



4. PROPERTIES of a pure substance

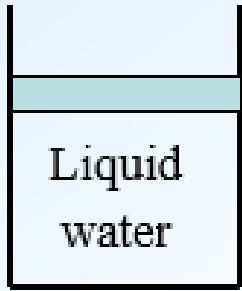
Pure substance has a homogeneous, non-varying chemical composition in all phases, for instance, ice, water, and steam.

Sometimes, a mixture of gases, such as air, is considered as a pure substance as long as there is no change of phase.

Simple compressible substance is a pure substance free of surface, magnetic, electrical, etc. effects.

4.1 Vapor-Liquid-Solid Phase Equilibrium in a Pure Substance

Let's do the following experiment in a closed piston-cylinder system that has a pure substance (water), adding heat and observing changes in the phases and the properties:



$P = 0.1 \text{ MPa}$

$T = 20 \text{ }^{\circ}\text{C}$

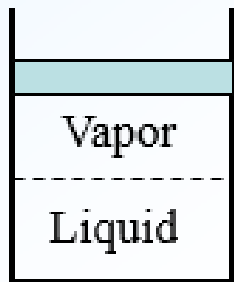
Add Heat



$P = 0.1 \text{ MPa}$ constant

T increases until $99.6 \text{ }^{\circ}\text{C}$

v increases very slightly



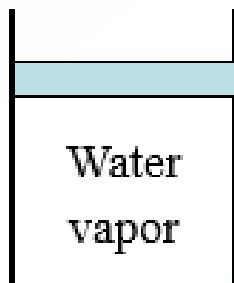
$P = 0.1 \text{ MPa}$ constant

$T = 99.6 \text{ }^{\circ}\text{C}$ constant

Change of phase occurs

v increases

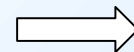
Keep adding heat



$P = 0.1 \text{ MPa}$ constant

$T > 99.6 \text{ }^{\circ}\text{C}$

Add Heat



$P = \text{constant}$

T increases

v increases



Saturation Temperature: Vaporization takes place at a given pressure

Saturation Pressure: The same at a given temperature at which vaporization occurs

Saturated Liquid or Vapor: at T_{sat} and P_{sat}

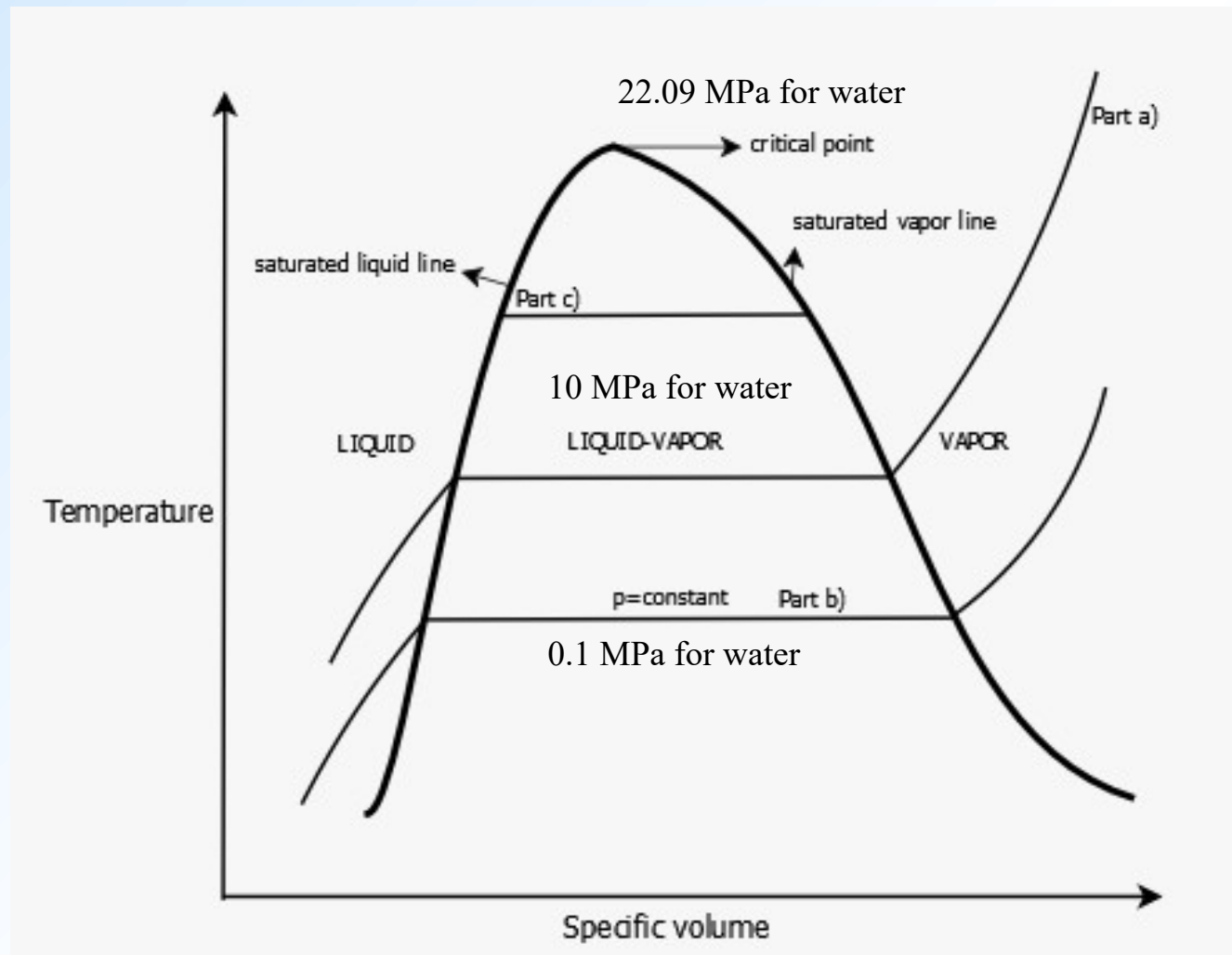
Compressed (or subcooled) Liquid: for a given pressure P , $T < T_{\text{sat}}$
or for a given temperature T , $P > P_{\text{sat}}$

Saturated State: at T_{sat} and P_{sat}

Supereated Vapor: for a given P , $T > T_{\text{sat}}$

Quality, x : an intensive property (independent of mass) $x = \frac{\text{Mass of vapor}}{\text{Total mass}}$

Critical Point: P_{cr} , T_{cr} Examine a T-v diagram of water



40 MPa for water. Is it liquid or vapor?



Consider another experiment:

1 kg Ice

$P = \text{constant}$

Ice melts at

$T < -20\text{ }^{\circ}\text{C}$

Add heat

T increases until $0\text{ }^{\circ}\text{C}$

$P = 100\text{ kPa}$ and $T = 0\text{ }^{\circ}\text{C}$

v decreases slightly

$P = 100\text{ kPa}$

v increases slightly

The reverse is true for most other substances

1 kg Ice

$P = \text{constant}$

Ice sublimates

$T < -10\text{ }^{\circ}\text{C}$

Add heat

T increases until $-10\text{ }^{\circ}\text{C}$

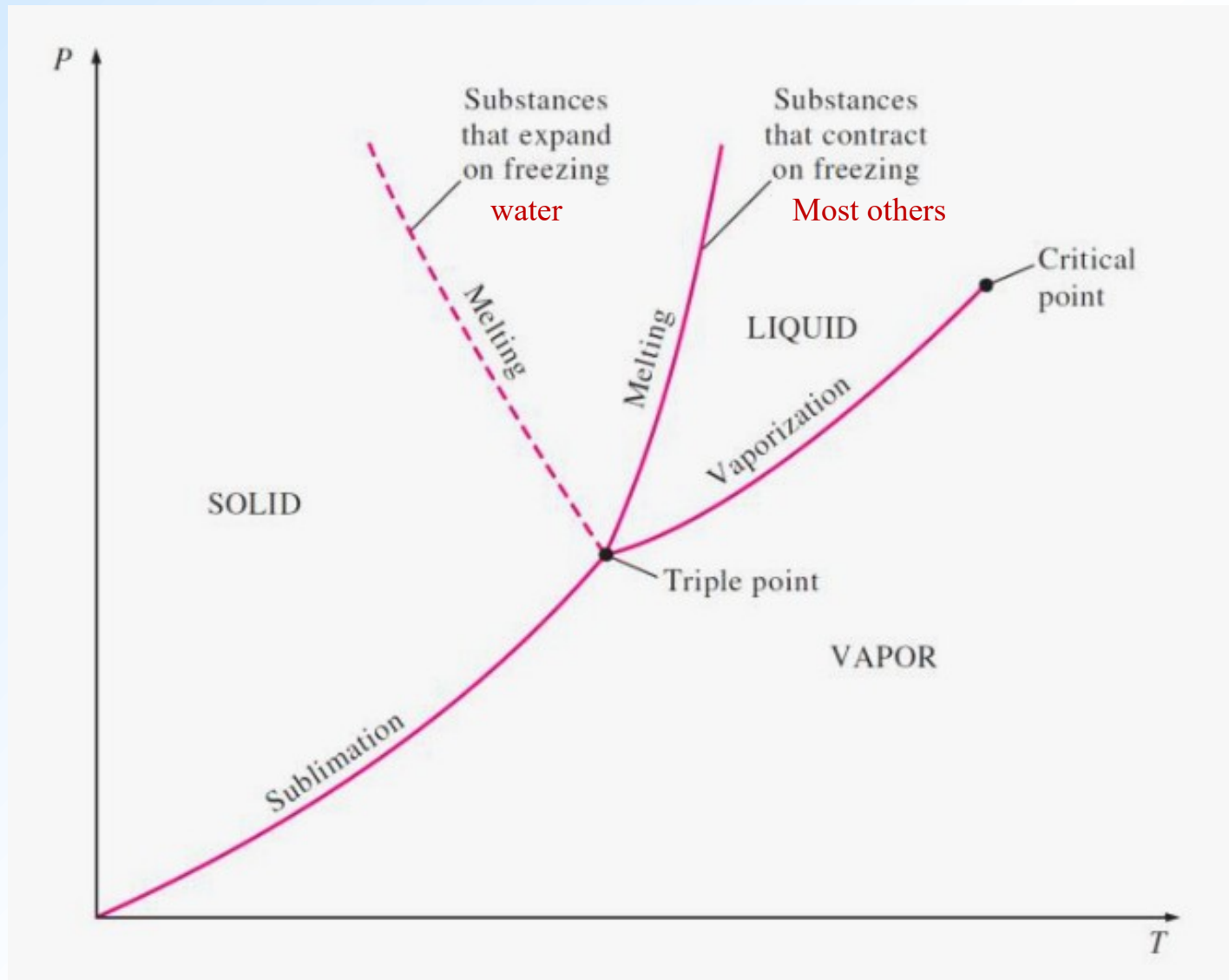
(becomes vapor directly at

$P = 0.26\text{ kPa}$

v increases slightly

$P = 0.26\text{ kPa}$ and $T = -10\text{ }^{\circ}\text{C}$

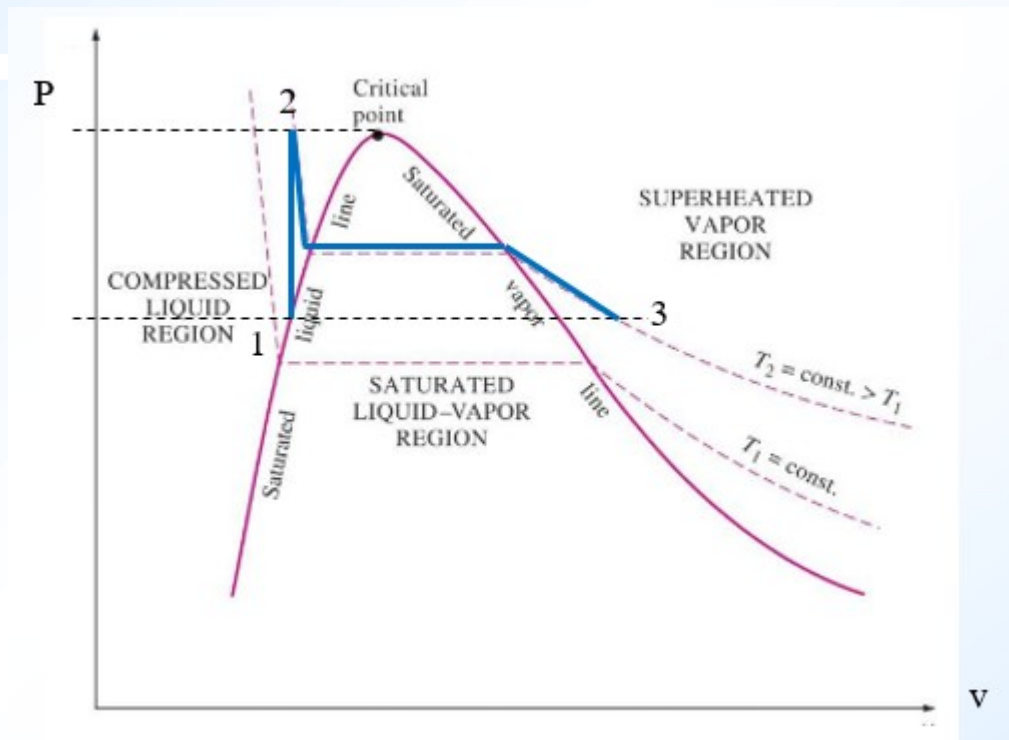
v stays constant

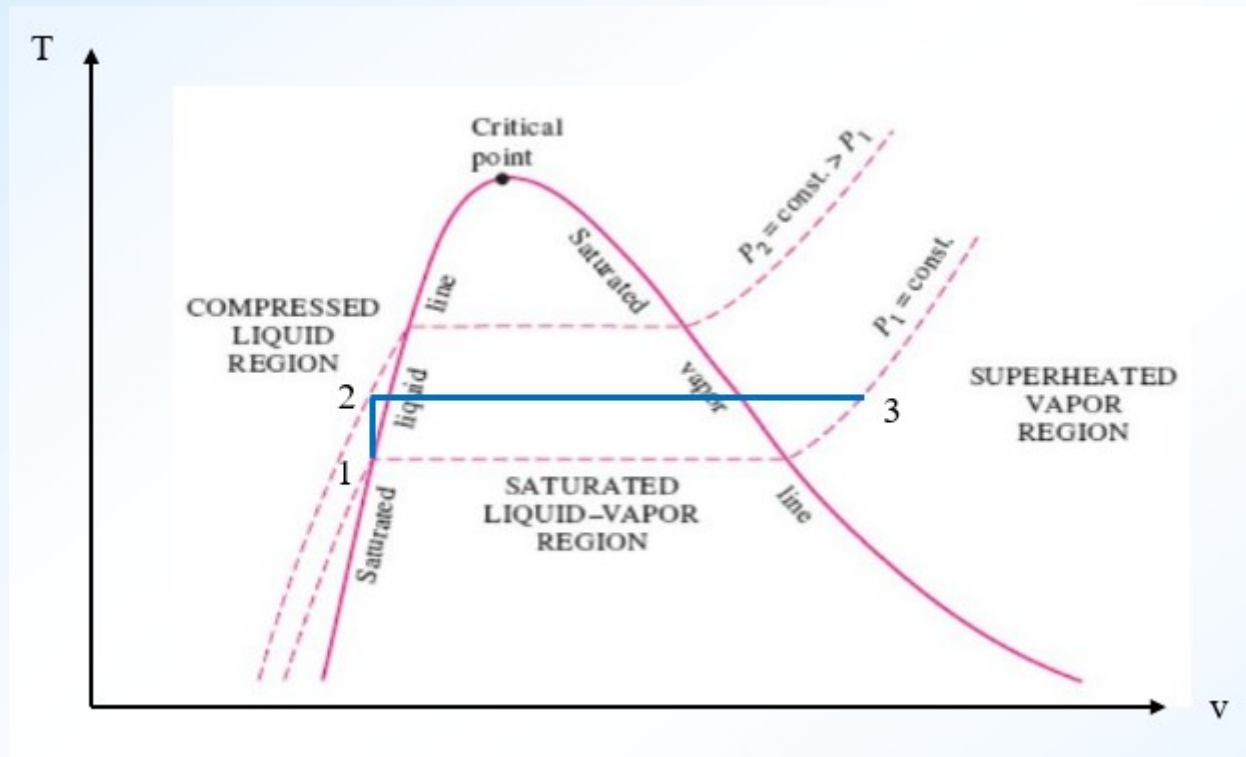


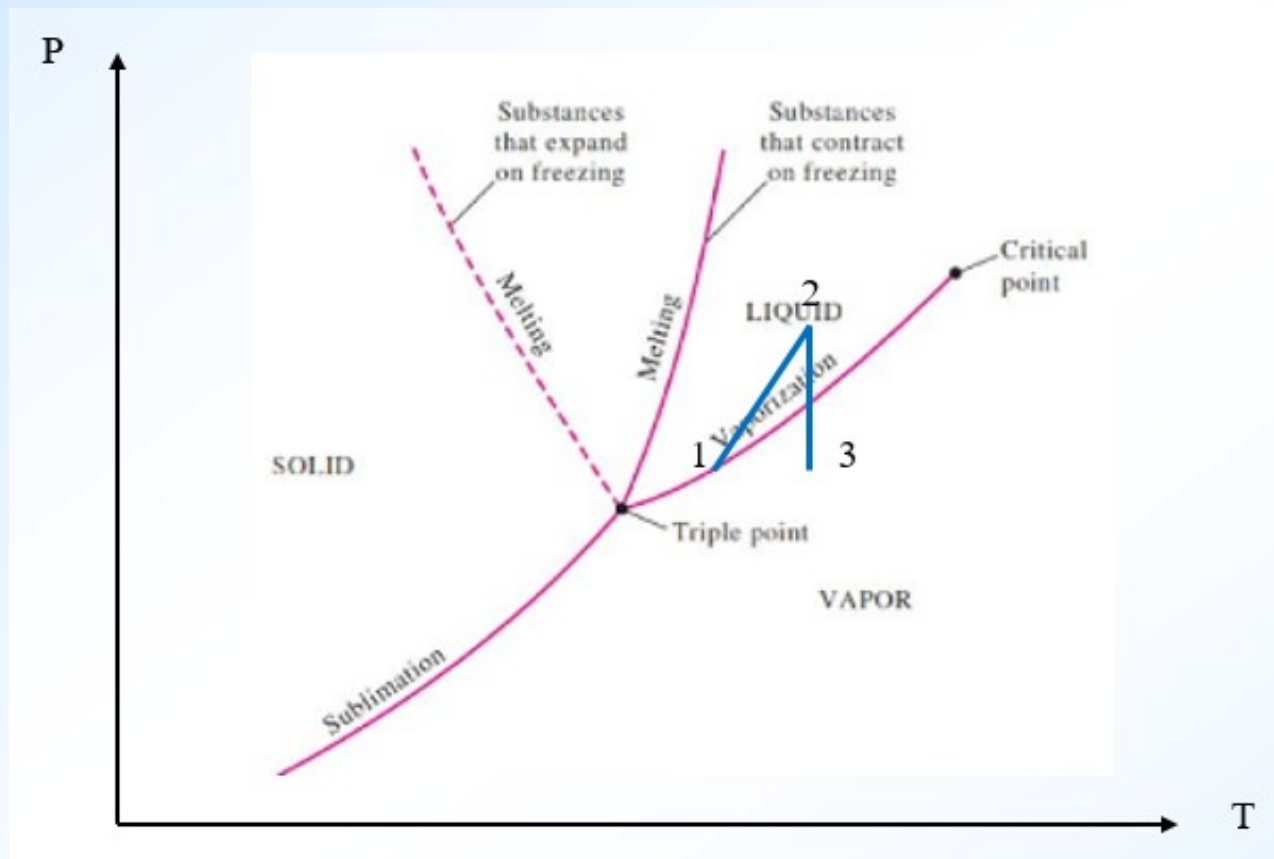


Example

Saturated liquid of a pure substance is heated at constant volume until the pressure is P_{crit} . Then, it is expanded at constant temperature back to its initial pressure. Show the processes on P-v, T-v, and P-T diagrams.







Note that P , T , and v are intensive properties. So are u and h . (not U and H)



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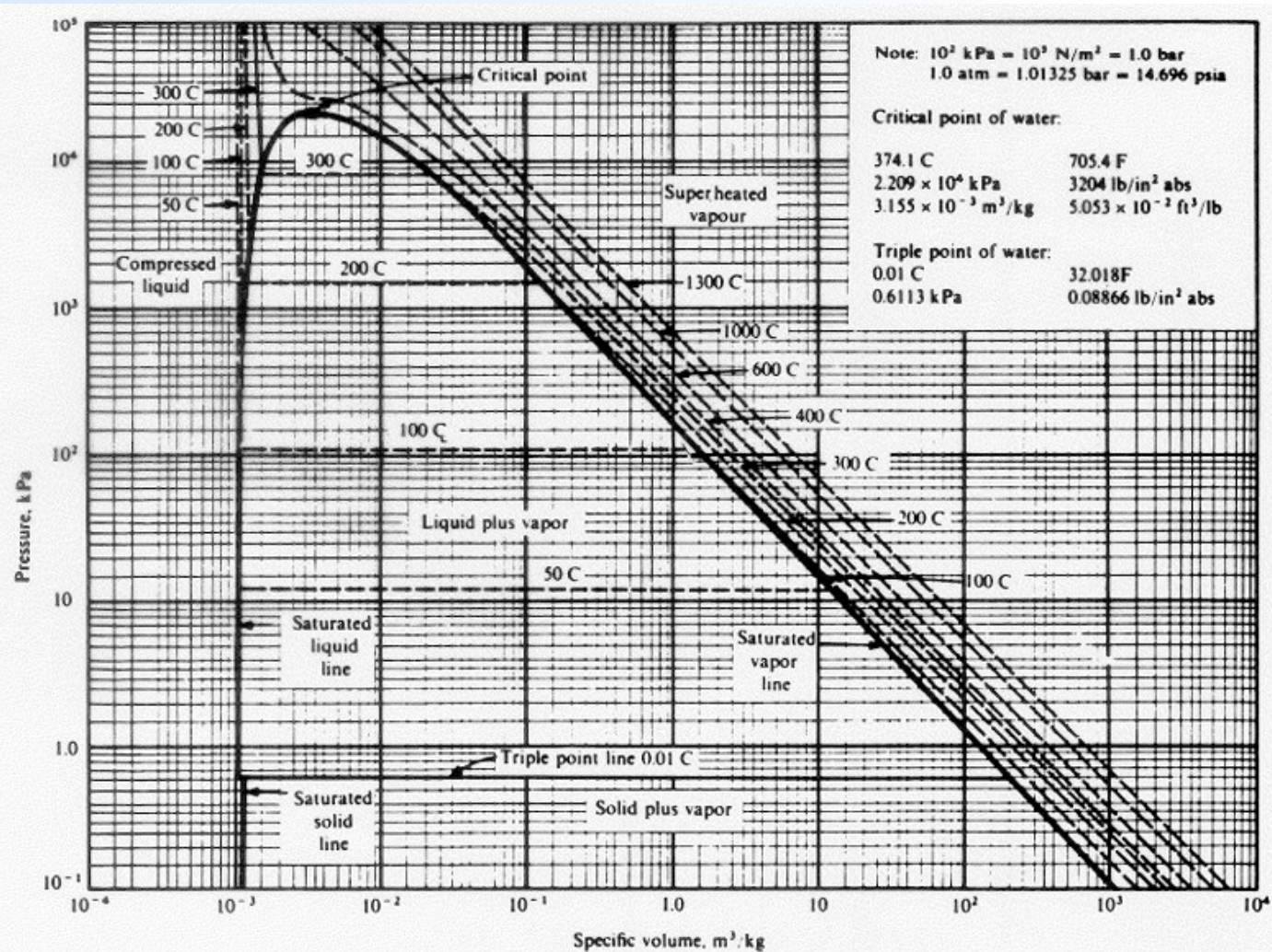


FIG. 1.5 Pressure-specific volume diagram for water (log scales).

Plotted from data in the *Steam Tables* by Keenan, Keyes, Hill and Moore, 1969 [B.16].



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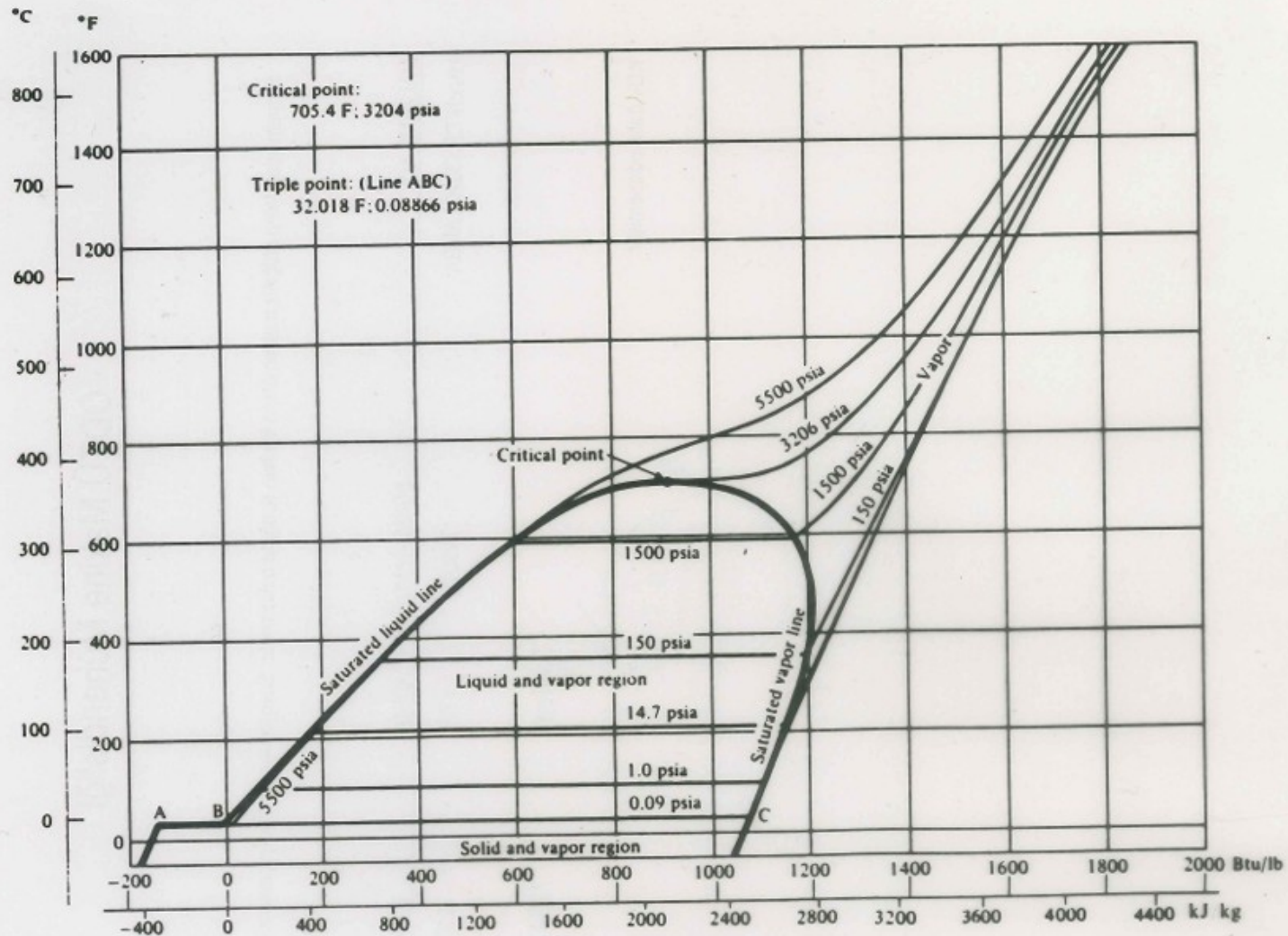


FIG. 1.4 Temperature-enthalpy diagram for water.

Note: 1.0 Btu/lb = 2.326 kJ/kg



4.2 Enthalpy as a Property

$P = 0 \text{ kPa}$ corresponds to vacuum (absolute zero pressure)

$T = 0 \text{ }^{\circ}\text{C} = 273 \text{ K}$ corresponds to saturation temperature of liquid water at 100 kPa

$T = 0 \text{ K}$ is the absolute zero temperature (of the vacuum)

$V = 0 \text{ m}^3$ or $v = 0 \text{ m}^3/\text{kg}$ obvious

What about u or $h = u + P v$?

The accepted «zero» for u and h depends on the substance (pure).

For example: $u = 0$ for water at $T = 0.01 \text{ }^{\circ}\text{C}$ and saturated liquid

$u = 0$ for freon at $T = -40 \text{ }^{\circ}\text{C}$ and saturated liquid

So, we may have negative values for u and h .



4.3 Properties of a Wet Mixture

$$v = x v_g + (1 - x) v_f$$

$$v = x v_f + x v_{fg}$$

$$v = v_g - (1 - x) v_{fg}$$

$$u = x u_g + (1 - x) u_f$$

$$u = x u_f + x u_{fg}$$

$$u = u_g - (1 - x) u_{fg}$$

$$u_f = h_f + P v_f$$

$$u_g = h_g + P v_g$$

$$u_{fg} = u_g - u_f$$

Subscript f : saturated liquid

Subscript g : saturated vapor

Subscript fg : difference, f - g

$x = \text{quality} = \text{dryness fraction} = \frac{\text{Mass of vapor}}{\text{Total mass}}$

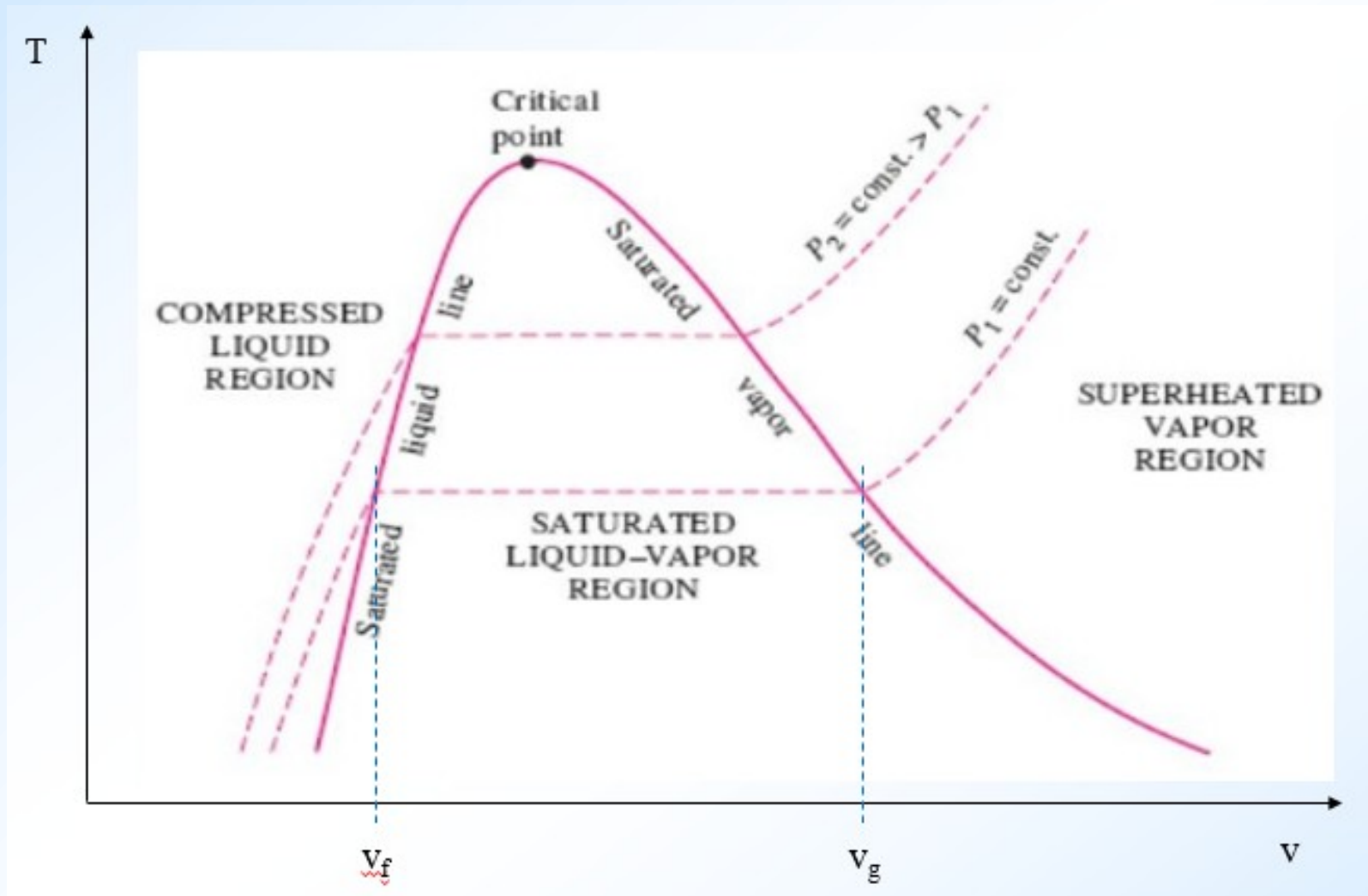
$$h = x h_g + (1 - x) h_f$$

$$h = h_f + x h_{fg}$$

$$h = h_g - (1 - x) h_{fg}$$

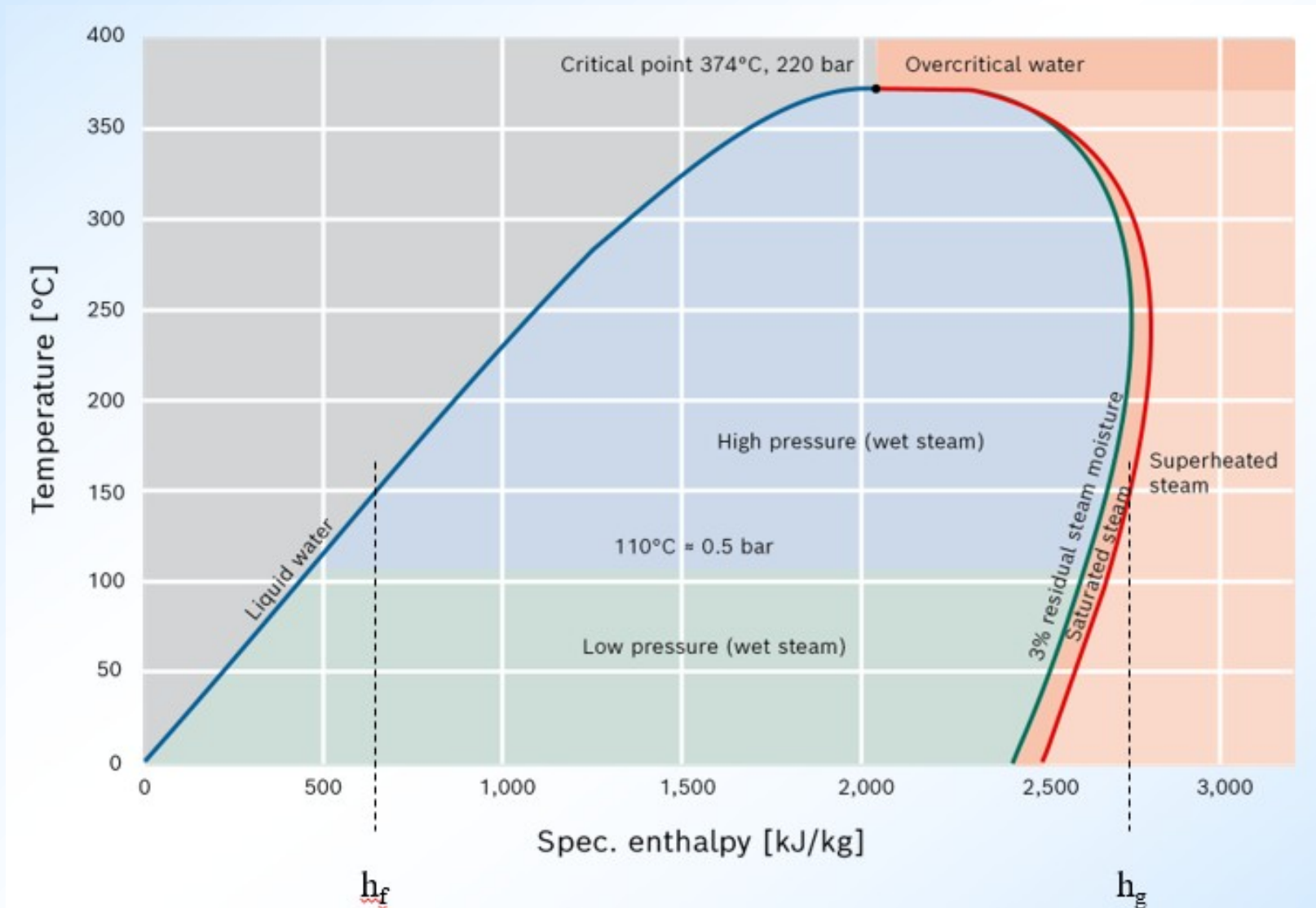
$$h_{fg} = h_g - h_f$$

$h_{fg} = \text{Latent heat (of vaporization)}$





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4.4 Properties of a Compressed (Subcooled) Liquid

Except at very high pressures, the properties vary only slightly with pressure. So we may use the properties of saturated liquid at the same temperature as the properties (u , h , v) of the compressed liquid.

Or, alternatively, to be more accurate, one may use thermodynamic tables.

4.5 Properties of a Superheated Vapor

An approximate formula:
$$v \cong \frac{0.233 (h \text{ in kJ/kg} - 1943)}{100 P \text{ in bars}}$$

To be more accurate, use thermodynamic tables.

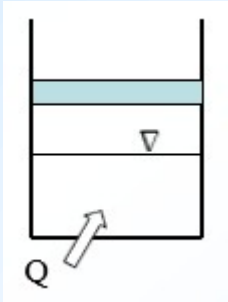


4.6 Independent Properties and State

Two independent properties determine the state of a pure substance.

P and T are NOT independent in the two-phase (saturation) region.

Example

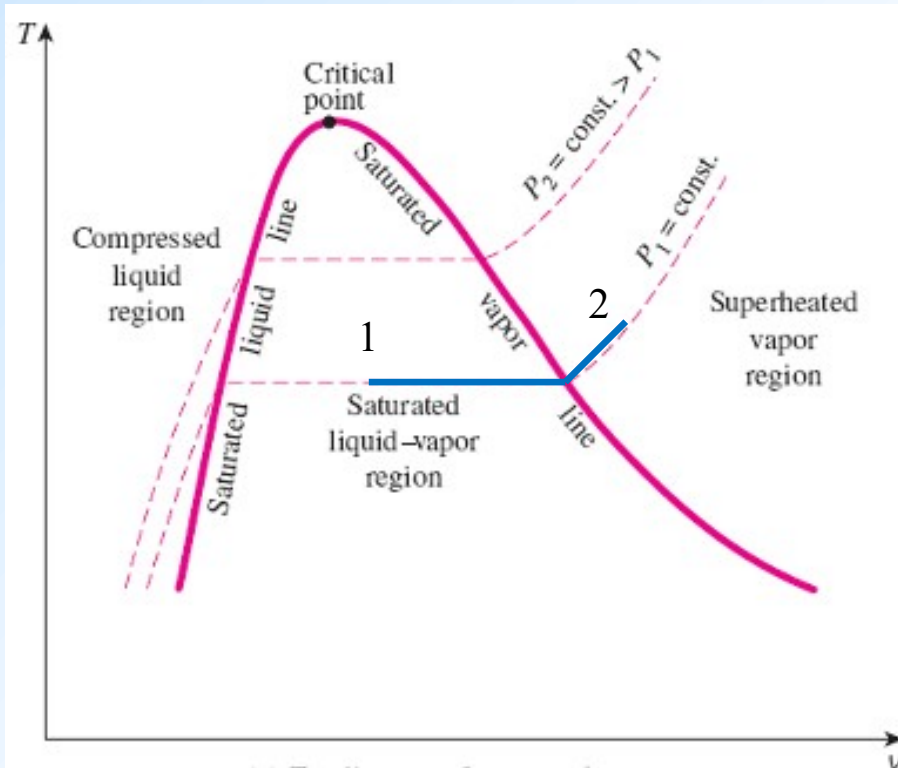


A cylinder fitted with a piston has a volume of 0.1 m^3 and contains 0.5 kg of water at 0.4 MPa . Heat is transferred to the steam until the temperature is 300°C . Determine ${}_1Q_2$ and ${}_1W_2$.

Constant-pressure process $\Rightarrow P_1 = P_2 = 0.4 \text{ MPa}$

Given : $m = 0.5 \text{ kg}$ $V_1 = 0.1 \text{ m}^3$

$$\left. \begin{array}{l} \text{Constant-pressure process} \\ \text{Given : } m = 0.5 \text{ kg} \quad V_1 = 0.1 \text{ m}^3 \end{array} \right\} \begin{aligned} {}_1W_2 &= \int_1^2 P \, dV = P_1 \int_1^2 dV \\ &= P m (v_2 - v_1) \end{aligned}$$



$$\begin{aligned} {}_1Q_2 &= {}_1W_2 + U_2 - U_1 \\ &= {}_1W_2 + m(u_2 - u_1) \end{aligned}$$

$$v_1 = \frac{V_1}{m} = \frac{0.1}{0.5} = 0.2 \text{ m}^3/\text{kg}$$

$$v_1 = v_{f1} + x v_{fg}$$

At $P = 0.4 \text{ MPa}$ two-phase region:

$$v_f = 0.001084 \text{ m}^3/\text{kg}$$

$$v_{fg} = 0.4614 \text{ m}^3/\text{kg}$$

$$x = \frac{v_1 - v_f}{v_{fg}} = \frac{0.2 - 0.001084}{0.4614} = 0.4311 \quad 43.11 \% \text{ quality}$$



$$\left. \begin{array}{l} h_1 = h_f + x h_{fg} \quad \text{At } P = 0.4 \text{ MPa} \\ \text{two-phase region} \end{array} \right\} \begin{array}{l} h_f = 604.74 \text{ kJ/kg} \\ h_{fg} = 2133.8 \text{ kJ/kg} \end{array}$$

$$h_1 = 604.74 + (0.4311) (2133.8) = 1524.6 \text{ kJ/kg}$$

$$\text{At } P = 0.4 \text{ MPa and } T_2 = 300 \text{ }^\circ\text{C} \Rightarrow h_2 = 3066.8 \text{ kJ/kg}$$

$${}_1W_2 = m P (v_2 - v_1) = (0.5) (400) (0.6548 - 0.2) = 91 \text{ kJ}$$

$$\begin{aligned} {}_1Q_2 &= m (u_2 - u_1) + m P (v_2 - v_1) = m (h_2 - h_1) = (0.5) (3066.8 - 1524.6) \\ &= 771.1 \text{ kJ} \end{aligned}$$

We may also determine u_1 and u_2 and then find ${}_1Q_2$.

$$\left. \begin{array}{l} \text{At } P = 0.4 \text{ MPa} \\ \text{saturation} \end{array} \right\} \begin{array}{l} u_f = 604.31 \text{ kJ/kg} \\ u_{fg} = 1949.3 \text{ kJ/kg} \end{array} \left\{ \begin{array}{l} u_1 = u_f + x u_{fg} \\ u_1 = 604.31 + (0.4311) (1949.3) \\ = 1444.6 \text{ kJ/kg} \end{array} \right.$$



$$\left. \begin{array}{l} \text{At } P = 0.4 \text{ MPa} \\ T_2 = 300 \text{ }^\circ\text{C} \end{array} \right\} \begin{array}{l} u_2 = 2804.8 \text{ kJ/kg} \\ {}_1Q_2 = (0.5) (2804.8 - 1444.5) + 91 = 771.1 \text{ kJ} \end{array}$$

4.7 Thermodynamic Tables

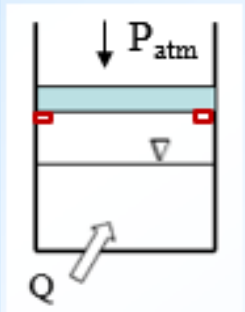
See the following Tables in the Appendices:

- Saturated steam temperature table
- Saturated steam pressure table
- Superheated vapor table
- Compressed liquid table
- Saturated-solid –vapor table (not much used)

Find the properties of the previous example problem using the Tables.



Example



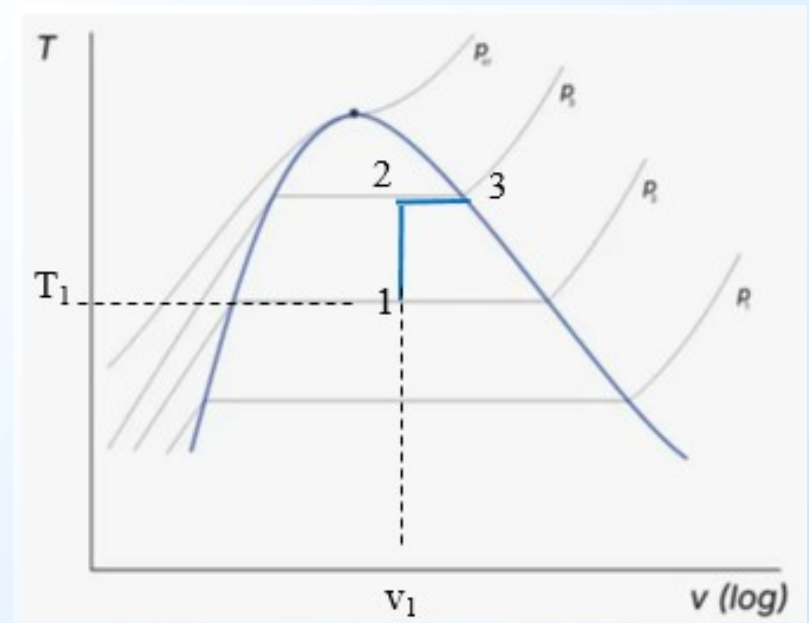
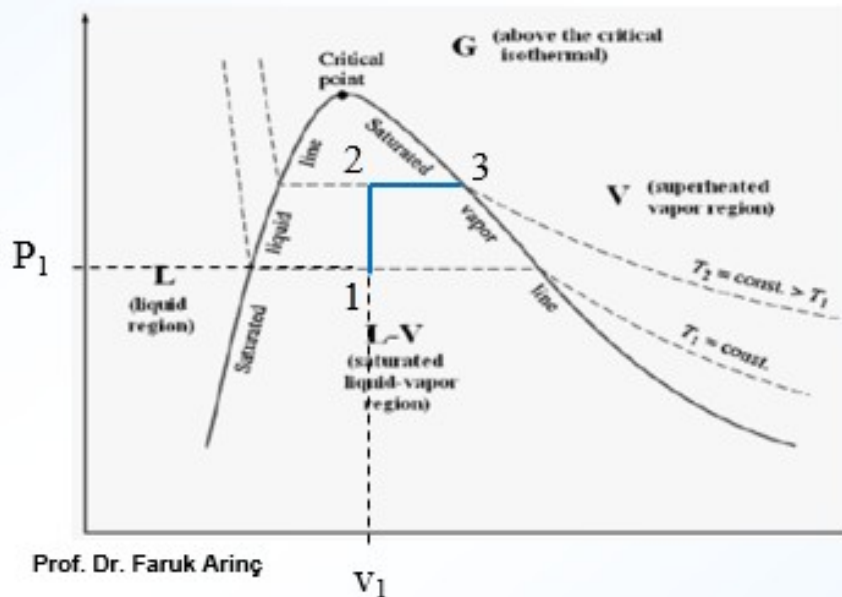
$m = 1 \text{ kg}$ saturated liquid water

$A_p = 0.065 \text{ m}^2$ $P_{\text{atm}} = 94 \text{ kPa}$ $g = 9.75 \text{ m/s}^2$

Initial state: $T_1 = 32^\circ \text{C}$ $V_1 = 0.1 \text{ m}^3$

Add Q until
all saturated vapor

- Find the temperature, T_2 , as the piston first rises from the pins
- Work done by the water





Choose the sytem as water only, and assume that the process is at semi-equilibrium.

First, find the pressure when the piston just starts moving: $P_2 = P_{\text{pist}} + P_{\text{atm}}$

$$\left. \begin{aligned} P_{\text{atm}} &= 94 \text{ kPa} \\ P_{\text{pist}} &= \frac{(40)(9.75)}{0.065} = 6000 \text{ Pa} = 6 \text{ kPa} \end{aligned} \right\} P_2 = 6 + 94 = 100 \text{ kPa}$$

$$\left. \begin{aligned} \text{(a)} \quad T_1 &= 32 \text{ }^\circ\text{C} \\ v_1 &= \frac{V_1}{m} = \frac{0.1}{1} = 0.1 \text{ m}^3/\text{kg} \end{aligned} \right\} \begin{aligned} &\text{Pressure is not given directly in the Table.} \\ &\text{Interpolation is necessary.} \end{aligned}$$

$$\left. \begin{aligned} T_{\text{sat},1} &= 30 \text{ }^\circ\text{C} \\ T_{\text{sat},2} &= 35 \text{ }^\circ\text{C} \end{aligned} \right\} \Delta T = 5 \text{ }^\circ\text{C} \quad \left. \begin{aligned} P_{\text{sat},1} &= 4.246 \text{ kPa} \\ P_{\text{sat},2} &= 5.628 \text{ kPa} \end{aligned} \right\} \Delta P = 1.382 \text{ kPa}$$

$$\text{For } T_1 = 32 \text{ }^\circ\text{C} \quad P_1 = P_{\text{sat},1} + \frac{P_{\text{sat},2} - P_{\text{sat},1}}{T_{\text{sat},2} - T_{\text{sat},1}} (T_1 - T_{\text{sat},1})$$



For $T_1 = 32\text{ }^{\circ}\text{C}$ $P_1 = 4.246 + \frac{1.382}{5} (32 - 30) = 4.8\text{ kPa}$

Let's find the quality at state 1 , again using interpolation in the Table:

$$\left. \begin{array}{l} T_{\text{sat},1} = 30\text{ }^{\circ}\text{C} \rightarrow v_f = 0.001004\text{ m}^3/\text{kg} \\ T_{\text{sat},2} = 35\text{ }^{\circ}\text{C} \rightarrow v_f = 0.001006\text{ m}^3/\text{kg} \end{array} \right\} \text{At } T = 32\text{ }^{\circ}\text{C}$$

$$v_f = 0.001004 + \frac{0.001006 - 0.001004}{35 - 30} (32 - 30) = 0.0010048\text{ m}^3/\text{kg}$$

$$\left. \begin{array}{l} T_{\text{sat},1} = 30\text{ }^{\circ}\text{C} \rightarrow v_g = 32.89\text{ m}^3/\text{kg} \\ T_{\text{sat},2} = 35\text{ }^{\circ}\text{C} \rightarrow v_g = 25.22\text{ m}^3/\text{kg} \end{array} \right\} \text{At } T = 32\text{ }^{\circ}\text{C}$$

$$v_g = 32.89 + \frac{25.22 - 32.89}{35 - 30} (32 - 30) = 29.822\text{ m}^3/\text{kg}$$

$$v_{fg} = 29.822 - 0.0010048 = 29.820955\text{ m}^3/\text{kg}$$



The quality or dryness fraction at State 1:
$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.1 - 0.0010048}{29.820955} = 0.0033196 \Rightarrow 0.33 \%$$

Add heat, at constant volume, until $P_2 = 100 \text{ kPa}$, $v_2 = v_1 = v_g = 0.1 \text{ m}^3/\text{kg}$

At $P_2 = 100 \text{ kPa}$, $T_2 = 99.63 \text{ }^\circ\text{C}$, $v_f = 0.001043 \text{ m}^3/\text{kg}$, $v_g = 1.6940 \text{ m}^3/\text{kg}$

Since $v_2 < v_g$, we are still in the two-phase region.

The quality or dryness fraction at State 2:
$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.1 - 0.0010043}{1.6440 - 0.001043} = 0.05845 \Rightarrow 5.8 \%$$

(b) Add more heat, at constant pressure, until all saturated vapor (steam)

State 3: $P_3 = 100 \text{ kPa}$, $T_3 = 99.63 \text{ }^\circ\text{C}$, $v_3 = v_g = 1.694 \text{ m}^3/\text{kg}$

$${}_1W_3 = {}_2W_3 = \int_2^3 P \, dV = P (V_3 - V_2) = (100) (1.694 - 0.1) = 159.4 \text{ kJ}$$



4.8 Perfect (Ideal) Gases

Boyle's Law or Boyle-Mariotte Law:

The absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies if the temperature and amount of gas remain unchanged within a closed system

$$\text{If } T \text{ is constant} \Rightarrow P \propto \frac{1}{V} \text{ or } P V = \text{constant}$$

Charles' Law:

When the pressure on a sample of a dry gas is held constant, the Kelvin temperature and the volume will be in direct proportion

$$\text{If } P \text{ is constant} \Rightarrow V \propto T \text{ or } \frac{V}{T} = \text{constant} \quad T \text{ is in Kelvins}$$



Robert Boyle FRS

Anglo-Irish Chemist

1627 - 1691



Jacques Charles

French Scientist

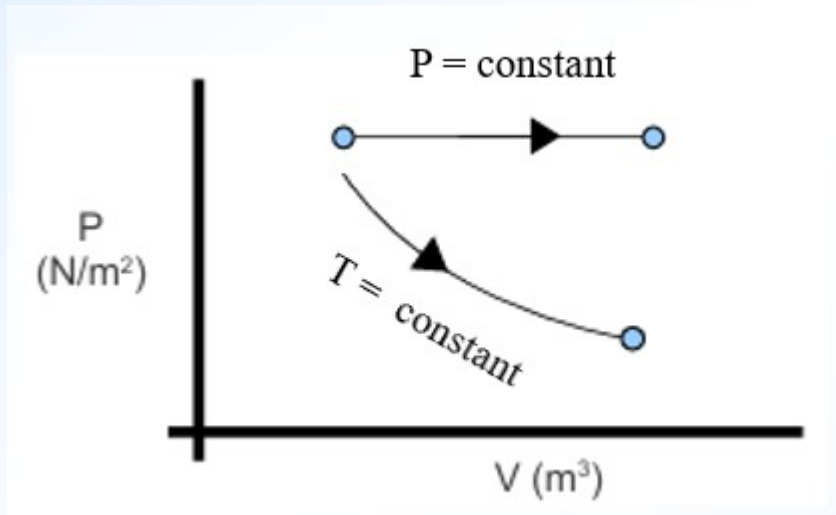
1746 - 1823



Combine the two laws: $\frac{P V}{T} = \text{constant}$

Include mass. m : $\frac{P V}{m T} = \text{constant} = R$ R is called the gas constant

Any gas with a state away from saturation and critical point that behaves close enough to this law is called a **perfect (ideal) gas**.



$P V = m R T$, T is in Kelvins

For air, $R = 0.287 \text{ kJ/kg.K}$. The gas constants for other gases are given in Tables.

Note that steam is NOT a perfect gas.



Define **molecular weight** of a gas = (12) $\left(\frac{\text{Mass of the gas molecule}}{\text{Mass of C-12 atom}} \right)$

Molecular weight of C-12 atom is exactly 12 (no units)

Molecular weight of O₂ molecule is approximately 32

Molecular weight of CO₂ molecule is approximately 44

Molecular weight of Air molecule is approximately 28.97

See the Thermodynamic tables for the others

Define **kg-mol or kmol**: It is the quantity of a gas whose mass is numerically equal to its molecular weight in kilograms.

O₂ => 32 kg / kmol

Air => 28.97 kg / kmol



Avagadro's Law: The total number of atoms (molecules) of a gas (i.e., the amount of gaseous substance) is directly proportional to the volume occupied by the gas at constant temperature and pressure.

Another statement of the **Avagadro's Law**: Equal volumes of different gasses at the same state (P and T) contain the same number of molecules (or moles), or vice versa.

This number is called the **Avagadro's number** = $N_A = 6.02214076 \times 10^{23}$

If M is the molecular weight of a gas, the mass (in kg's) of the gas has n number of kmol's.

$$n \text{ (number of kmol's)} = \frac{M \text{ (molecular weight)}}{m \text{ (mass in kg)}}$$



Lorenzo Romano Amedeo Carlo Avogadro

Count of Quaregna and Cerreto

Italian Chemist

1776 - 1826



Ideal gas law: $\frac{P V}{m T} = R \Rightarrow \frac{P V / M}{m T / M} = \frac{P V / M}{(m / M) T} = \frac{P V / M}{n T}$

$$\frac{P V}{n T} = M R = \text{Constant for all perfect (ideal) gasses}$$

Universal gas constant

$$\frac{P V}{n T} = M R = R_0 = 8.3143 \text{ kJ/kmol.K}$$

$$P V = m R T$$

Diagram showing units for $P V = m R T$:
- P (kPa) points to the first P
- V (m^3) points to the first V
- m (kg) points to m
- R (kJ/kg.K) points to R
- T (K) points to T

$$P V = n R_0 T$$

Diagram showing units for $P V = n R_0 T$:
- P (kPa) points to the first P
- V (m^3) points to the first V
- n (kmol) points to n
- R_0 (kJ/kmol.K) points to R_0
- T (K) points to T



Example Room: 6 m x 4 m x 10 m $P = 100 \text{ kPa}$, $T = 25 \text{ }^{\circ}\text{C}$

Find the mass of the air in the room.

$$m = \frac{P V}{R T} = \frac{(100) (6) (4) (10)}{(0.287) (25 + 273)} \cong 280 \text{ kg}$$

Example

Piston area = 0.2 m^2

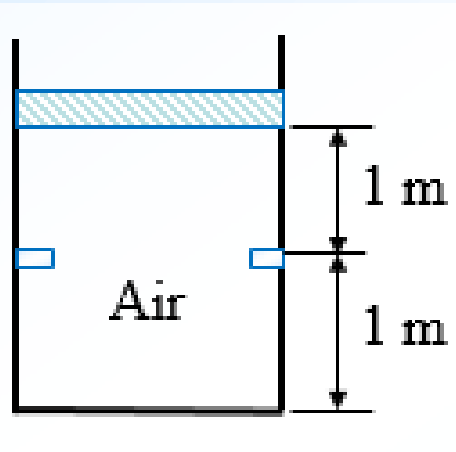
Initial state: $P_1 = 200 \text{ kPa}$ $T_1 = 500 \text{ }^{\circ}\text{C} = 773 \text{ K}$

(a) Air is cooled until the stops (Pressure remains constant)

Find T_2 , ${}_1Q_2$, ${}_1W_2$

(b) Air is cooled until $T_3 = 20 \text{ }^{\circ}\text{C}$ (Volume remains constant)

Find ${}_2Q_3$, P_3



State 1



(a) Constant pressure process

Initial volume: $V_1 = (2) (0.2) = 0.4 \text{ m}^3$

Volume at State 2: $V_2 = (1) (0.2) = 0.2 \text{ m}^3$

Constant pressure process: $P_2 = P_1 = 200 \text{ kPa}$

$$\left. \begin{array}{l} P_1 V_1 = m R T_1 \\ P_2 V_2 = m R T_2 \end{array} \right\} \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow T_2 = T_1 \frac{V_1}{V_2} = 773 \frac{0.4}{0.2} = 1546 \text{ K} = 1273 \text{ } ^\circ\text{C}$$

$${}_1W_2 = \int_1^2 P dV = P_1 (V_2 - V_1) = (200) (0.2 - 0.4) = -40 \text{ kJ}$$

Work is done on the system

First Law: ${}_1Q_2 = m (u_2 - u_1) + {}_1W_2$

$$u_2 - u_1 = ?$$

$$m = \frac{P_1 V_1}{R T_1} = \frac{(200) (0.4)}{(0.287) (773)} = 0.3606 \text{ kg of air}$$



$u_2 - u_1 = ?$ } Two methods: }
Use the air table
Use the definition of heat capacity

Use the air table

For an ideal gas, $u = f(T)$, i.e, internal energy is a function of temperature only.

This can be proved, experimentally.

At $T_1 = 773 \text{ K}$ $\Rightarrow h_1 = 792.379 \text{ kJ/kg}$
At $T_2 = 386.6 \text{ K}$ $\Rightarrow h_2 = 387.443 \text{ kJ/kg}$ } from air table

$${}_1Q_2 = m (u_2 - u_1) + {}_1W_2 = m (u_2 - u_1) + P m (v_2 - v_1)$$

$${}_1Q_2 = m [P v_2 + u_2 - (P v_1 + u_1)] = m (h_2 - h_1)$$

$${}_1Q_2 = m (h_2 - h_1) = (0.3606) (387.443 - 792.379) = -146 \text{ kJ}$$

Heat flows out of the system



Use the definition of heat capacity

Definition of specific heat capacity:

Amount of heat (thermal energy) required to raise the temperature of a unit mass of a substance through 1 degree under specified conditions.

Distinguish two conditions: $\left\{ \begin{array}{l} \text{Constant pressure process} \Rightarrow c_p \\ \text{Constant volume process} \Rightarrow c_v \end{array} \right.$

$${}_1Q_2 = m c_p (T_2 - T_1) = m (h_2 - h_1) \Rightarrow \underbrace{c_p (T_2 - T_1)}_{h_2 - h_1}$$

If c_p remains constant during the process

$${}_1Q_2 = m c_p (T_2 - T_1) = \underbrace{(1.0035)}_{\text{From air table in kJ/kg.K}} (386.6 - 773) = - 139.82 \text{ kJ}$$

From air table in kJ/kg.K



Use the air table: ${}_1Q_2 = -146 \text{ kJ}$

Use the definition of heat capacity: ${}_1Q_2 = -139.82 \text{ kJ}$

The result using the air table must be more accurate. The reason why there is a discrepancy is that c_p is not quite constant and changes with temperature.

$P = \text{constant: } \int c_p dT = \int dh \Rightarrow c_p \Delta T = \Delta h$ if c_p is assumed to be constant

$V = \text{constant: } \int c_v dT = \int du \Rightarrow c_v \Delta T = \Delta u$ if c_v is assumed to be constant

For a given gas, $R = c_p - c_v$. See the textbook for the proof.

For an isentropic process (reversible and adiabatic): $\frac{c_p}{c_v} = k$

Isentropic index of expansion



(b) Constant volume process: $V_2 = V_3 = 0.2 \text{ m}^3$ and

$$T_2 = 386.6 \text{ K}$$

$$\left. \begin{array}{l} P_2 V_2 = m R T_2 \\ P_3 V_3 = m R T_3 \end{array} \right\} \frac{P_2}{P_3} = \frac{T_2}{T_3} \Rightarrow P_3 = P_2 \frac{T_3}{T_2} = 200 \frac{293}{386.6} = 151.6 \text{ kPa}$$

$$\begin{aligned} {}_2Q_3 &= m (u_3 - u_2) + {}_2W_3 & {}_2W_3 &= 0 & \Rightarrow & {}_2Q_3 &= m (u_3 - u_2) \\ & & & & & {}_2Q_3 &= m c_v (T_3 - T_2) \end{aligned}$$

$${}_2Q_3 = m c_v (T_3 - T_2) = (0.7165) (0.3606) (293 - 386.6) = -24.2 \text{ kJ}$$

↑
Heat flows out of the system

$$\text{Using Table A-10: } {}_2Q_3 = m (u_3 - u_2) = (0.7165) (209.071 - 276.5) = -24.3 \text{ kJ}$$



$${}_2Q_3 = m (u_3 - u_2) = -24.3 \text{ kJ}$$

$${}_2Q_3 = m c_v (T_3 - T_2) = -24.2 \text{ kJ}$$

} The difference is due to the fact that c_v is assumed to remain constant during the constant-volume process

$$\text{Actually: } \int c_v dT = \int du \quad \Rightarrow \quad c_v \Delta T = \Delta u \quad \text{iff } c_v \text{ is constant}$$

For the variation of c_v and c_p with temperature, see the Appendix 8 for real gasses.

The air table is derived using these equations.



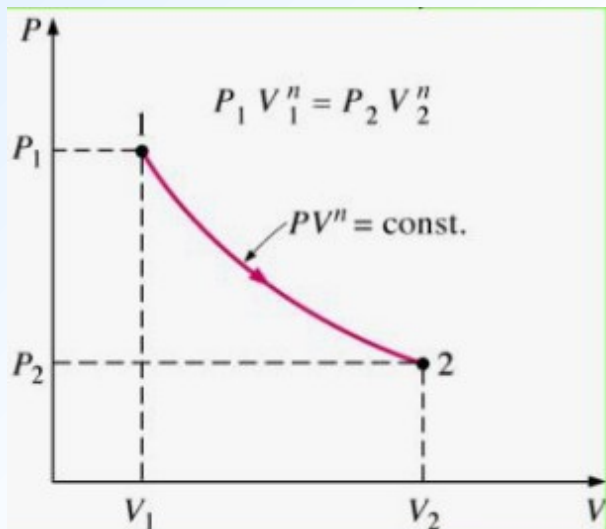
4.8.1 Polytropic Process in an Ideal Gas (reversible)

General Relation: $PV = m R T$ R : Gas constant = $c_p - c_v$

$PV = n R_0 T$ R_0 : Universal gas constant = 8.3143 kJ/kmol.K

Constant volume process: $\int c_v dT = \int du = \int \delta Q$

Constant pressure process: $\int c_p dT = \int dh = \int \delta Q$



Define n : Index of expansion or compression

For a process where both P and V are changing:
(Polytropic Reversible Process)

$$P V^n = \text{Constant}$$



During such a polytropic reversible process in an ideal gas with n :

$${}_1W_2 = \frac{P_1 V_1 - P_2 V_2}{n - 1} \quad \text{This we proved before.}$$

$${}_1W_2 = \frac{m R (T_1 - T_2)}{n - 1} \quad \text{Since } P V = m R T$$

$${}_1Q_2 = m R (T_1 - T_2) \left(\frac{c_p - n c_v}{n - 1} \right)$$

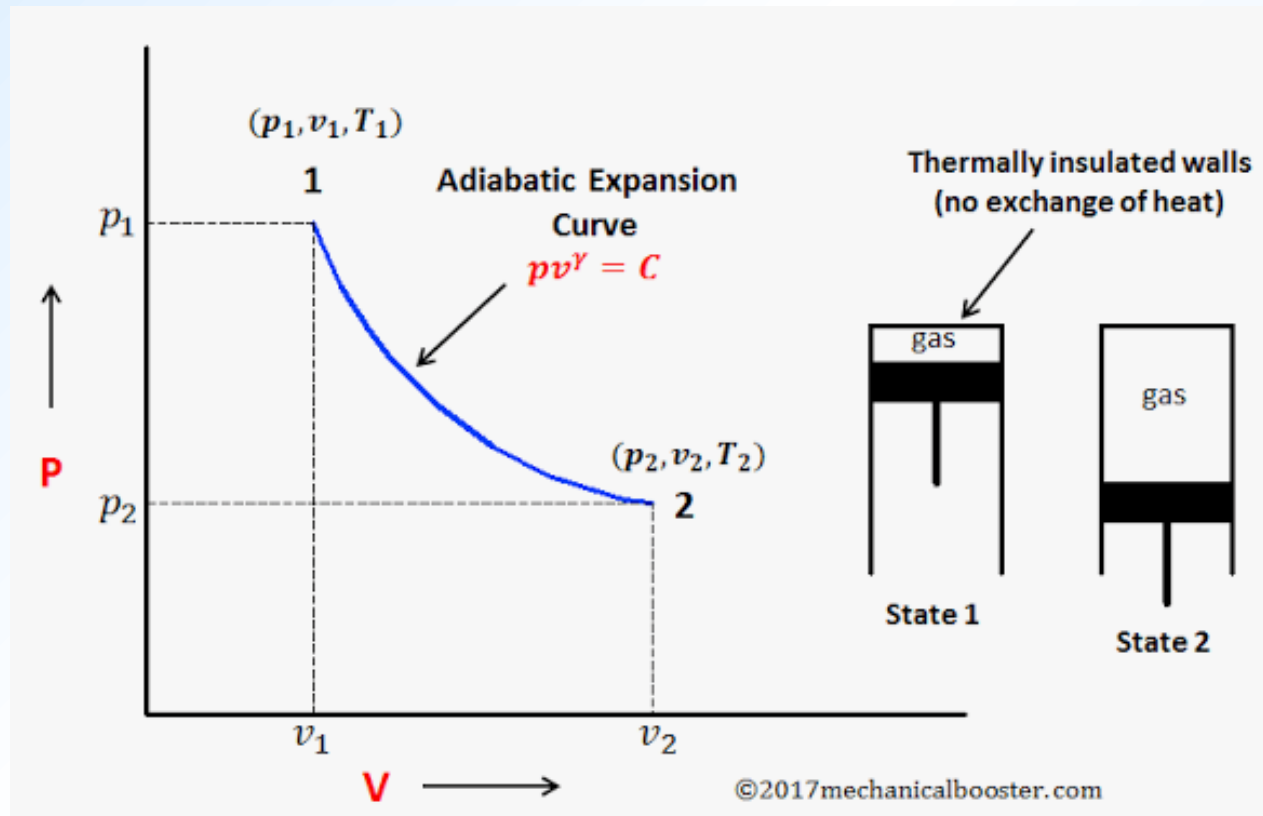
$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{n}{n-1}}$$

See the text book for the proofs.



4.8.2 Adiabatic Process in an Ideal Gas (reversible)

An adiabatic process is a type of thermodynamic process that occurs without transferring heat between the thermodynamic system and its environment.





$${}_1Q_2 = 0 = m (T_1 - T_2) \left(\frac{c_p - n c_v}{n - 1} \right) \Rightarrow c_p - n c_v = 0 \Rightarrow n = \frac{c_p}{c_v}$$

↑

True only if reversible and adiabatic

n is called isentropic index of expansion

