2. WORK and HEAT

2.1 Work

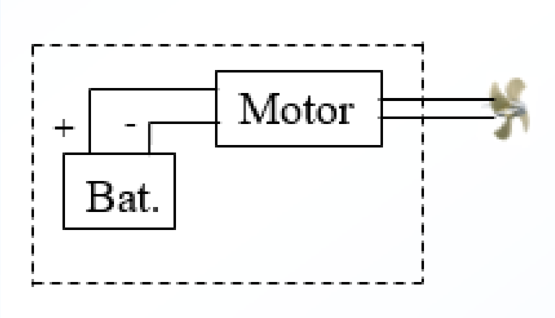
Formal definition: $${}_1W_2 = \int_1^2 F\,dx$$

F: Force acting through a distance

dx: Displacement in the direction of the force

Work is done by a system if the sole effect on the surroundings could be the raising of a weight.

Work done by a system is conventionally considered to be positive (+).

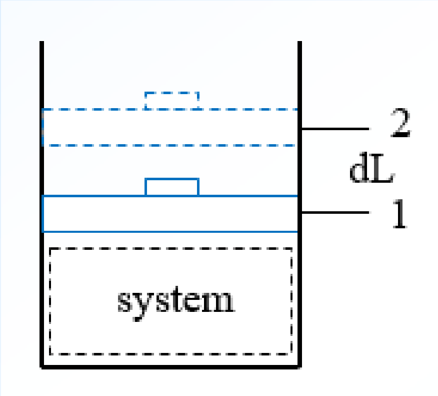


If the system does not include the propeller, then the work is crossing the boundary. Assume no heat is generated. Otherwise, with the propeller, there is no work.

Unit for power: $\dot{W} = \frac{\delta W}{\partial t} \rightarrow$ Joule / second $\rightarrow$ Watt

Work is a path function and δW is an exact differential: $\int_1^2 \delta W \neq W_2 - W_1$

Example: Given a frictionless system, remove an infinitesimal quantity of weight.



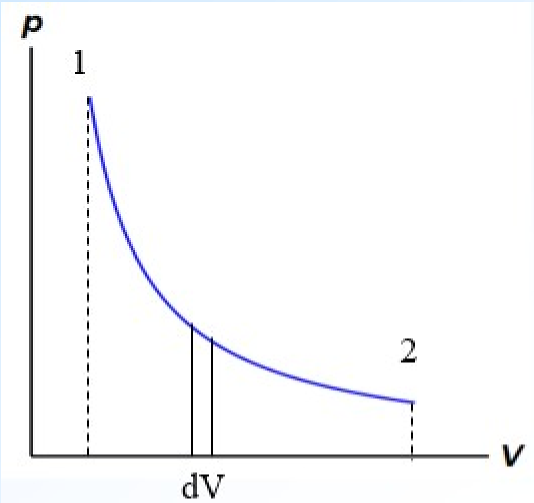
$\delta W = P\, A\, dL$

pressure, area → force; dL → distance moved

$A\, dL = dV$ change in the volume $\Rightarrow$ $\delta W = P\, dV$

Total work: ${}_1W_2 = \int_1^2 \delta W = \int_1^2 P\, dV$ $P(V) = ?$

If P(V) is known, one can evaluate the integral.



On a P-V diagram: $_1W_2 = \int_1^2 P\,dV$ represents area under the curve

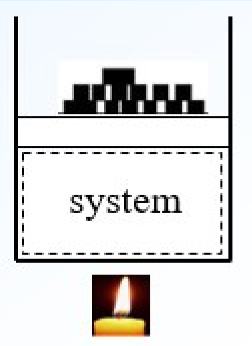
Note that $_1W_2$ depends on both the end states 1 and 2 and the path followed.

Therefore, work is a path function and δW is an inexact differential. (That's why the notation δ)

$$_1W_2 = \int_1^2 \delta W \neq W_2 - W_1$$ This is meaningless

However $$\int_1^2 dV = V_2 - V_1$$ Volume change depends on the end states, only.

Example: Consider a frictionless piston with tiny weights on top as shown.



Initial state: P_1 = 200 kPa, V_1 = 0.04 m^3

(a) Add Q with a bunsen burner such that P remains constant but V_2 becomes 0.1 m^3. W = ? (at an extremely slow rate)

(b) Add Q and also remove weights such that V_2 becomes 0.1 m^3 but P V = constant. W = ?

(c) Add Q and also remove weights such that V_2 becomes 0.1 m^3 but $P\,V^{1.3}$ = constant. W = ?

(d) $V_2 = V_1$ Volume remains constant while Q is removed (by a pin or cooled) and P_2 = 100 kPa. W = ?

Solution:

(a) $${}_1W_2 = \int_1^2 P\,dV = P\int_1^2 dV = (200)\,(0.1 - 0.04) = 12 \text{ kJ}$$

kPa m³

(b) $$P_1 V_1 = P_2 V_2 = P V \quad \Rightarrow \quad P = \frac{P_1 V_1}{V} = \frac{P_2 V_2}{V}$$

$${}_1W_2 = \int_1^2 P\,dV = \int_1^2 \frac{P_1 V_1}{V}\,dV = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = (200)\,(0.04)\ln\left(\frac{0.1}{0.04}\right) = 7.33 \text{ kJ}$$

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{(200)\,(0.04)}{0.1} = 80 \text{ kPa}$$

(c) $$P V^{1.3} = P_1 V_1^{1.3} = P_2 V_2^{1.3} \quad \Rightarrow \quad P = P_1 V_1^{1.3}\,\frac{1}{V^{1.3}}$$

$${}_1W_2 = \int_1^2 P\,dV = P_1 V_1^{1.3}\int_1^2 \frac{1}{V^{1.3}}\,dV = P_1 V_1^{1.3}\left(\frac{V_2^{1-1.3} - V_1^{1-1.3}}{1 - 1.3}\right) = P_2 V_2^{1.3}\left(\frac{V_2^{1-1.3} - V_1^{1-1.3}}{1 - 1.3}\right)$$

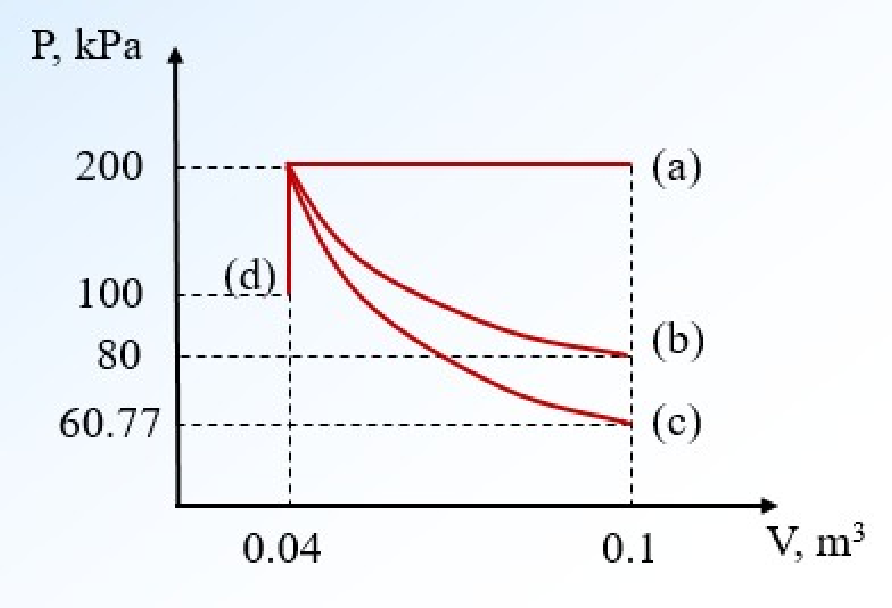
A more general expression: $_1W_2 = \dfrac{P_2 V_2 - P_1 V_1}{1 - 1.3}$

Final pressure: $P_2 = \dfrac{P_1 V_1^{1.3}}{V_2^{1.3}} = \dfrac{(200)(0.04)^{1.3}}{(0.1)^{1.3}} = 60.77 \text{ kPa}$

$$_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1 - 1.3} = \frac{(60.77)(0.1) - (200)(0.04)}{1 - 1.3} = 6.41 \text{ kJ}$$

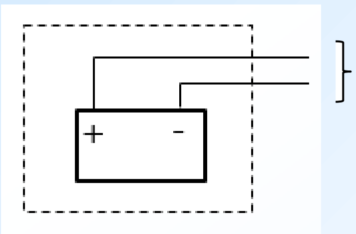
(d) $_1W_2 = 0$ since $dV = 0$

Question: Where does the removed energy come from?



Compare the numerical values for work with the areas under the curves.

Electrical mode of work:



Potential difference,

E Volts

$$\delta W = -E\,dz$$

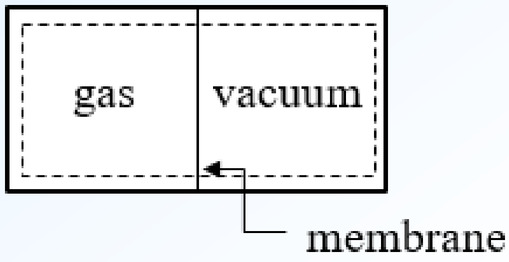
Flowing into the system → δW; Volts → E; Coulomb → dz

Define: $i = \dfrac{dz}{dt}$, Current $\rightarrow$ $dz = i\,dt$

$$\delta W = -E\,i\,dt \;\Rightarrow\; \frac{\delta W}{dt} = -E\,i \;\Rightarrow\; {}_1W_2 = -\int_1^2 E\,i\,dt$$

J/s = W → $\frac{\delta W}{dt}$; Volts → E; Amperes → i

Question:



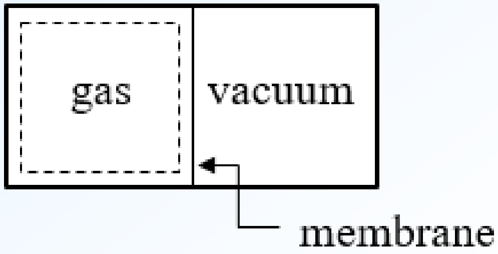
System is both sides of the chambers

Membrane ruptures

W = ?

Answer: W = 0 because dV = 0.

Question:



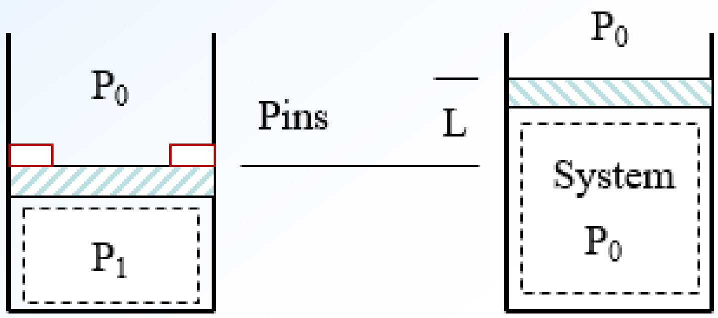
System is the gas chamber only

Membrane ruptures

W = ?

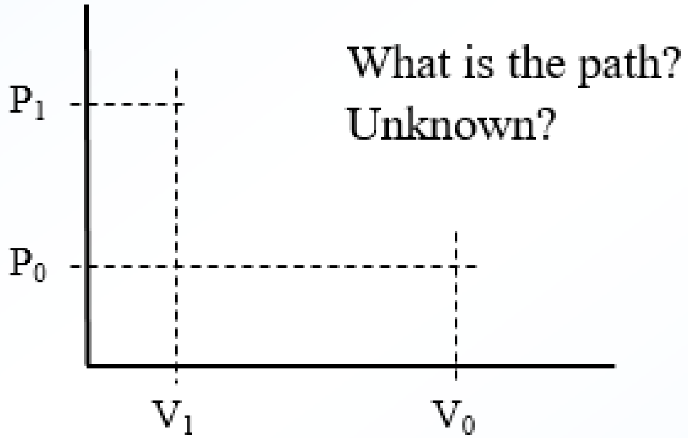
Answer: W = 0 because there is no external force.

Example:



Remove the pins.

W = ?



Force = $P_0\, A_{piston} + m_{piston}\, g$

Distance moved = L

Work = Force x Distance

$= (P_0\, A_{piston} + m_{piston}\, g)\, L$ in Joules

2.2 Heat

It is a form of energy transfer.

It is transfered across the boundary of a system due to temperature differential between the system and its surroundings.

It is identified only when it is crossing a boundary.

A system cannot contain heat. It may contain energy.

Note that all these are also true for work.

Heat, Q, (in Joules) is positive if it is transfered to the system.

Work, W (in Joules) is positive if it is transfered from the system.

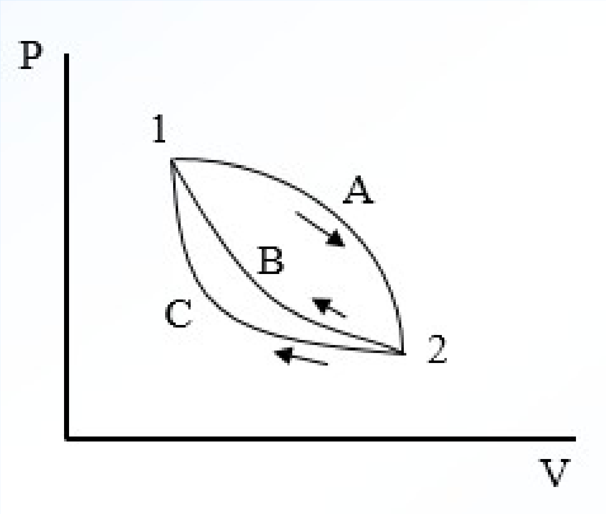
When Q = 0, it is called adiabatic process.

Heat is also a path function (inexact differential)

2.3 First Law of Thermodynamics

There is no rigirous mathematical proof of the laws of thermodynamics.

For a system undergoing a cycle where the sytem returns to the original position } $\oint \delta W = \oint \delta Q$

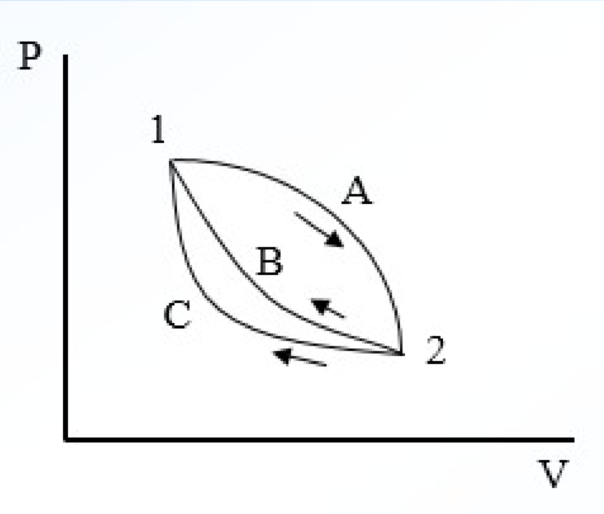


$$\int_{1A}^{2A} \delta Q + \int_{2B}^{1B} \delta Q = \int_{1A}^{2A} \delta W + \int_{2B}^{1B} \delta W$$

$$\int_{1A}^{2A} \delta Q + \int_{2C}^{1C} \delta Q = \int_{1A}^{2A} \delta W + \int_{2C}^{1C} \delta W$$

Substract:

$$\int_{2B}^{1B} (\delta Q - \delta W) = \int_{2C}^{1C} (\delta Q - \delta W)$$



$$\int_{2B}^{1B} (\delta Q - \delta W) = \int_{2B}^{1B} (\delta Q - \delta W)$$

This proves that $\delta Q - \delta W$ is independent of path, but depends on the end states, 1 and 2

Therefore: $\delta Q - \delta W = dE$ or $\delta Q = \delta W + dE$

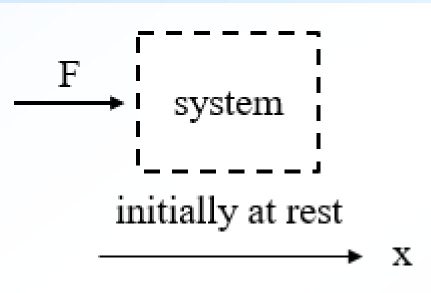
Integrate: ${}_1Q_2 = {}_1W_2 + (E_2 - E_1)$ E represents all the energy of the system

Define: $E = KE + PE + U$

E = Kinetic Energy + Potential Energy + Internal Energy

$$\delta Q = \delta W + d(KE) + d(PE) + dU$$

Kinetic Energy



It is the energy in a moving object.

$$\underbrace{\delta Q}_{0} = \underbrace{\delta W}_{-F\,dx} + d(KE) + \underbrace{d(PE)}_{0} + \underbrace{dU}_{0} \quad \Rightarrow \quad d(KE) = F\,dx$$

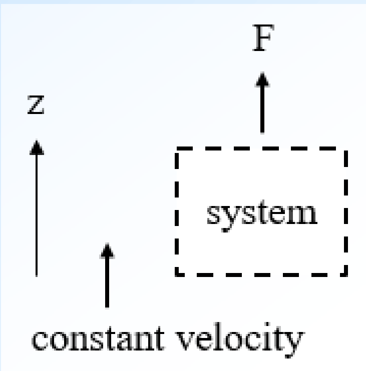
$$F = m\,a = m\,\frac{dV}{dt} = m\,\frac{dx}{dt}\,\frac{dV}{dx} = m\,V\,\frac{dV}{dx}$$

$$d(KE) = F\,dx = m\,V\,dV$$

V is the velocity.

Integrate: $\int d(KE) = \int m\,V\,dV \quad \Rightarrow \quad KE = \frac{1}{2}\,m\,V^2$

Potential Energy

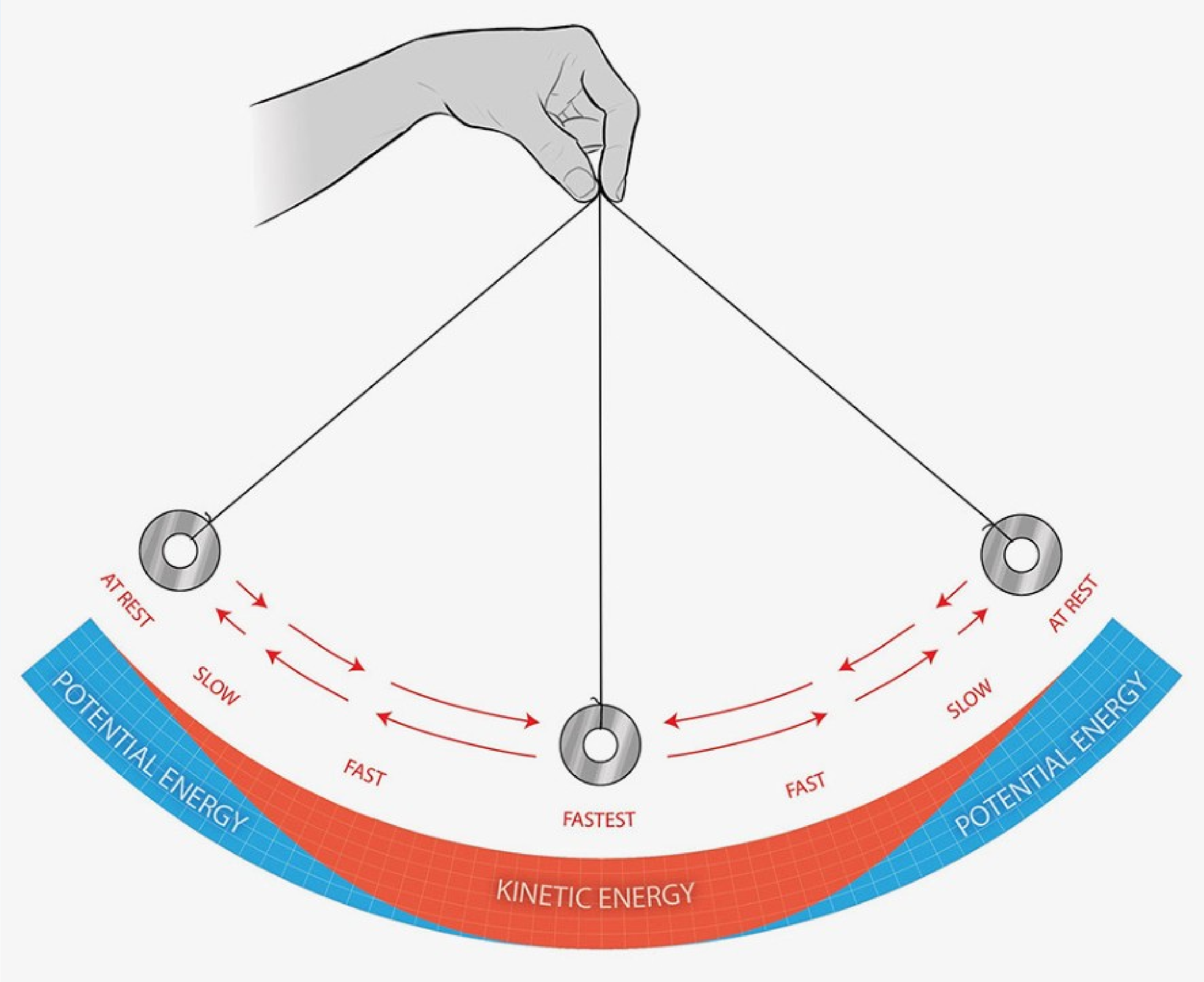


It is the energy held by an object because of its position relative to other objects, internal stress, electric charge, or other factors.

$$\underbrace{\delta Q}_{0} = \underbrace{\delta W}_{-\,F\,dz} + \underbrace{d(KE)}_{0} + d(PE) + \underbrace{dU}_{0} \quad \Big\} \quad d(PE) = F\,dz$$

$$F = m\,a = m\,g \quad \Rightarrow \quad d(PE) = m\,g\,dz \quad \Rightarrow \quad \int_{1}^{2} d(PE) = m \int_{z_1}^{z_2} g\,dz$$

If g is constant: $PE_1 - PE_2 = m\,g\,(z_2 - z_1)$


AT REST
SLOW
FAST
FASTEST
FAST
SLOW
AT REST
POTENTIAL ENERGY
KINETIC ENERGY
POTENTIAL ENERGY

First law: $\delta Q = dU + \delta W + d(KE) + d(PE)$

$$\delta Q = dU + \delta W + m\,V\,dV + d(m\,g\,z)$$

$$ {}_1Q_2 = U_2 - U_1 + {}_1W_2 + \frac{m\left(V_2^2 - V_1^2\right)}{2} + m\,g\,(z_2 - z_1) $$

Three observations:

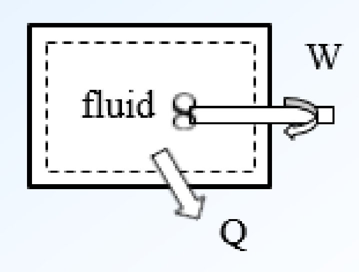
1. $dE = dU + d(KE) + d(PE)$ More convenient in this form
2. Conservation of energy
3. Equation deals with changes in KE, PE, and U.

   What about their absolute values?

For KE: velocity relative to earth; For PE: any reference elevation from earth

For U: ? It is an extensive property. Define specific U: $u = \frac{U}{m}$

Example



$_1W_2 = -5090$ kJ Work is done on the system

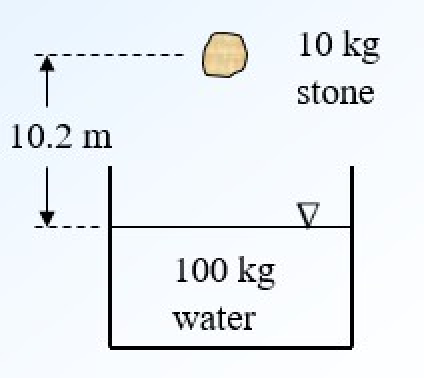
$_1Q_2 = -1500$ kJ Heat is transfered from the system

$U_2 - U_1 = ?$

First law: $$_1Q_2 = U_2 - U_1 + {_1W_2} + \underbrace{\Delta(KE)}_{0} + \underbrace{\Delta(PE)}_{0}$$

$$U_2 - U_1 = 1500 - 5090 = -3590 \text{ kJ}$$

Example



Initially. the stone and the water are at the same temperature.

The stone falls into the water; g = 9.81 m²/s

Find: ΔU, Δ(KE), Δ(PE), Q, and W

(a) The stone is abut to enter the water

(b) The stone comes to rest in the bucket

(c) Q is transferred such that the stone and the water are at the initial temperature.

(a)

$${}_1Q_2 = 0$$

$$\Delta U = 0$$

$${}_1W_2 = 0$$

$$\Delta(KE) = \frac{m\left(V_2^2 - V_1^2\right)}{2} = \frac{10\left(V_2^2\right)}{2} = 5\,V_2^2$$

$$\Delta(PE) = m\,g\,(z_2 - z_1) = (10)\,(9.81)\,(-\,10.2)$$

$$= -\,1000\ J = -\,1\ kJ$$

First law: $0 = \Delta(KE) + \Delta(PE) \Rightarrow \Delta(KE) = 1\ kJ$

(b)
$$\left.\begin{array}{l} {}_1Q_2 = 0 \\ \Delta(PE) = 0 \\ {}_1W_2 = 0 \end{array}\right\} \quad \begin{array}{l} \Delta(KE) = 1\ kJ \\ \Delta U = 1\ kJ \end{array}$$

(c)
$$\left.\begin{array}{l} {}_1W_2 = 0 \\ \Delta(PE) = 0 \\ \Delta(KE) = 0 \end{array}\right\} \quad \begin{array}{l} \Delta U = -1\ kJ \\ {}_1Q_2 = -1\ kJ \end{array}$$