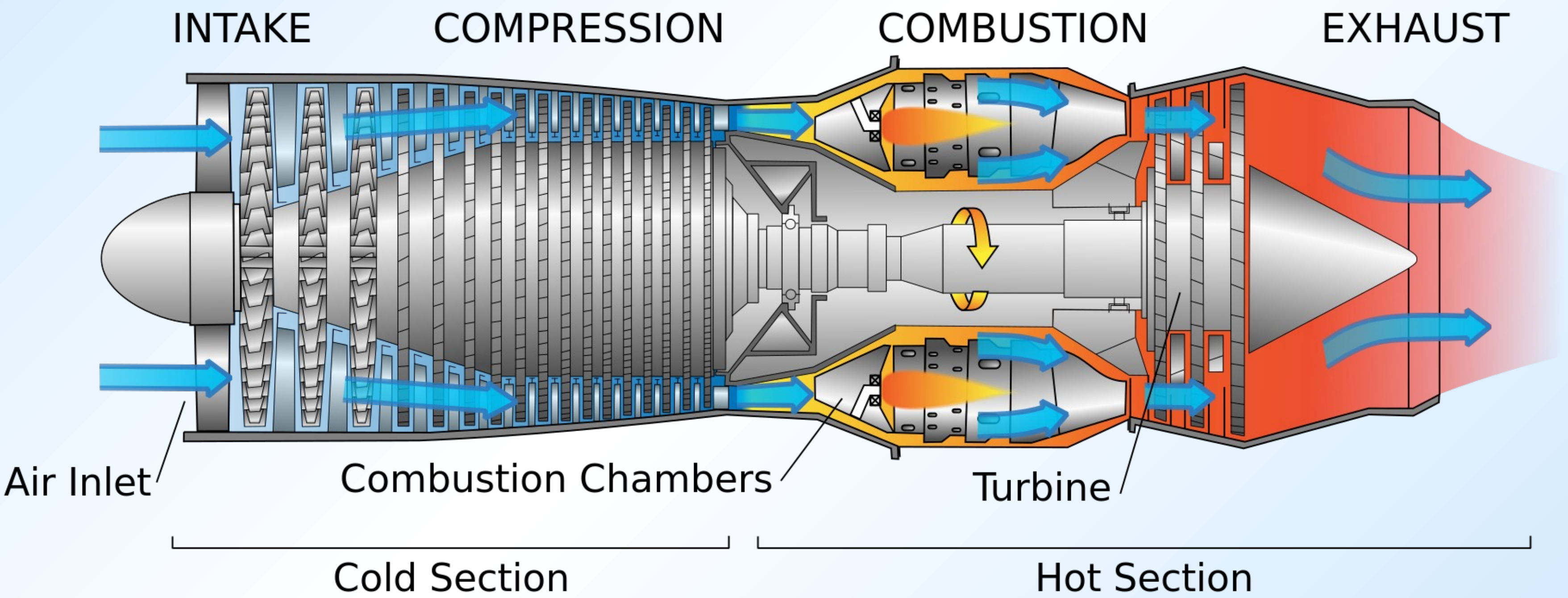
Advantages of Brayton Cycle:

- High thermal efficiency: The Brayton Cycle can achieve high thermal efficiencies, especially in gas turbine engines, due to the high temperatures at which the combustion process occurs.
- Flexibility: The Brayton Cycle can be used with a variety of fuels, including natural gas, diesel, and even hydrogen. This flexibility makes it suitable for a wide range of applications, from power generation to aircraft propulsion.
- Quick startup and shutdown: Gas turbine engines based on the Brayton Cycle can be started and stopped quickly, making them ideal for applications where rapid response is required.

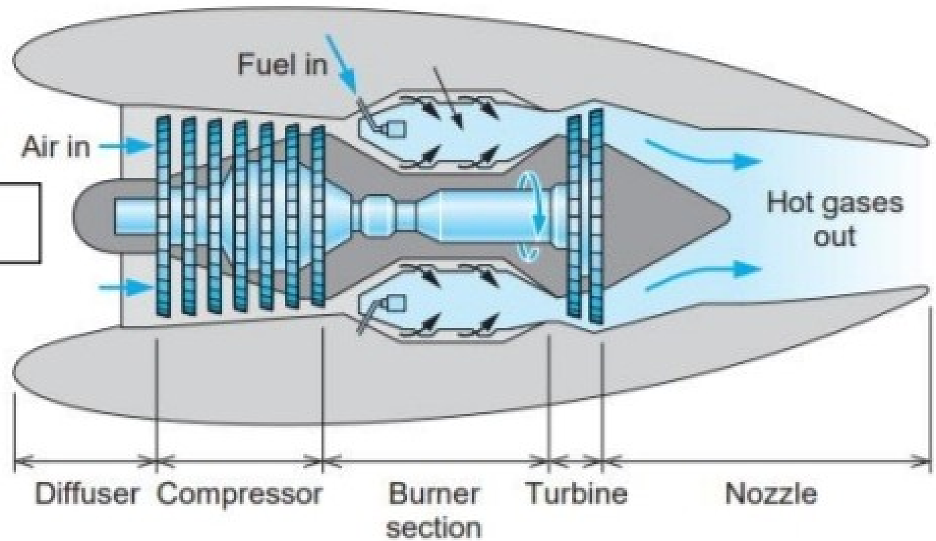
Turbofan Engine



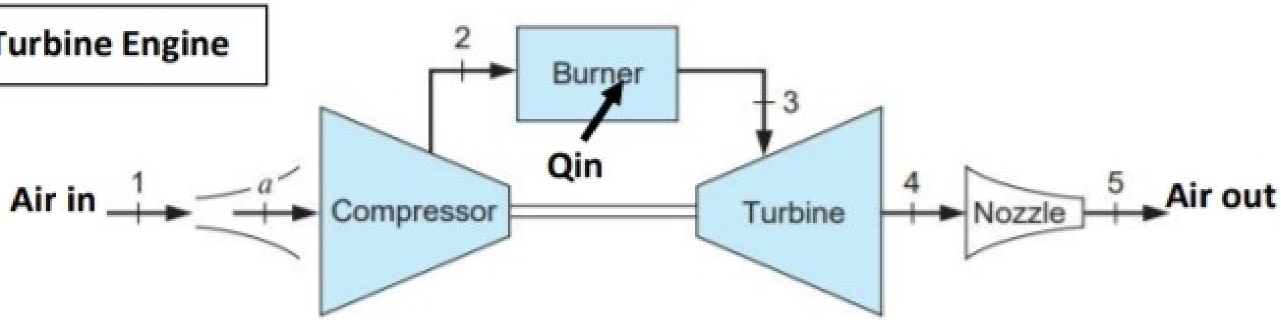
Turbojet Engine



Actual engine



Modelled Gas Turbine Engine



First law for each process:

$$\dot{Q} - \dot{W} = H_{exit} - H_{inlet}$$
$$= \dot{m}(h_{exit} - h_{inlet})$$

Uniform, steady flow

neglecting changes in KE and PE

$$q - w = h_{exit} - h_{inlet}$$
$$= c_p (T_{exit} - T_{inlet})$$

→ Ideal gas

Thermal Efficiency:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{q_L}{q_H}$$

$\dot{m}$ is constant for the cycle

$$q_L = -\,{}_4q_1 = -c_p (T_1 - T_4)$$
$$q_H = +\,{}_2q_3 = c_p (T_3 - T_2)$$

$$\eta_{th} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2}\right) = 1 - \frac{T_1}{T_2}\left[\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right]$$

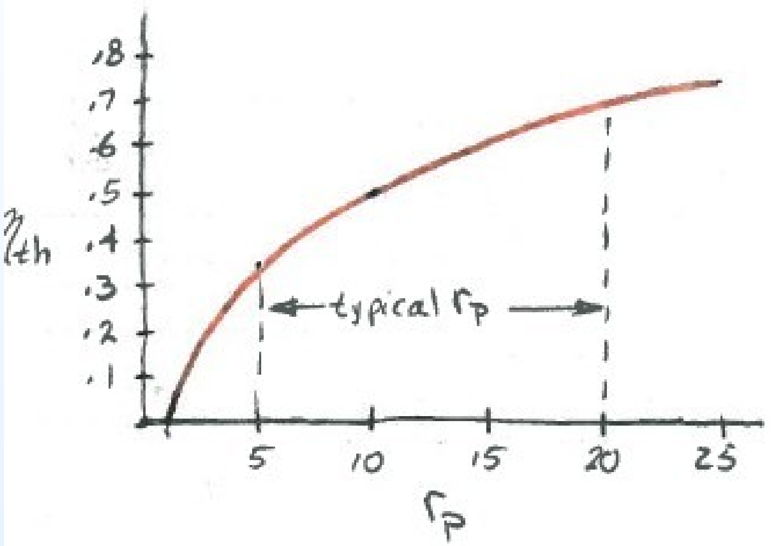
Recall, process 1 – 2 is an isentropic compression

Therefore, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$ r_p is the pressure ratio

Also, $P_2 = P_3$ and $P_1 = P_4$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ Then $\frac{T_4/T_1 - 1}{T_3/T_2 - 1} = 1$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

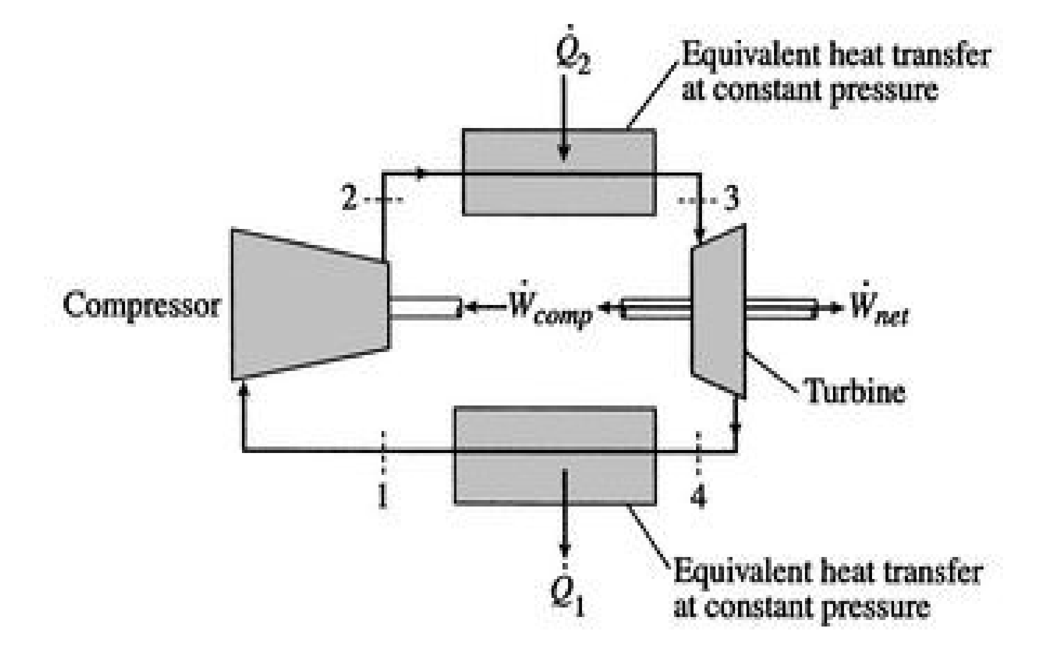


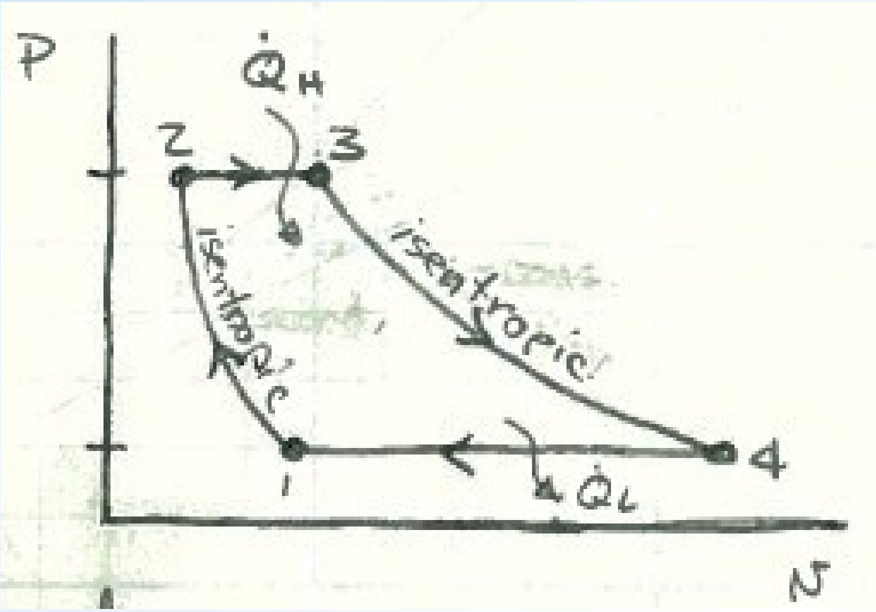
Example 4

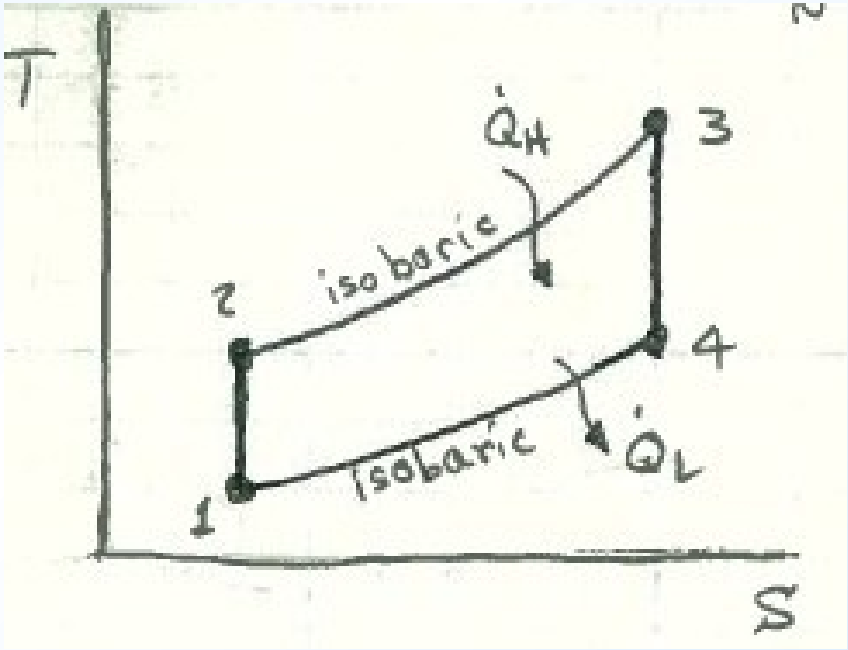
In an air-standard Brayton cycle, the air enters the compressor at 0.1 MPa, 15 °C. The pressure leaving the compressor is 1.0 MPa and the maximum temperature in the cycle is 1100 °C. Determine

(a) The pressure and temperature at each point in the cycle

(b) The compressor work, turbine work, and cycle efficiency







Prepare a table that looks like this:

	State 1	State 2	State 3	State 4
Pressure, MPa				
Temperature, K				

- Assume steady, uniform flow
- Neglect changes in KE and PE

For an isentropic process, 1 to 2: $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

For an ideal gas: $\Delta h = c_p \, \Delta T$

State 1: $T_1 = 15 + 273 = 288$ K and $P_1 = 0.1$ MPa

State 2: $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (15 + 273) \left(\frac{1}{0.1}\right)^{\frac{1.4-1}{1.4}} = 556.8 \text{ K}$

Compressor work: $_1w_2 = h_1 - h_2 = c_p\ (T_1 - T_2) = (1.005)\ (288 - 556.8) = -\ 270\ \text{kJ/kg}$

State 3: $P_3 = P_2 = 1\ \text{MPa}$ and $T_3 = 1100 + 273 = 1373\ \text{K}$

Heat Transfer: $_2q_3 = h_3 - h_2 = c_p\ (T_3 - T_2) = (1.005)\ (1373 - 556.8) = 821\ \text{kJ/kg}$

State 4: $P_4 = P_1 = 0.1\ \text{MPa}$ $\quad T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = (1373) \left(\frac{0.1}{1}\right)^{\frac{1.4-1}{1.4}} = 710.8\ \text{K}$

Turbine work: $_3w_4 = h_3 - h_4 = c_p\ (T_3 - T_4) = (1.005)\ (1373 - 710.8) = 665\ \text{kJ/kg}$

Net work: $w_{net} = 665 - 270 = 395\ \text{kJ/kg}$

Heat transfer: $_4q_1 = q_H = h_4 - h_1 = c_p\ (T_4 - T_1) = (1.005)\ (710.8 - 288) = 425\ kJ/kg$

Efficiency: $\eta = \dfrac{w_{net}}{q_H} = \dfrac{395}{821} = 0.48$

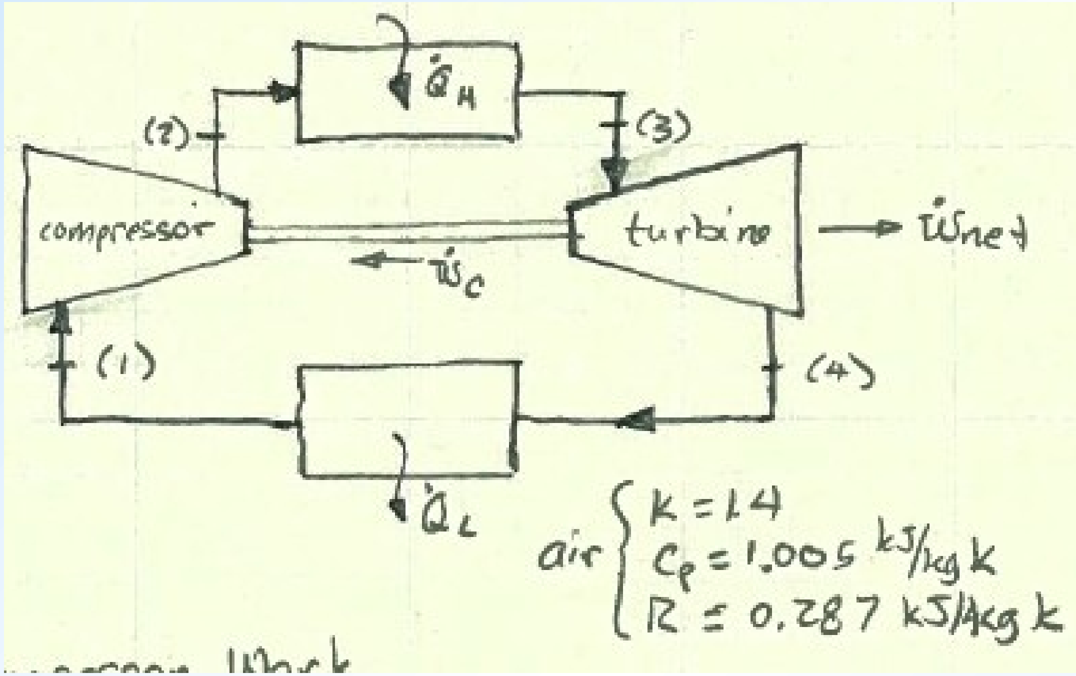
	State 1	State 2	State 3	State 4
Pressure, MPa	0.1	1	1	0.1
Temperature, K	288	556.8	1373	711
	↑ Inlet to Compressor	↑ Inlet to Combustor	↑ Inlet to Turbine	↑ Exit to Atmosphere

Example 5

An air-standard Brayton cycle operates with a compression ratio of 5.0. The actual expansion & compression efficiencies of the gas processes are 88 % and 82 %, respectively, and the maximum and minimum temperatures are 750 °C and 16 °C, respectively. Compute

- the compression work;
- the expansion work;
- the ratio of compression work to expansion work (back-work ratio); and
- the actual and theoretical thermal efficiencies.

If the power output of the installation is 8 MW, determine the mass flow rate, kg/min.



$r_p = 5.0$

$T_{max} = 750\ °C = T_3$

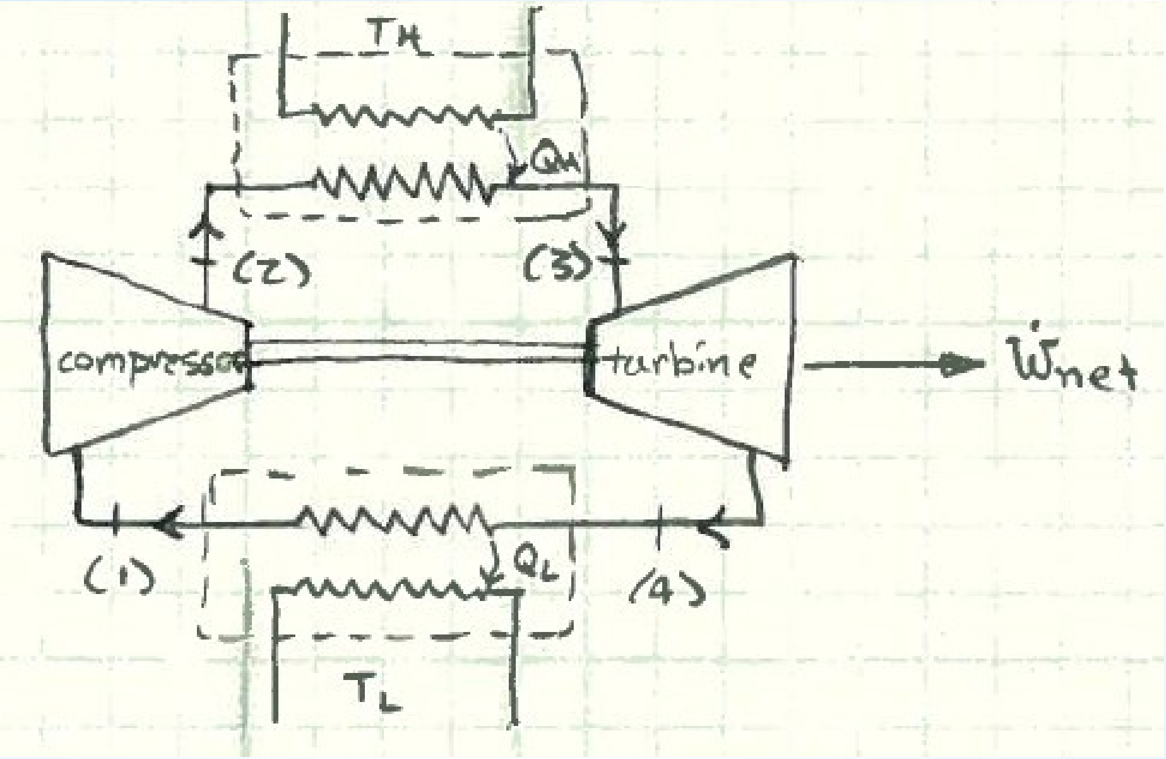
$T_{min} = 16\ °C = T_1$

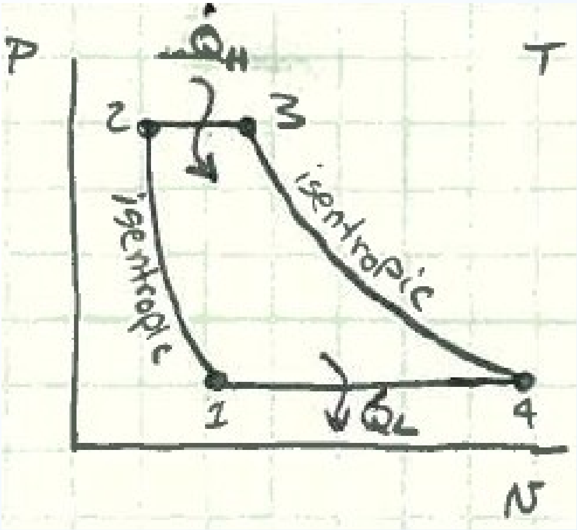
$\eta_T = 0.88$

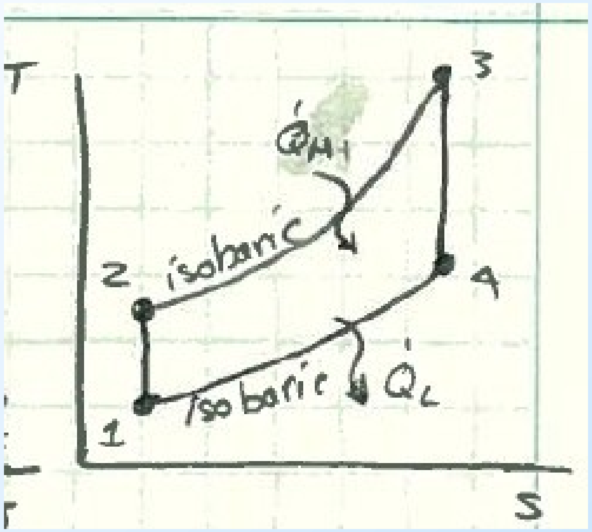
$\eta_c = 0.82$

$\dot{W}_{net} = 8\ MW$

$w_c = ?$ $w_T = ?$ BWR = ? $\eta_{ac} = ?$ $\eta_{id} = ?$ $\dot{m} = ?$


TH
QH
(2)
(3)
compressor
turbine
Wnet
(1)
QL
(4)
TL


P
QH
T
2
3
isentropic
isentropic
1
QL
4
v


T
3
QH
isobaric
2
4
isobaric
QL
1
s

Turbine work: $\dot{W}_T = \dot{m} c_p (h_3 - h_4) = \dot{m} \int_{T_4}^{T_3} c_p(T)\, dT$

For constant specific heat: $\dot{W}_T = \dot{m} c_p (T_3 - T_4)$

Isentropic process & ideal gas: $\dfrac{T_3}{T_4} = \left(\dfrac{P_3}{P_4}\right)^{\frac{k-1}{k}}$

Pressure ratio is defined as: $r_{p_T} = \dfrac{P_3}{P_4}$

Therefore: $\dot{W}_T = \dot{m} c_p T_3 \left(1 - \dfrac{1}{r_{p_T}^{\frac{k-1}{k}}}\right)$

Compressor work: $\dot{W}_C = \dot{m} c_p (h_2 - h_1) = \dot{m} c_p T_2 \left(1 - \dfrac{1}{r_{p_C}^{\frac{k-1}{k}}} \right)$

For an ideal Bryton cycle: $r_{p_T} = r_{p_C} = r_p$

Net work: $\dot{W}_{net} = \dot{W}_T - \left| \dot{W}_C \right| = \underbrace{\dot{m} c_p (T_3 - T_2)}_{\dot{Q}_H} \left(1 - \dfrac{1}{r_p^{\frac{k-1}{k}}} \right)$

Thermal efficiency: $\eta_{th} = \dfrac{\dot{W}_{net}}{\dot{Q}_H} = \left(1 - \dfrac{1}{r_p^{\frac{k-1}{k}}} \right)$

$$\frac{\dot{W}_{net}}{\dot{m}} = w_{net} = c_p\,(T_3 - T_2)\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

$$w_{net} = c_p\left[T_3 - T_1\, r_p^{\frac{k-1}{k}}\right]\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$$

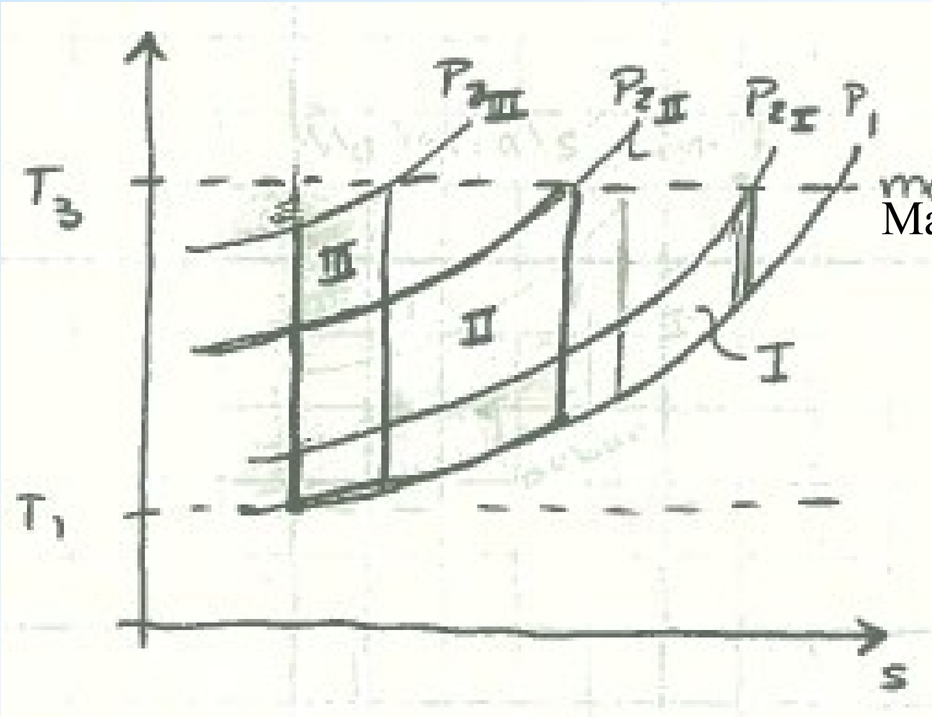
Or

$$w_{net} = c_p\left\{\underbrace{T_1\left(1 - r_p^{\frac{k-1}{k}}\right)}_{T_1 = T_{in}} + \underbrace{T_3\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)}_{T_3 = T_{max}}\right\}$$

Decreases with increase in r_p (first term, T_{in}) — Increases with increase in r_p (second term, T_{max})

Note the following:

1. All other things being equal (T_1, T_3, r_p, k), the specific work of the cycle is proportional to c_p; the higher the c_p the higher the specific work. Thus, helium can produce five times as much work as can air at low temperatures.
   - Specific heats for monatomic gases (He, Ar) are relatively constant and independent of temperature.
   - Specific heats for diatomic gases (O_2, N_2, air) increase with temperature.
   - Specific heats for triatomic gases (CO_2) increase with temperature faster than diatomic gases.
2. All other things being equal, gases with higher values of k, produce more work than gases with lower values of k.
3. For any particular gas, an increase in r_p from 1.0 (no work) decreases one part of w_{net}, and increases the other part.



Materials limit

$Area_{II} > Area_{III}$ & $Area_{I}$

$r_{pI} < r_{pIII} < r_{pI}$

The optimum pressure ratio for an ideal Bryton cycle can be found by differentiating the net work with respect to r_p and setting the derivative to zero

$$r_{p,\ optimum} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

$$\dot{W}_{net} = Q_1 - Q_2 = \dot{m} c_p (T_3 - T_2) - \dot{m} c_p (T_4 - T_1)$$

$$\dot{W}_{net} = \dot{m} c_p \left[(T_3 - T_4) - (T_2 - T_1)\right]$$

$$\dot{W}_{net} = \dot{m} c_p \left[T_3 \left(1 - \frac{T_4}{T_3}\right) - T_1 \left(\frac{T_2}{T_1} - 1\right)\right]$$

$$\dot{W}_{net} = \dot{m} c_p \left[T_3 \left(1 - \frac{1}{r_p}\right)^{\frac{k-1}{k}} - T_1 \left(\left(r_p\right)^{\frac{k-1}{k}} - 1\right)\right]$$

For optimum pressure ratio: $\dfrac{d\dot{W}_{net}}{dr_p} = 0$

$$r_{p,\ optimum} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

Derivation of the optimum pressure ratio as a function of initial (T_1) and maximum (T_3) temperature of an for an ideal Bryton cycle:

$$w_{net} = c_p \left\{ T_1 \left(1 - r_p^{\frac{k-1}{k}} \right) + T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right) \right\}$$

Define $\beta = \dfrac{k-1}{k}$ $\qquad w_{net} = c_p \left\{ T_1 \left(1 - r_p^{\beta} \right) + T_3 \left(1 - \dfrac{1}{r_p^{\beta}} \right) \right\}$

$$\frac{w_{net}}{dr_p} = c_p \left\{ T_1 \left(- \beta \, r_p^{\beta-1} \right) + T_3 \left(1 - \beta \, r_p^{-\beta-1} \right) \right\} = 0$$

$$\frac{w_{net}}{dr_p} = c_p \left\{T_1 \left(- \beta \, r_p^{\beta-1}\right) + T_3 \left(1 - \beta \, r_p^{-\beta-1}\right)\right\} = 0$$

$$- T_1 \, r_p^{\beta-1} + T_3 \frac{1}{r_p^{\beta+1}} = 0 \quad => \quad T_1 \, r_p^{\beta-1} \, r_p^{\beta+1} = T_3 \quad => \quad r_p^{\beta-1} \, r_p^{\beta+1} = \frac{T_3}{T_1}$$

$$r_p^{2\beta} = \frac{T_3}{T_1} \quad => \quad r_p = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2\beta}}$$

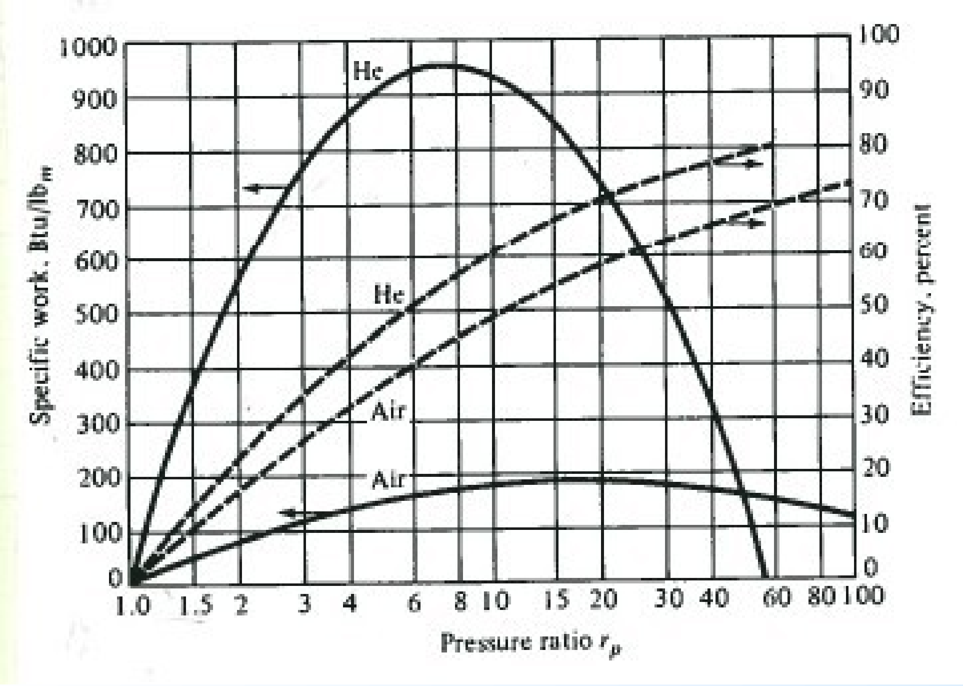
$$2\,\beta = \frac{2\,(k-1)}{k}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

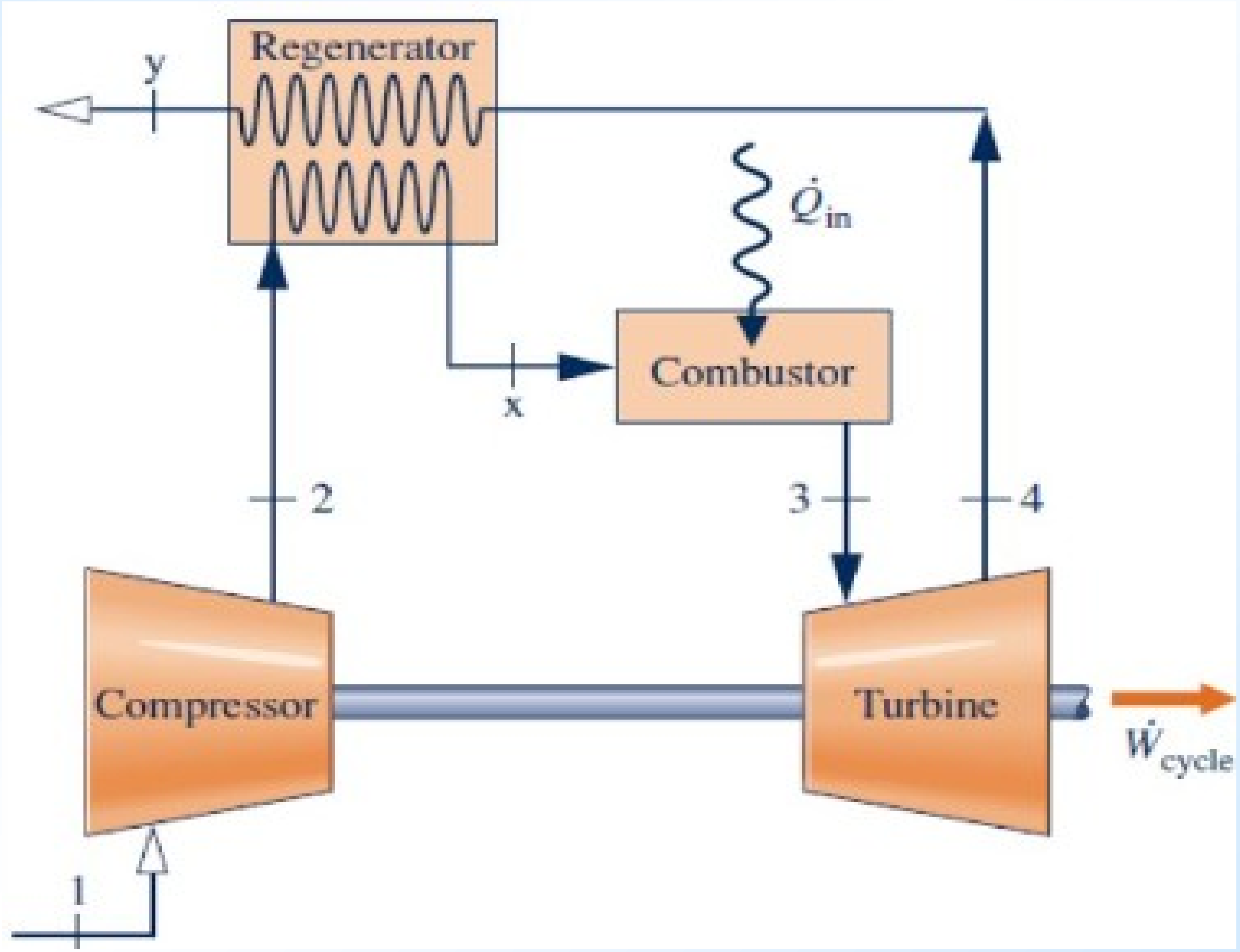
Optimum pressure ratio for simple Bryton cycle

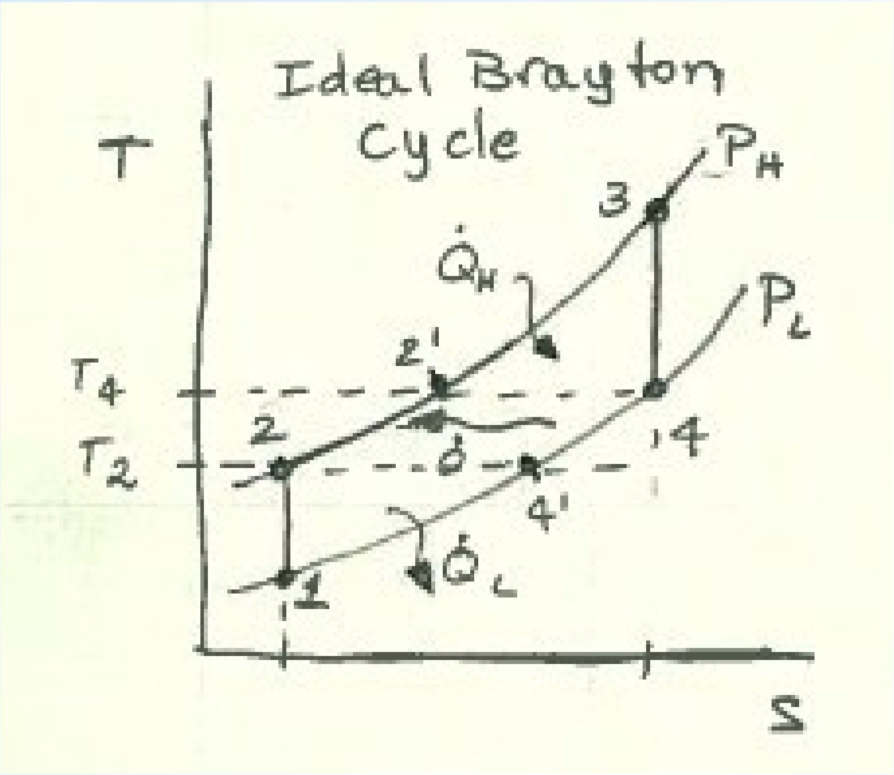
Example 6

Find the pressure ratio required by an ideal Bryton cycle to produce a net work of 600 Btu/lbm of (i) helium and (ii) air with constant specific heats. The cycle has initial and maximum temperatures of 500 R and 2500 R, respectively. Also, calculate the optimum pressure ratio for both gases.



Brayton Cycle – Regeneration



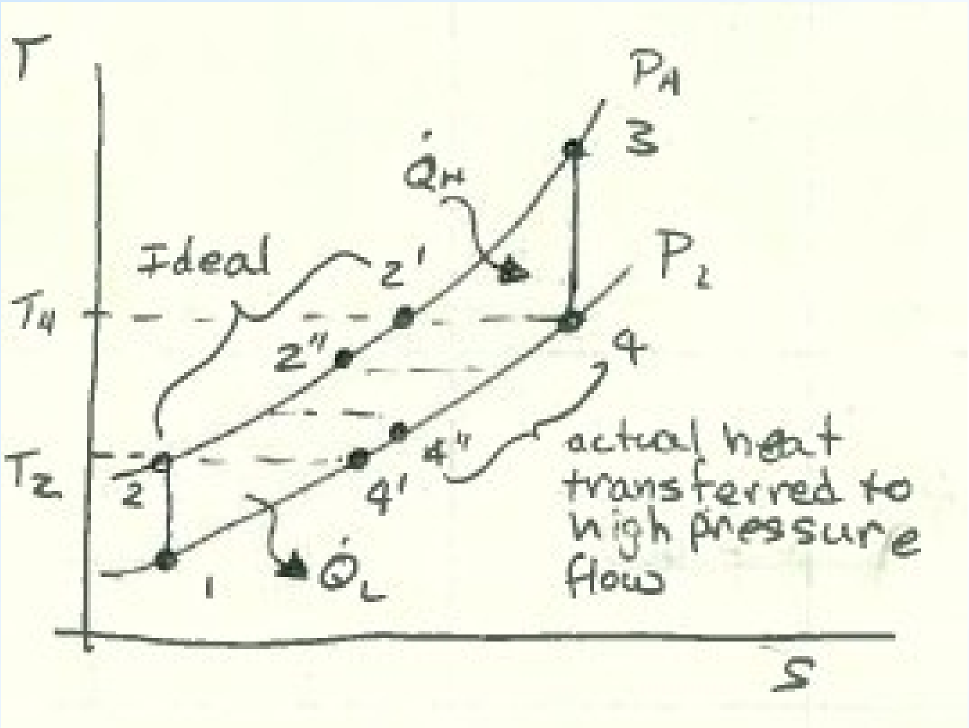


$T_4 > T_2$

Thermal energy from the turbine exhaust can be transferred to the compressor exhaust thereby saving energy in the combustor.

Highest temperature in the regenerator is T_4 (turbine exhaust)

In practice, the exit temperature of the high pressure flow is less than T_4



$$q_{req}\Big|_{max} = h_{2'} - h_2 = h_4 - h_{4'}$$

$$q_{req}\Big|_{act} = h_{2''} - h_2 = h_4 - h_{4''}$$

Regenerator effectiveness:

$$\varepsilon_R = \frac{q_{req}\big|_{act}}{q_{req}\big|_{max}}$$

$$\varepsilon_R = \frac{q_{req}\big|_{act}}{q_{req}\big|_{max}} = \frac{h_{2''} - h_2}{h_{2'} - h_2} = \frac{h_4 - h_{4''}}{h_4 - h_{4'}}$$

$$\varepsilon_R = \frac{q_{req}\big|_{act}}{q_{req}\big|_{max}} = \frac{h_{2''} - h_2}{h_{2'} - h_2} = \frac{h_4 - h_{4''}}{h_4 - h_{4'}}$$

For an ideal gas with constant specific heats: $\varepsilon_R = \dfrac{T_{2''} - T_2}{T_{2'} - T_2} = \dfrac{T_4 - T_{4''}}{T_4 - T_{4'}}$

Note that $T_{4'} = T_2$ and $T_{2'} = T_4$ $\varepsilon_R = \dfrac{T_{2''} - T_2}{T_4 - T_2} = \dfrac{T_4 - T_{4''}}{T_4 - T_2}$

Typical effectiveness $<$ 0.85

Thermal efficiency $\eta_{th} = 1 - \dfrac{T_{4''} - T_1}{T_3 - T_{2''}} = 1 - \dfrac{T_{4''} - T_{min}}{T_{max} - T_{2''}}$

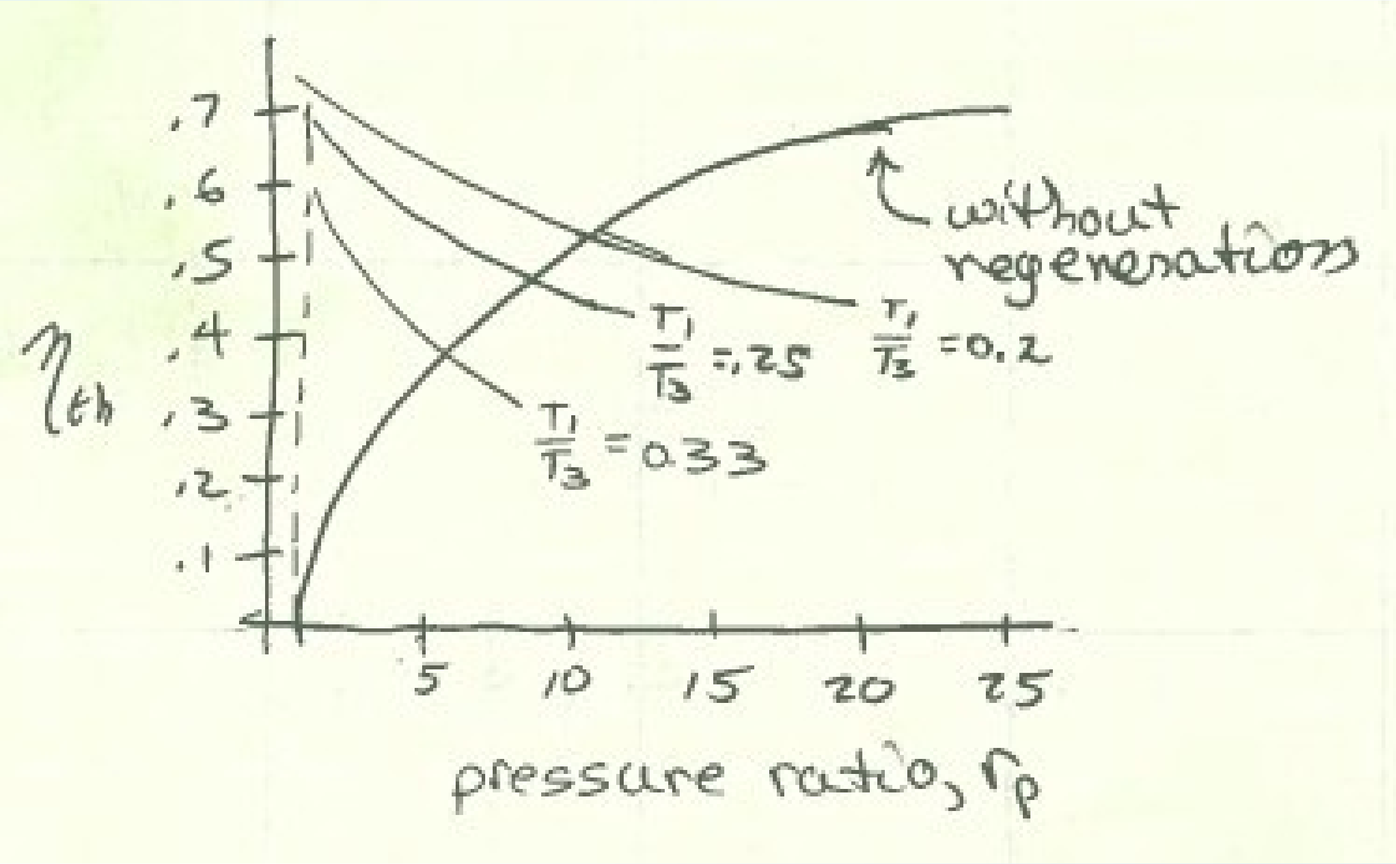
For an ideal cycle $T_{4''} = T_{4'} = T_2$ and $T_{2''} = T_{2'} = T_4$

The thermal efficiency with regeneration becomes: $\eta_{th} = 1 - \left(\dfrac{T_1}{T_3}\right) r_p^{\frac{k-1}{k}}$

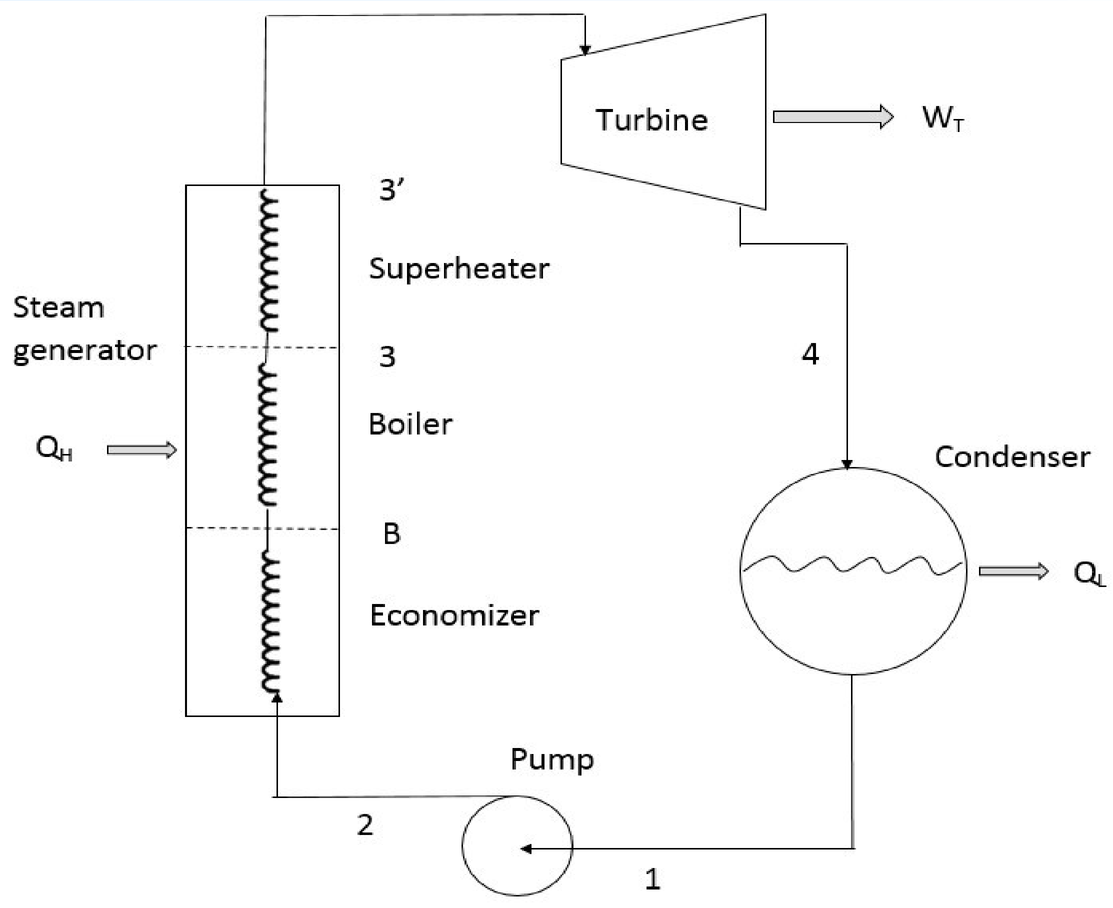
($T_1 = T_{min}$, $T_3 = T_{max}$)

Without regeneration: $\eta_{th} = 1 - \dfrac{1}{r_p^{\frac{k-1}{k}}}$

Note that the effects of k and r_p are reversed with regeneration as compared to a Bryton cycle without regeneration.



.7
.6
.5
.4
.3
.2
.1
η_{th}
without regeneration
$\frac{T_1}{T_3}$ =.25
$\frac{T_1}{T_3}$ =0.2
$\frac{T_1}{T_3}$ =0.33
5
10
15
20
25
pressure ratio, r_p



Rankine Cycle

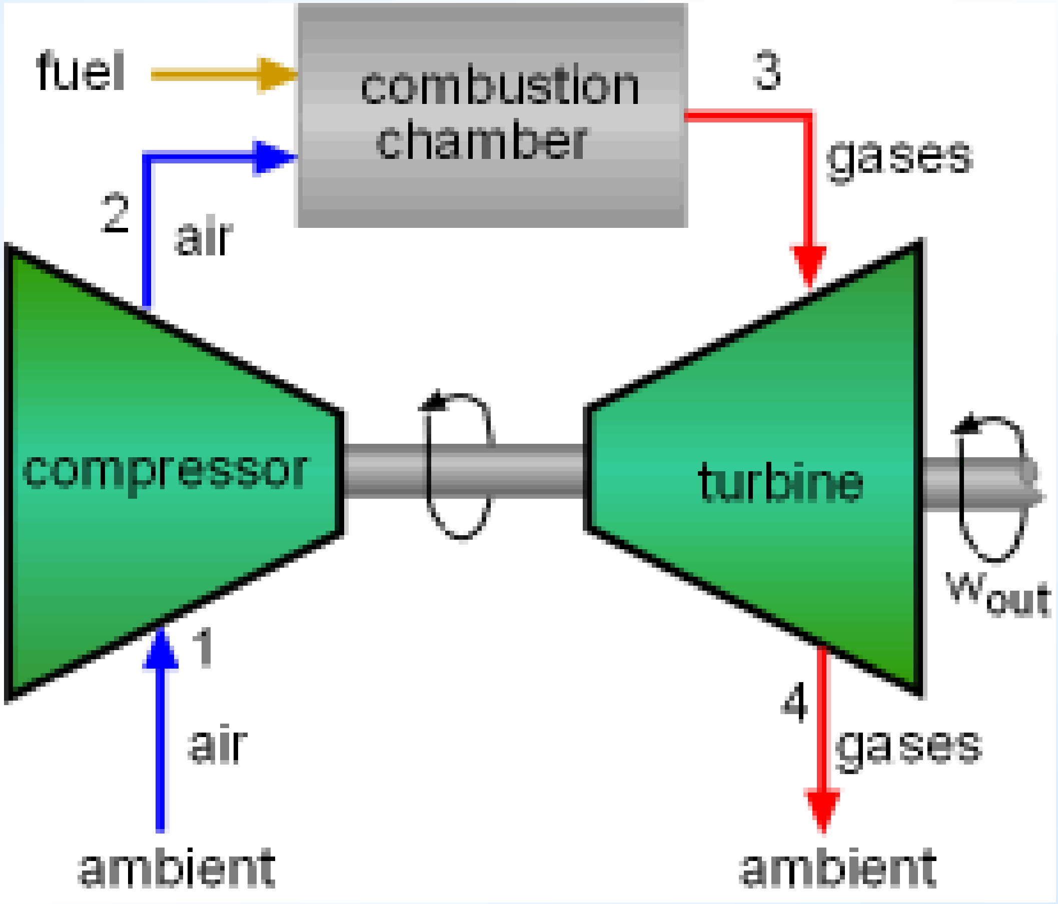
Steam Generator
- Economizer
- Boiler
- Superheater

Turbine
- LPT
- HPT

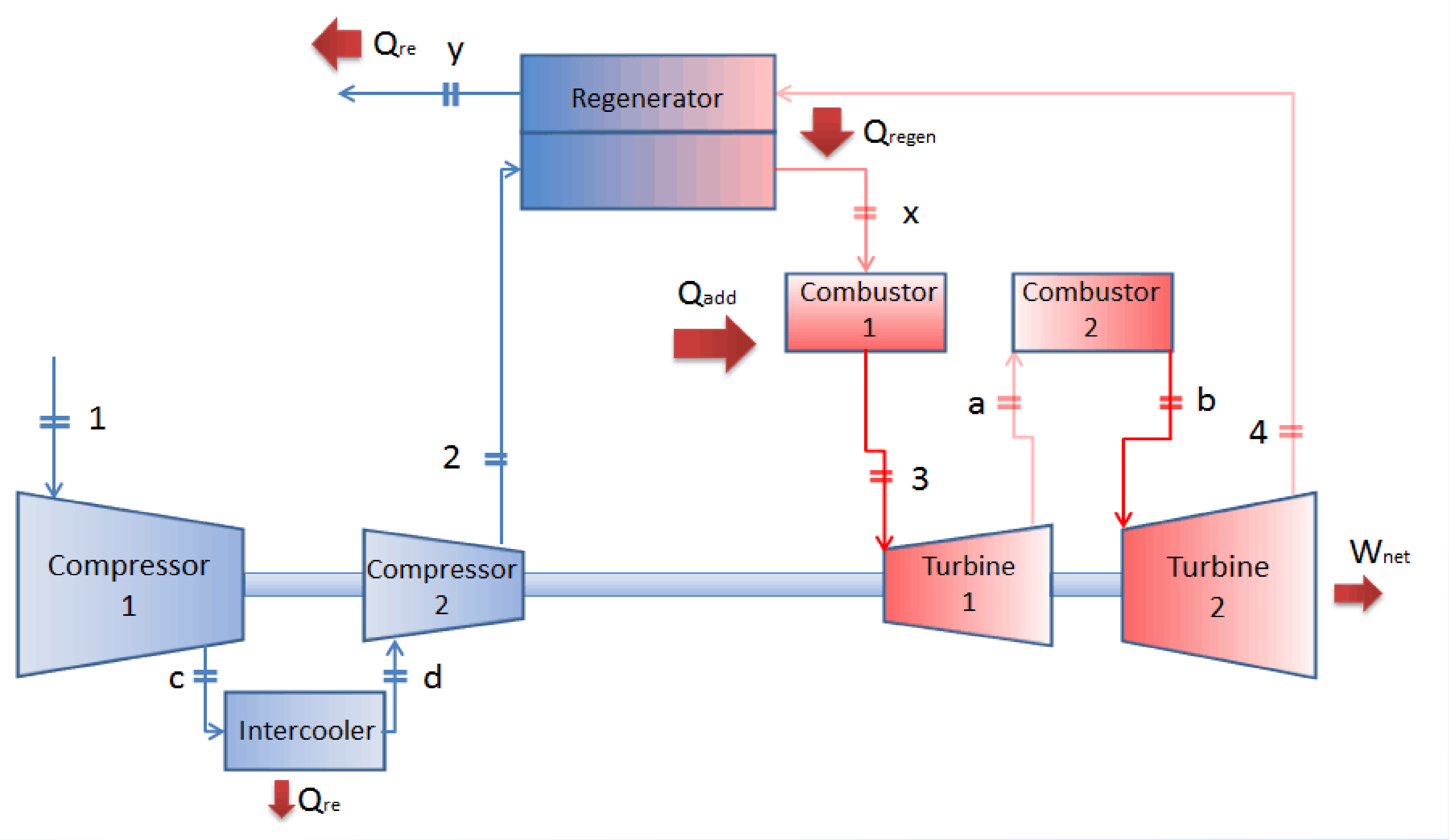
Reheater

Feed Water Heater
- Open
- Closed

Brayton (Gas) Cycle



Brayton Cycle – Regeneration, Intercooling, Reheating



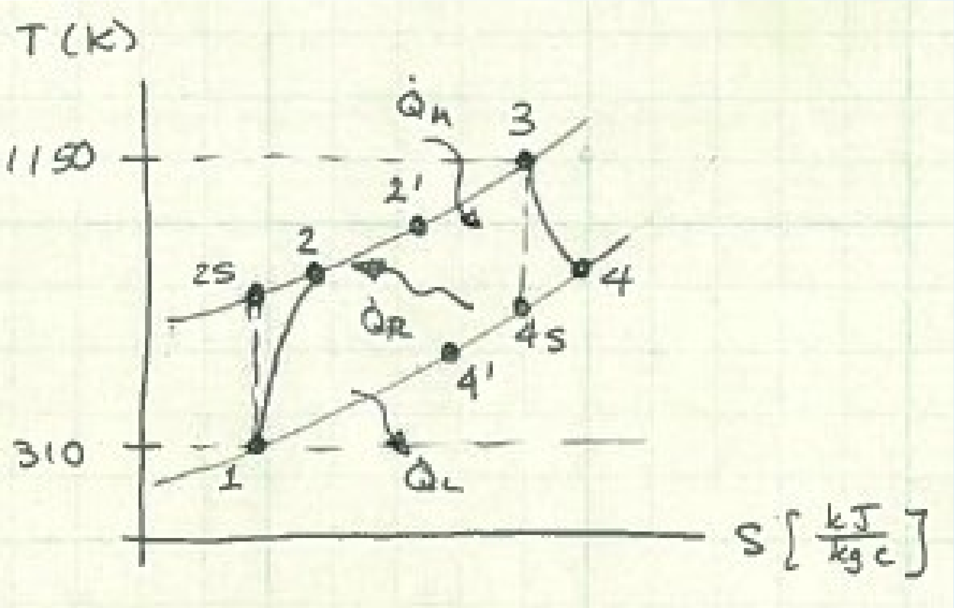
Example 7

A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 K and 1150 K. Assuming an isentropic efficiency of 75 % for the compressor and 82 % for the turbine and a regenerator effectiveness of 65 %, determine

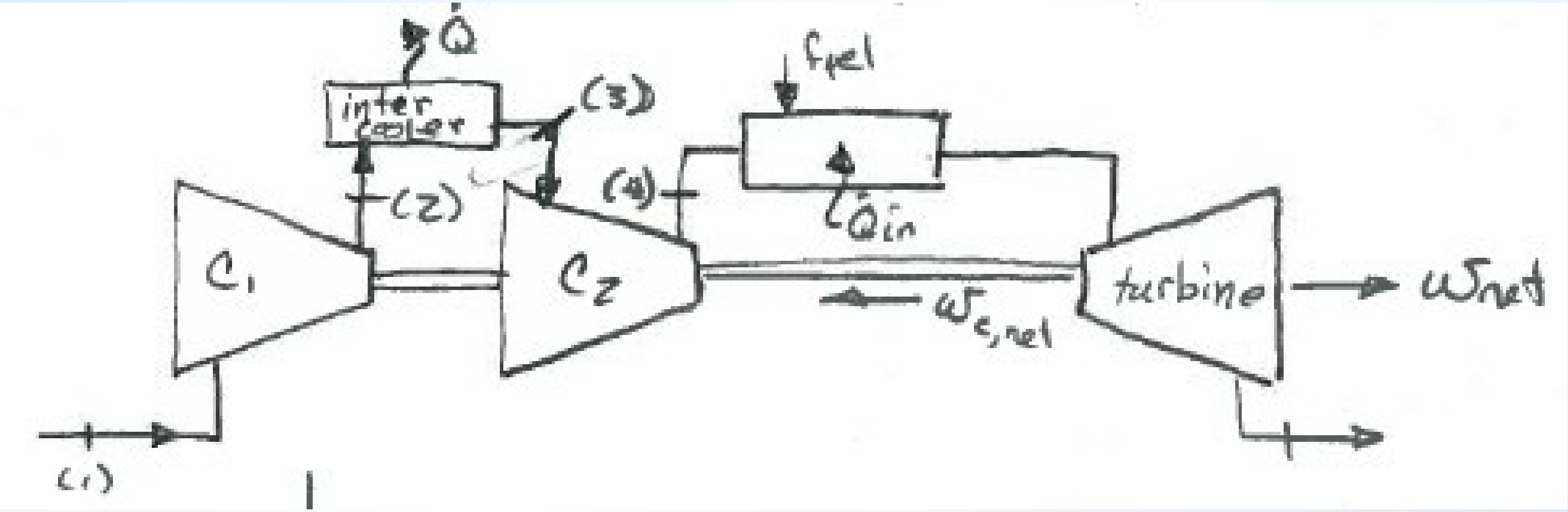
a) The air temperature at the turbine exit,

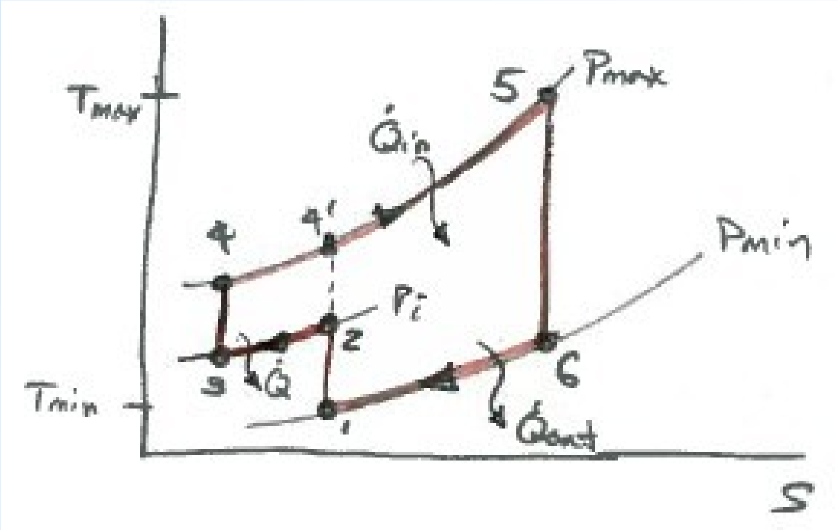
b) The net work output, and

c) The thermal efficiency



Brayton Cycle – Multistage Compression with Intercooling





Multistage compression with intercooling produces a lower discharge temperature than that resulting from single-stage compression; $T_4 < T_{4'}$

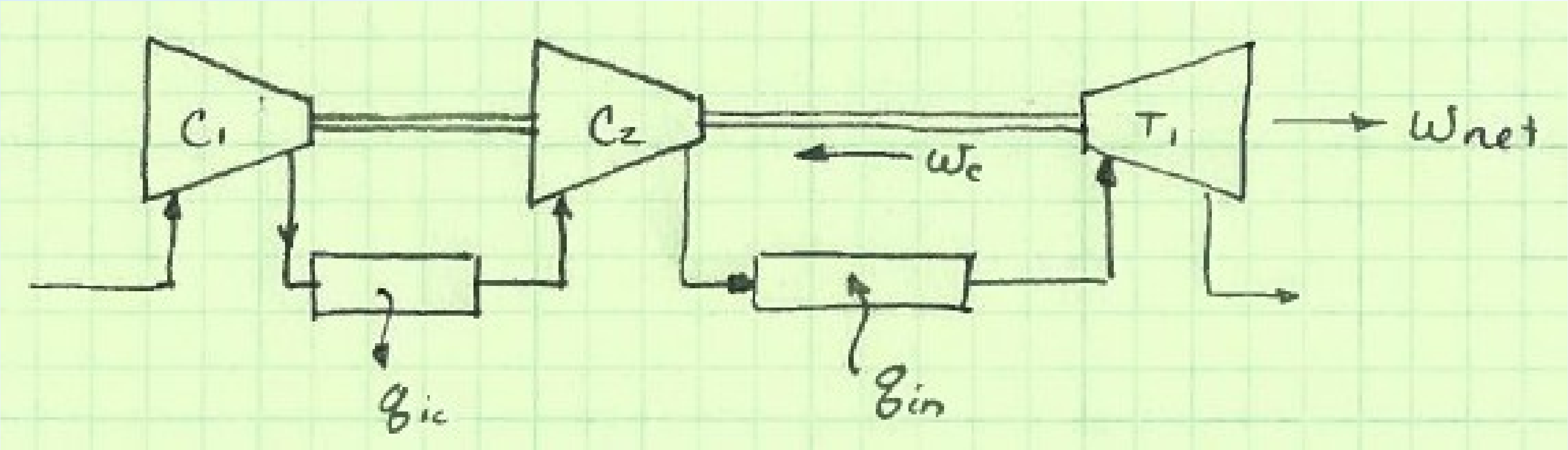
More heat is required with multistage compression with intercooling;

$$h_5 - h_4 > h_5 - h_{4'}$$

Unless regeneration is used, the theoretical efficiency of the cycle will be lower.

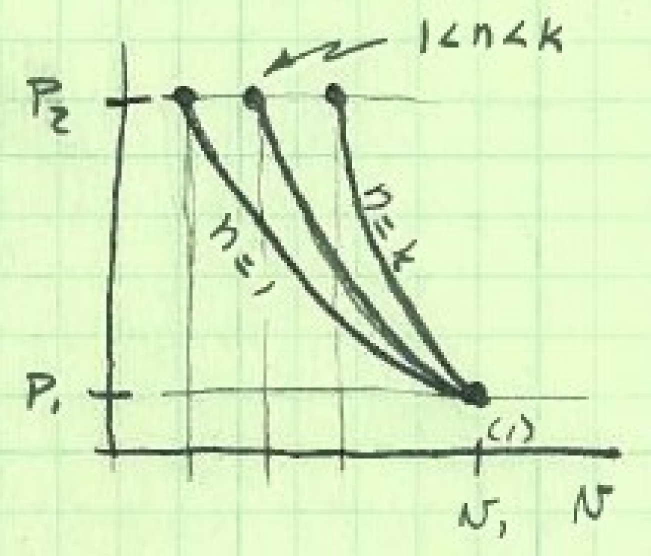
In practice, the cycle efficiency may improve due to improved compressor efficiency over the smaller ΔP.

Compressor Work – Multistage with Intercooling



- Which is more efficient, single-stage compression or multi-stage compression with intercooling?

- Polytropic process, Pv^n = constant



For an isentropic process, n = k

- Smallest area under the curve; least amount of work required to compress from P1 to P2.

For an isothermal process, n = 1

- largest area under the curve;
- P v = R T = constant => T must be constant

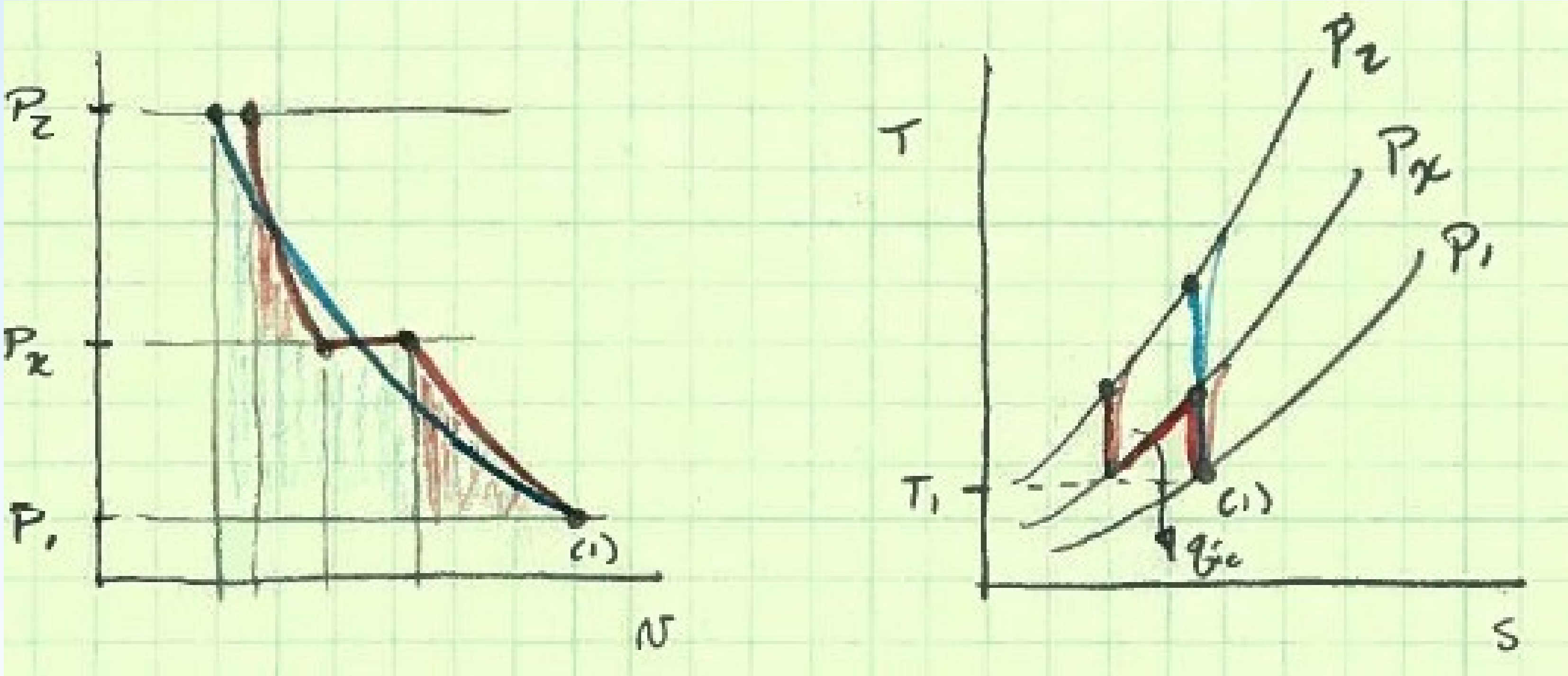
For an ideal gas with constant specific heats:

$$w_c\Big|_{reverssible} = -\frac{R\,(T_2 - T_1)}{(1 - n)\,/\,n} = -\frac{R\,T_1}{(n - 1)\,/\,n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \quad , \quad n \neq 1$$

r_p

Isentropic compression, n = k

For an isothermal compression (n = 1) $$w_c = -R\,T\,\ln\left(\frac{P_2}{P_1}\right)$$



$$w_c = w_{c1} + w_{c2} = -\frac{R\,T_1}{\frac{(n-1)}{n}}\left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{R\,T_1}{\frac{(n-1)}{n}}\left[\left(\frac{P_2}{P_x}\right)^{\frac{n-1}{n}} - 1\right]$$

What value of P_x minimizes w_c? => when $\frac{P_x}{P_1} = \frac{P_2}{P_x}$ => $r_{p1} = r_{p2}$

Pressure rise per stage: $r_{p,\ stage} = \sqrt[N_c]{r_{p,total}}$ N_c = No of compressor stages

P1 = 1 bar
P3 = 9 bar
2 stages of compression

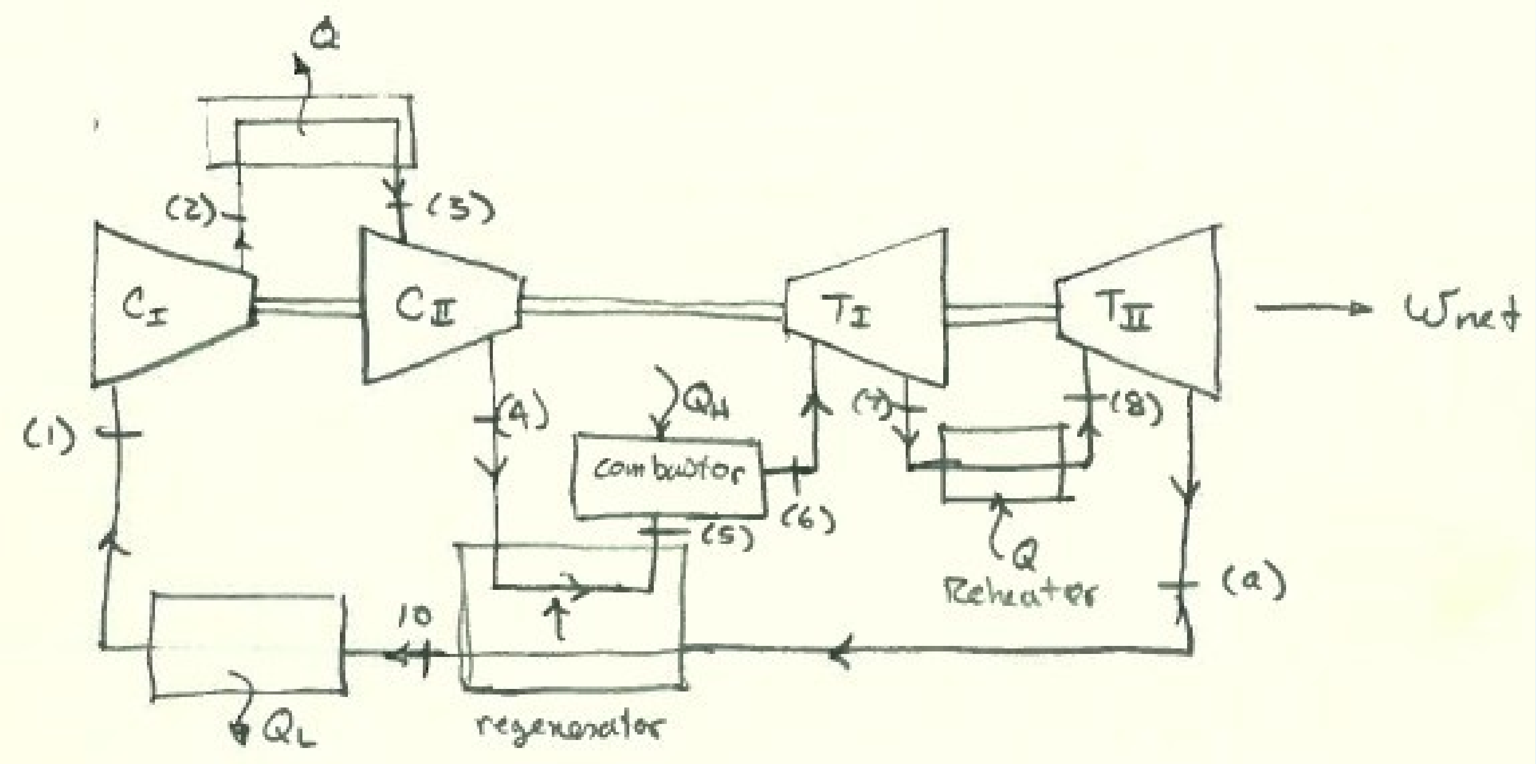
$$r_{p,s} = \sqrt{\frac{9}{1}} = 3$$

Stage 1: 1 bar x $r_{p,s}$ = 3 bar

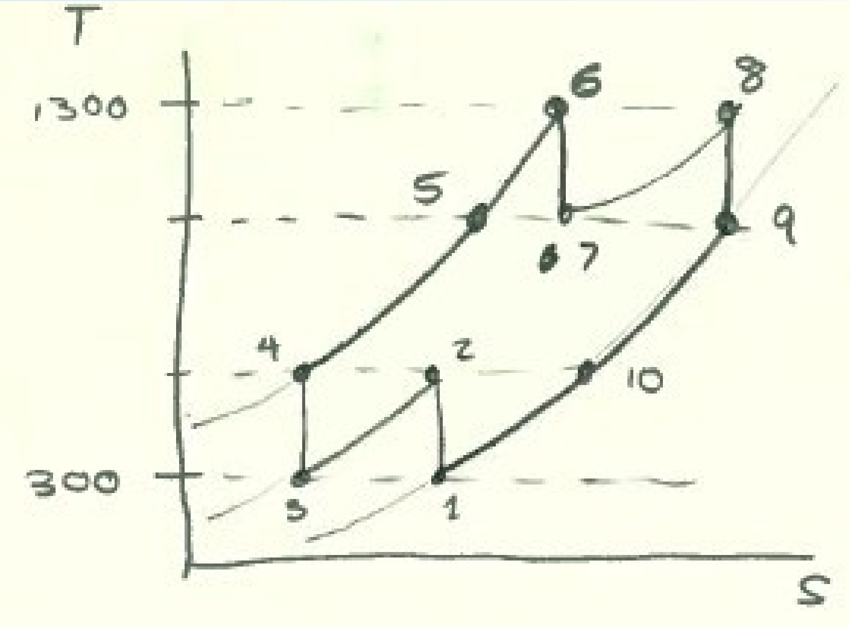
Stage 2: 3 bar x $r_{p,s}$ = 9 bar

Example: Ideal gas turbine, 2 stages of compression, 2 stages of expansion,

$r_{p,total}$ = 8, $T_{c,in}$ = 300 K (both stages), $T_{T,in}$ = 1300 K (both stages)



Back work ratio = ?, η_{th} = ? (a) no regenerator; (b) ideal regenerator (ε_R = 1)



$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{8} = 2.83$$

$$\frac{P_6}{P_7} = \frac{P_8}{P_9} = \sqrt{8} = 2.83$$

$T_1 = T_3$, $h_1 = h_3$ compressor inlets

$T_2 = T_4$, $h_2 = h_4$ compressor exits

$T_6 = T_8$, $h_6 = h_8$ turbine inlets

$T_7 = T_9$, $h_2 = h_9$ turbine exits

Therefore, $w_{cI} = w_{cII}$

$w_{TI} = w_{TII}$

Intercooling, reheat, and regeneration can be combined in the same cycle

$$w_{net} = w_T - |w_c| = c_p \left\{ T_3 \, \eta_T \, (N_T + 1) \left[1 - \frac{1}{r_{p,c}^{\frac{k-1}{k}}} \right] - T_1 \left(\frac{N_c + 1}{\eta_c} \right) \left[r_{p,c}^{\frac{k-1}{k}} - 1 \right] \right\}$$

$$q_{in} = c_p \, T_3 \left\{ (N_T + 1) - (N_T + \varepsilon_R) \left[1 - \eta_c \left(1 - \frac{1}{r_{p,c}^{\frac{k-1}{k}}} \right) \right] \right\} - c_p \, T_1 \, (1 - \varepsilon_R) \left[1 + \frac{1}{\eta_c} \left(r_{p,c}^{\frac{k-1}{k}} - 1 \right) \right]$$

η_T: turbine efficiency

η_c: compressor efficiency

N_T: no of reheat processes

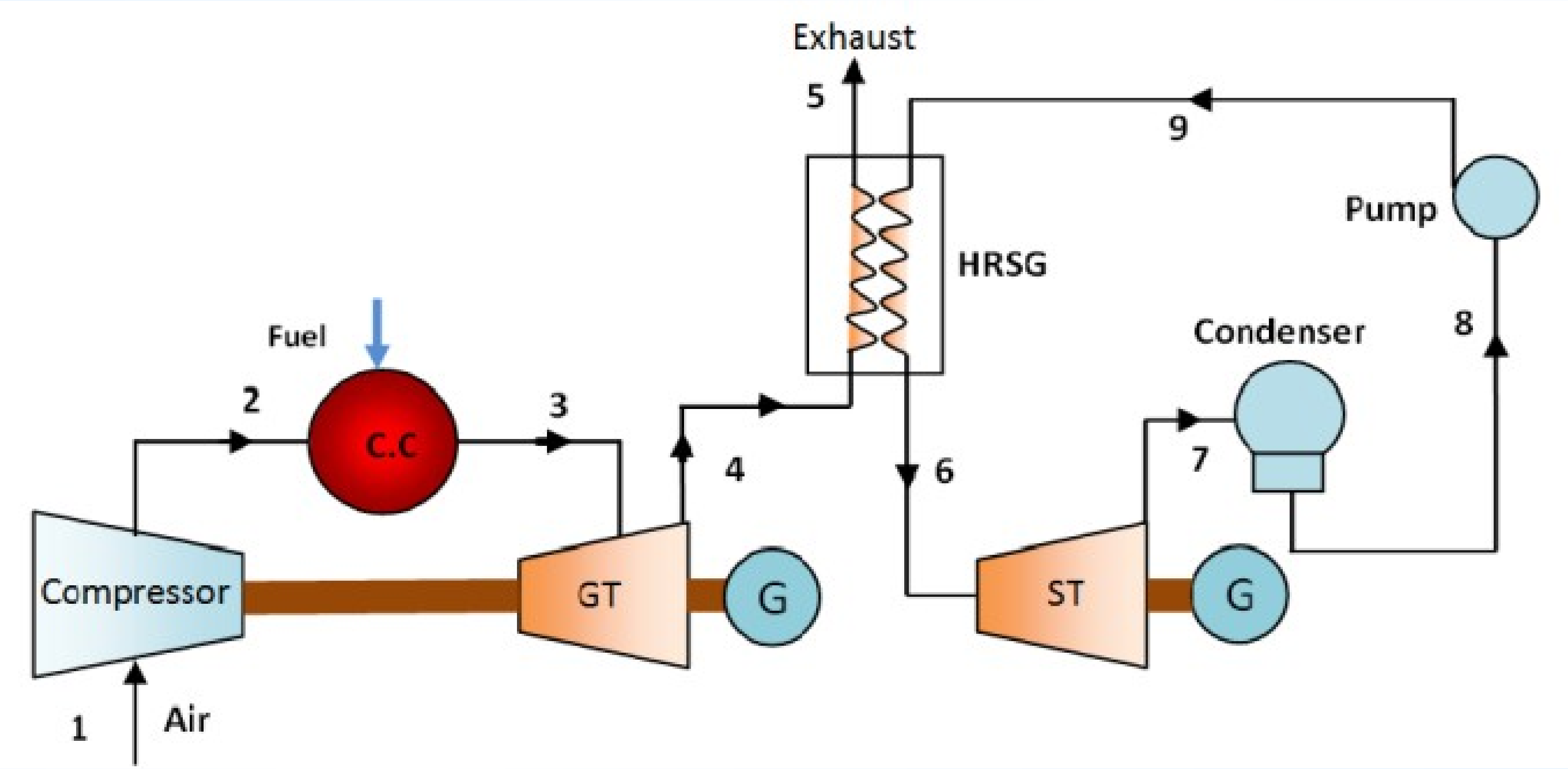
N_c: no of intercoolers

Note the change in definitions of N_T and N_c

$$r_{p,c} = r_{p,T} \qquad r_{p,T_i} = \sqrt[N_T+1]{r_{p,T}}$$

$$r_{p,c_i} = \sqrt[N_c+1]{r_{p,c}}$$

Combined-Cycle Power Plant



HRSG: Heat Recovery Steam Generator

Example

A combined gas-steam turbine power plant is designed with four 50 MWe gas turbines and one 120 MWe steam turbine.

Gas turbines:

compressor inlet temperature = 505 R

turbine inlet temperature = 2450 R

pressure ratio (compressor and turbine) = 5

$\eta_c = \eta_T = 0.87$

$\eta_m = 0.96$

The gases leave the turbine and go to a heat-recovery boiler then to a regenerator

$\varepsilon_R = 0.87$

200 % theoretical air (use CH2.145 as fuel)

Steam cycle:

turbine inlet pressure = 1200 psia

turbine inlet temperature = 1460 R

1 open feedwater heater, 920 R

$\eta_T = 0.87$

$\eta_m = 0.96$

ignore pump work

generator: $\eta_G = 0.96$

Supplemental firing raises gas temperature to 2000 F (full load)

Draw the flow and T-s diagrams, find:

(a) $\dot{m}_{steam}$, lbm/hr

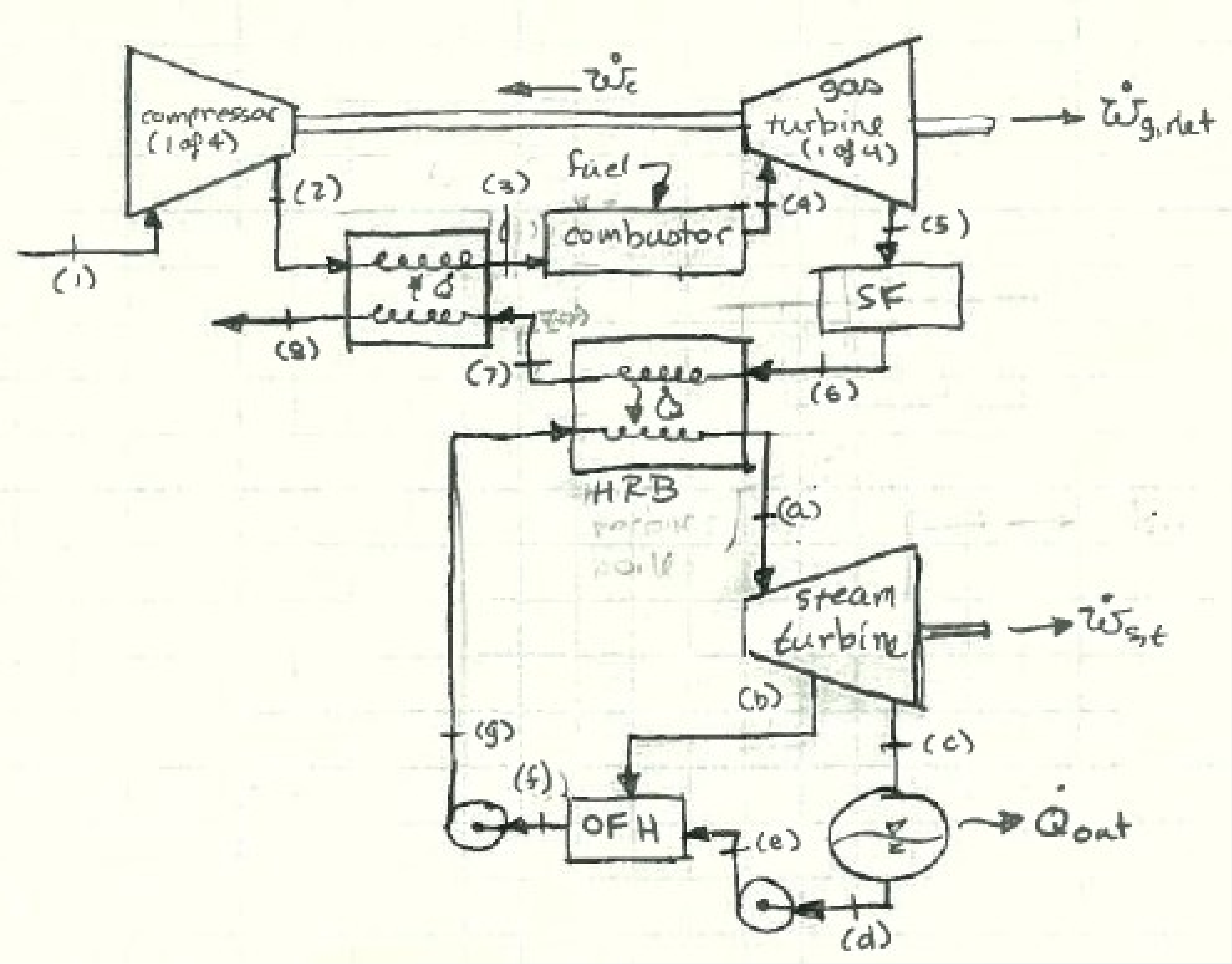
(b) $\dot{m}_{gas}$, lbm/hr/turbine

(c) $\dot{Q}_{in}$ (combustor and supplemental firing), Btu/hr

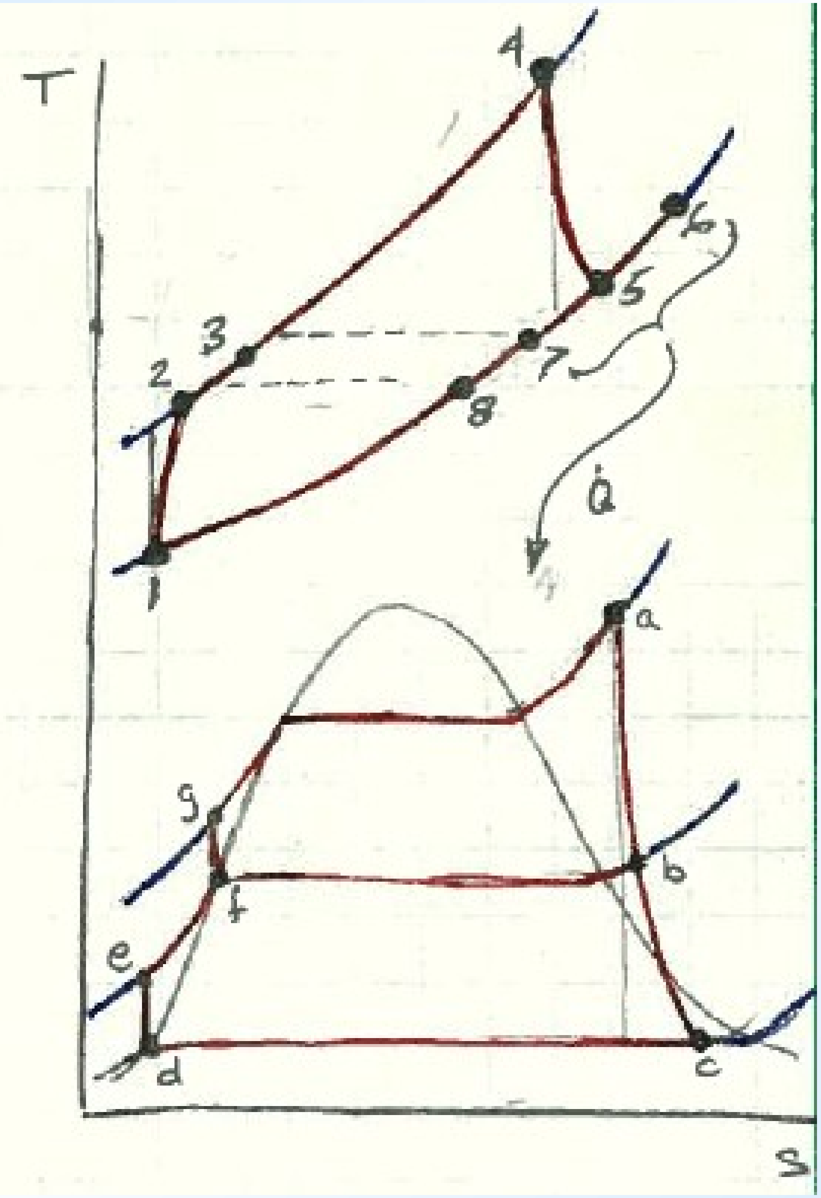
(d) T_{stack} of exhaust gas, F

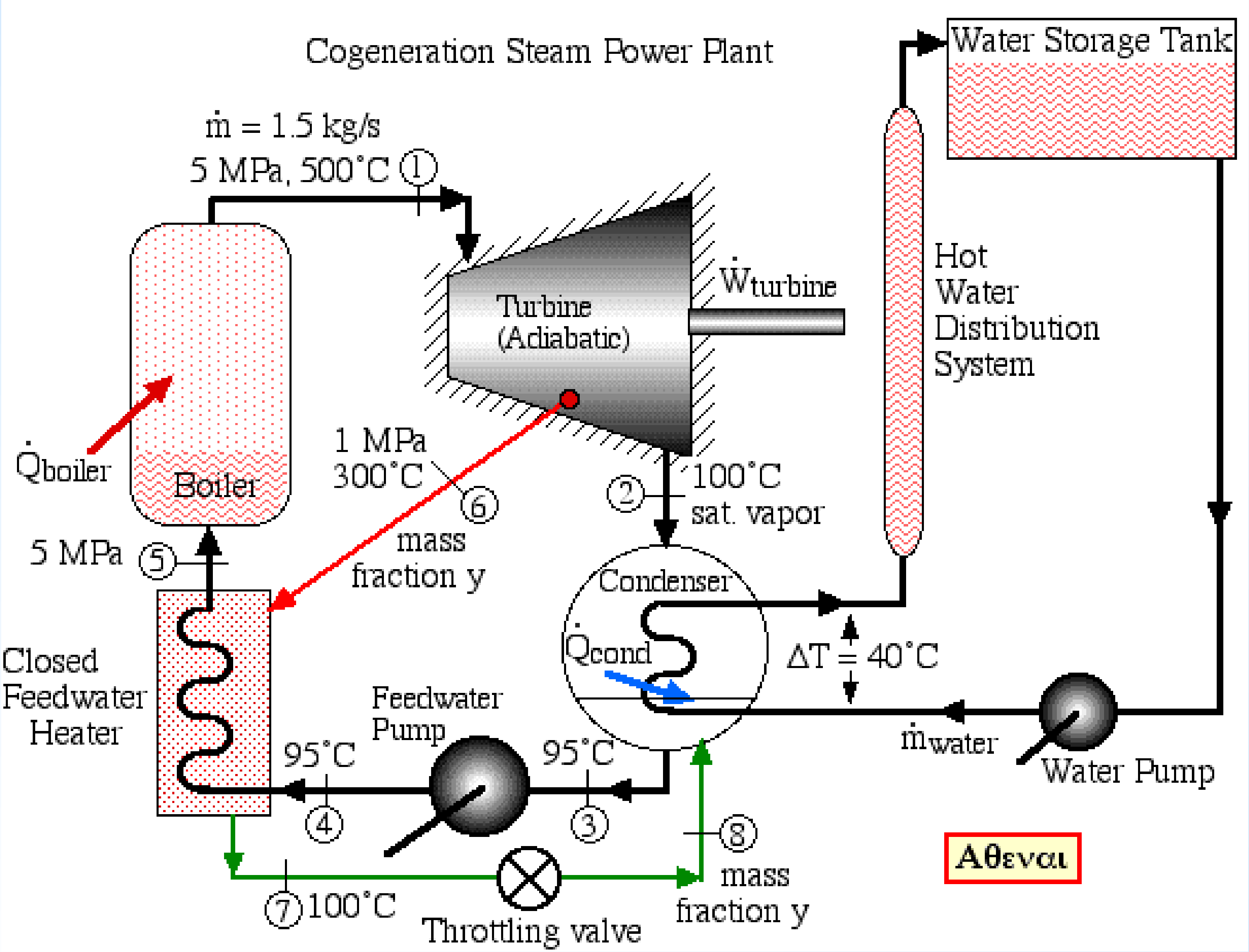
(e) η_{th} at full load for combined cycle

(f) η_{th} with 1 gas turbine & no supplemental firing or regeneration

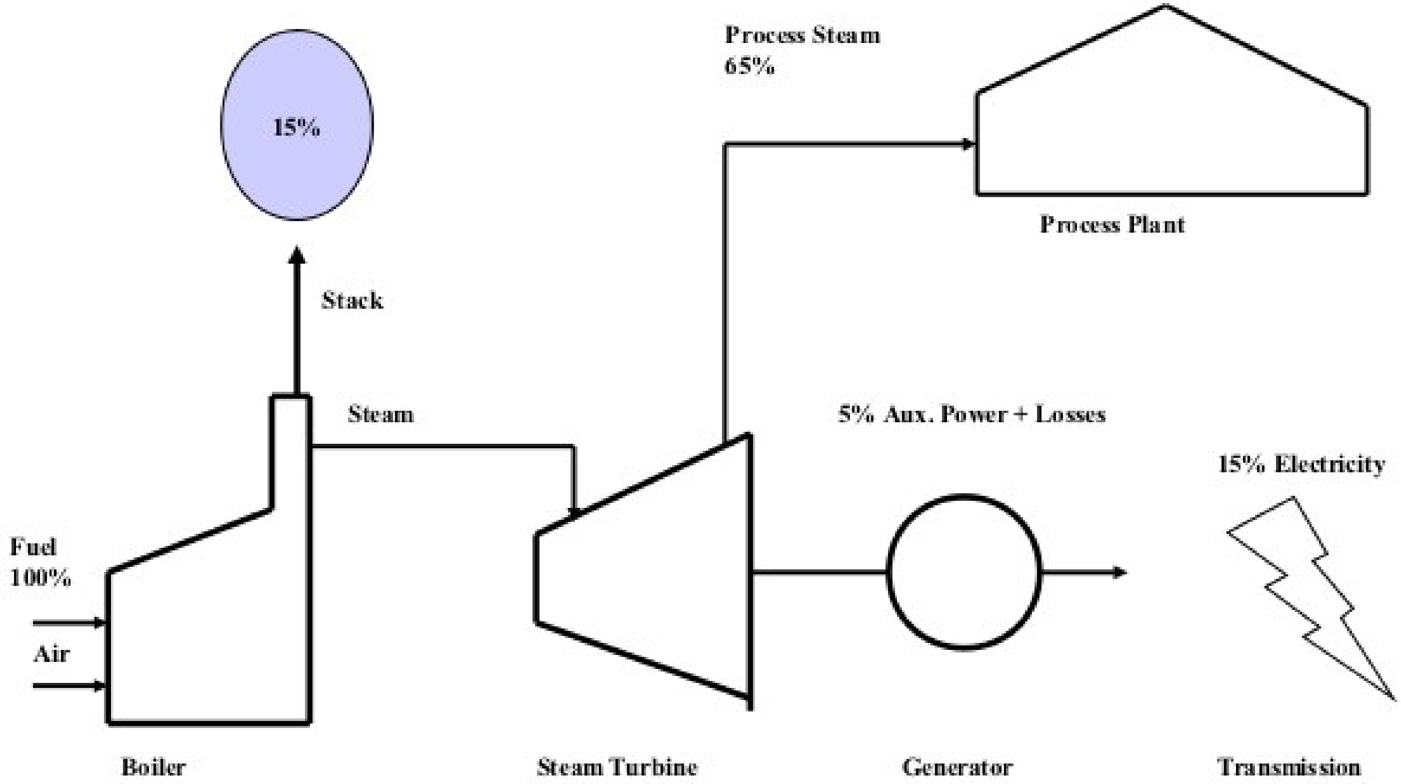


compressor (1 of 4)
gas turbine (1 of 4)
fuel
combustor
SF
HRB
steam turbine
OFH
(1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(a)
(b)
(c)
(d)
(e)
(f)
(g)


T
A
6
5
3
7
2
8
1
Q̇
a
g
b
f
e
d
c
s


Cogeneration Steam Power Plant
Water Storage Tank
ṁ = 1.5 kg/s
5 MPa, 500˚C
1
Turbine
(Adiabatic)
Ẇturbine
Hot
Water
Distribution
System
Q̇boiler
Boiler
1 MPa
300˚C
6
mass
fraction y
100˚C
sat. vapor
2
5 MPa
5
Condenser
Q̇cond
ΔT = 40˚C
Closed
Feedwater
Heater
Feedwater
Pump
95˚C
95˚C
ṁwater
Water Pump
4
3
8
mass
fraction y
7
100˚C
Throttling valve
Αθεναι

BOILER / STEAM TURBINE-GENERATOR INDUSTRIAL CO-GENERATION



THERMAL EFFICIENCY - 80%