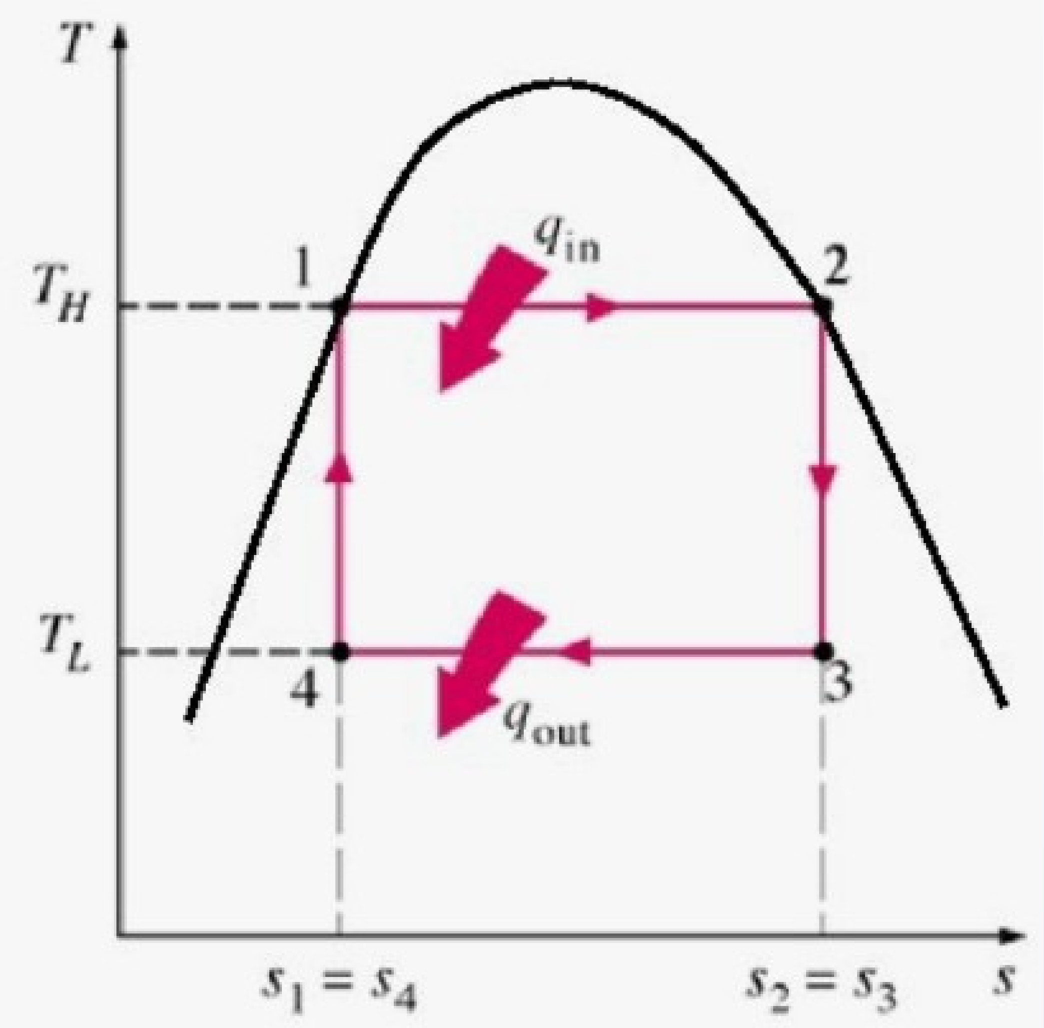
# 6. Thermodynamic Cycles (Power Cycles)




## 6.1 Carnot Cycle

4-1: Isentropic compression

1-2: Isothermal heat addition

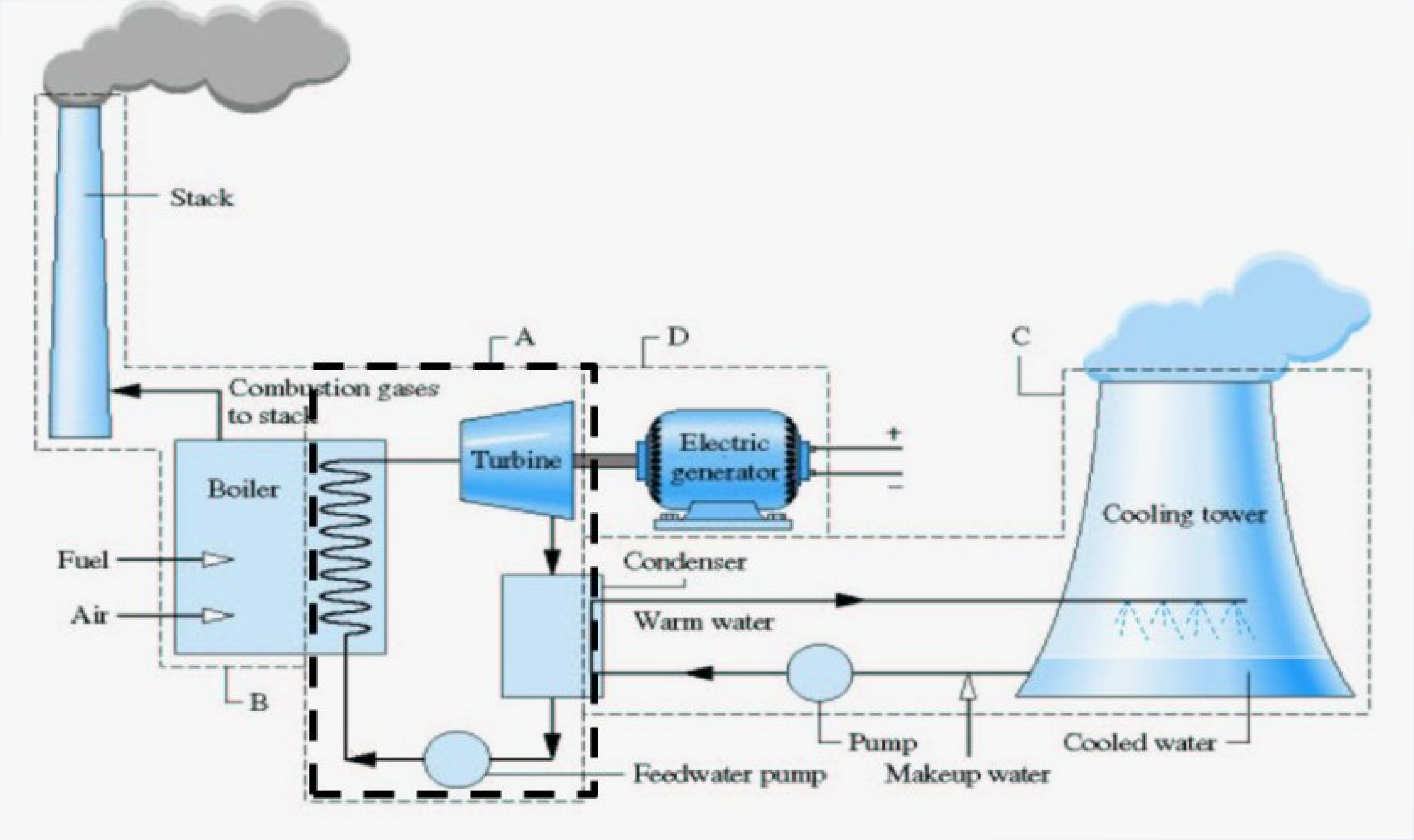
2-3: Isentropic expansion

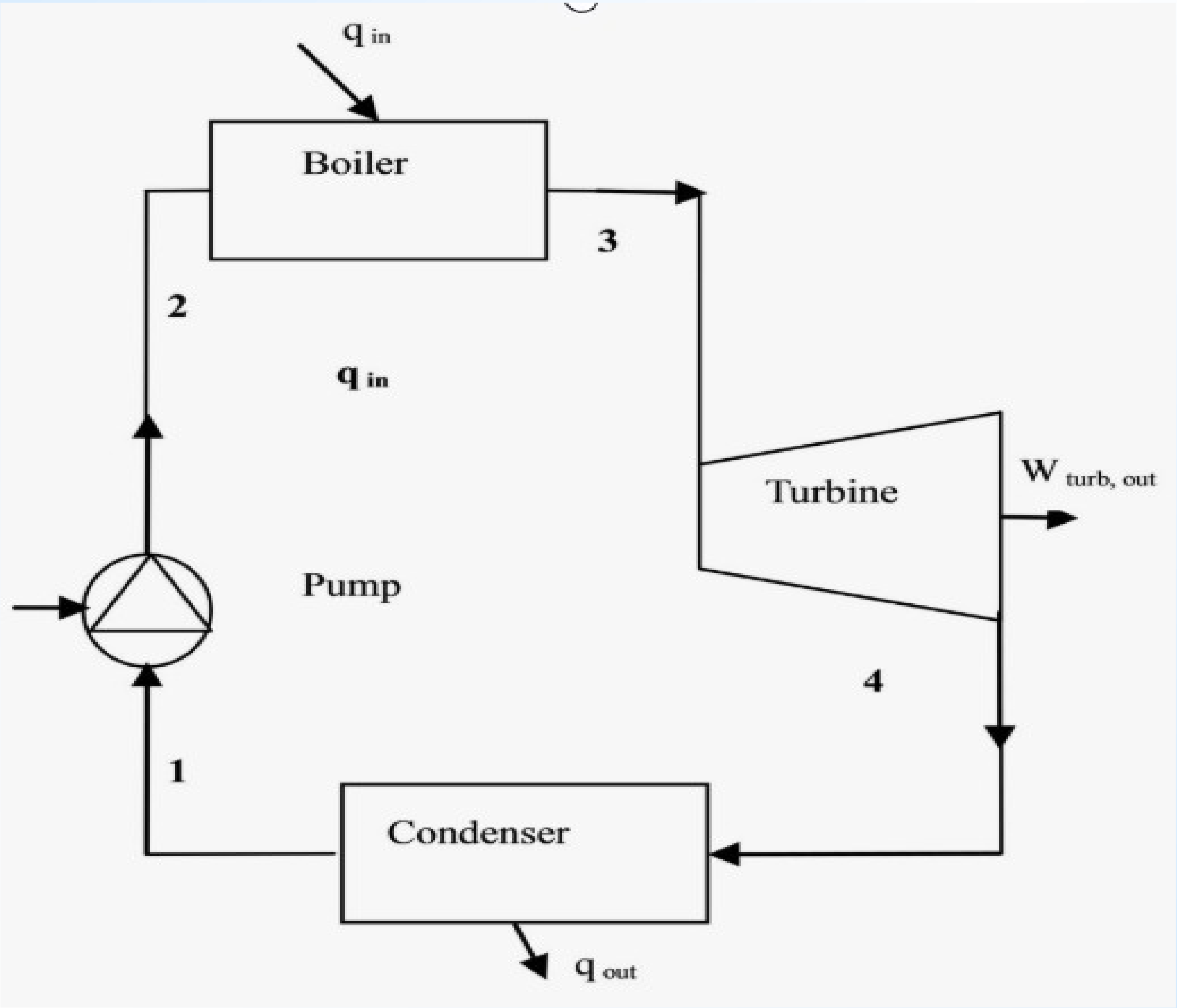
3-4:Isothermal heat substraction

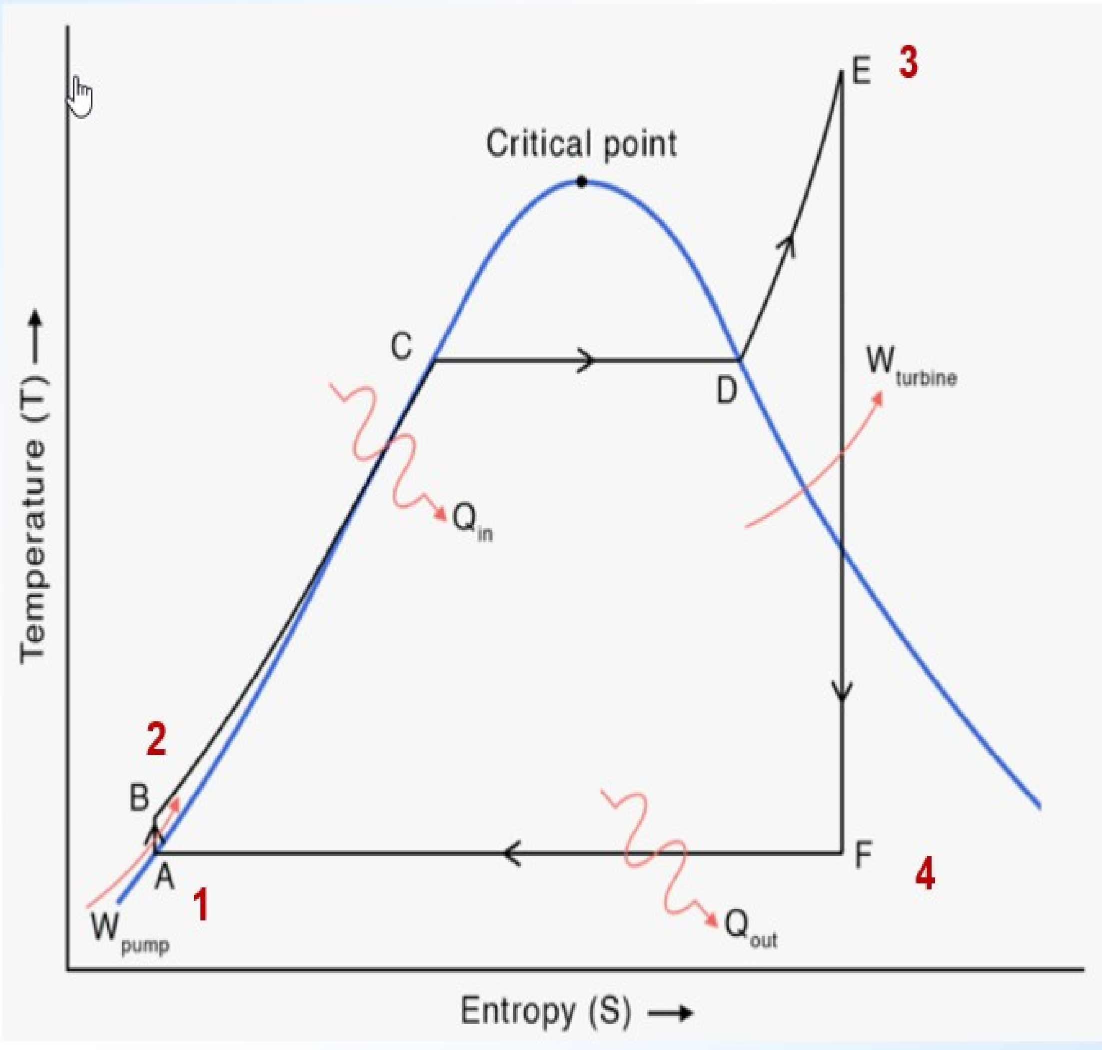
Problems:

- How to pump a two-phase mixture
- How to handle the liquid at the turbine exit

## 6.2 Steam Power Plant – Rankine Cycle




q in
Boiler
3
2
q in
Turbine
W turb, out
Pump
4
1
Condenser
q out



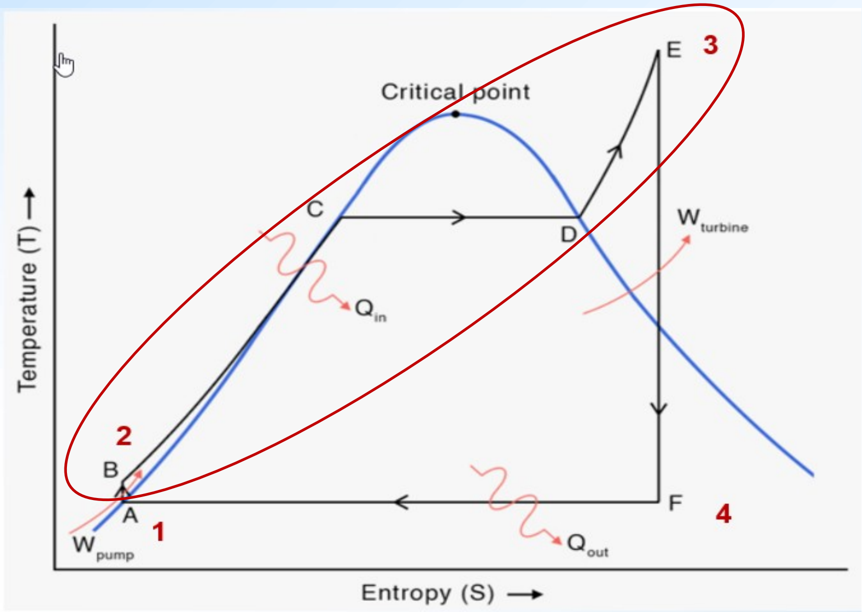

## Ideal Rankine Cycle:

1 – 2: Isentropic (reversible, adiabatic) compression in the pump,

2 – 3: Isobaric (constant pressure) heat addition in the boiler

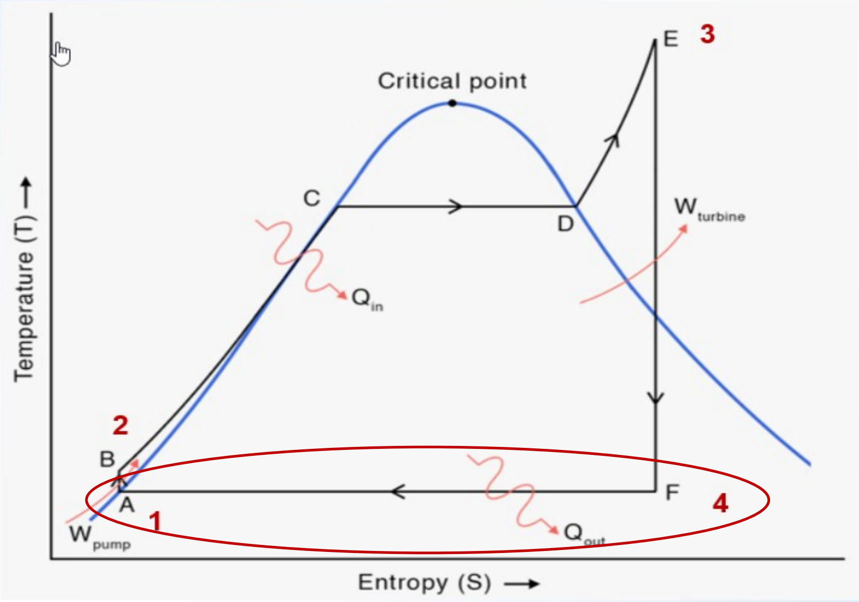
3 – 4: Isentropic (reversible, adiabatic) expansion in the turbine

4 – 1: Isobaric heat removal in the condensor



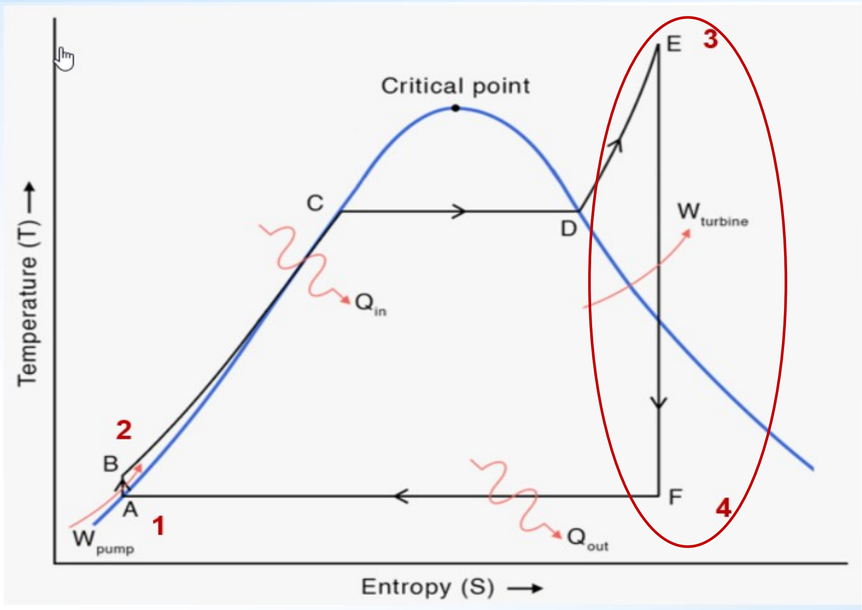

Process 2 – 3 : Isobaric (constant pressure) heat addition in the boiler (or evaporator)

$$\dot{Q}_H = \dot{m}\ (h_3 - h_2) \qquad \text{positive}$$



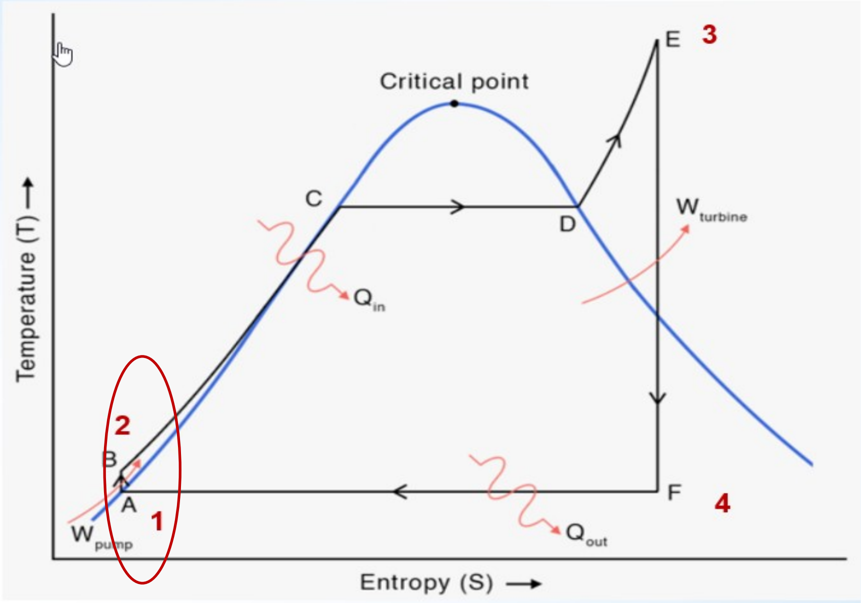

Process 4 – 1: Isobaric heat removal in the condensor

$$\dot{Q}_L = \dot{m}\ (h_1 - h_4) \qquad \text{negative}$$



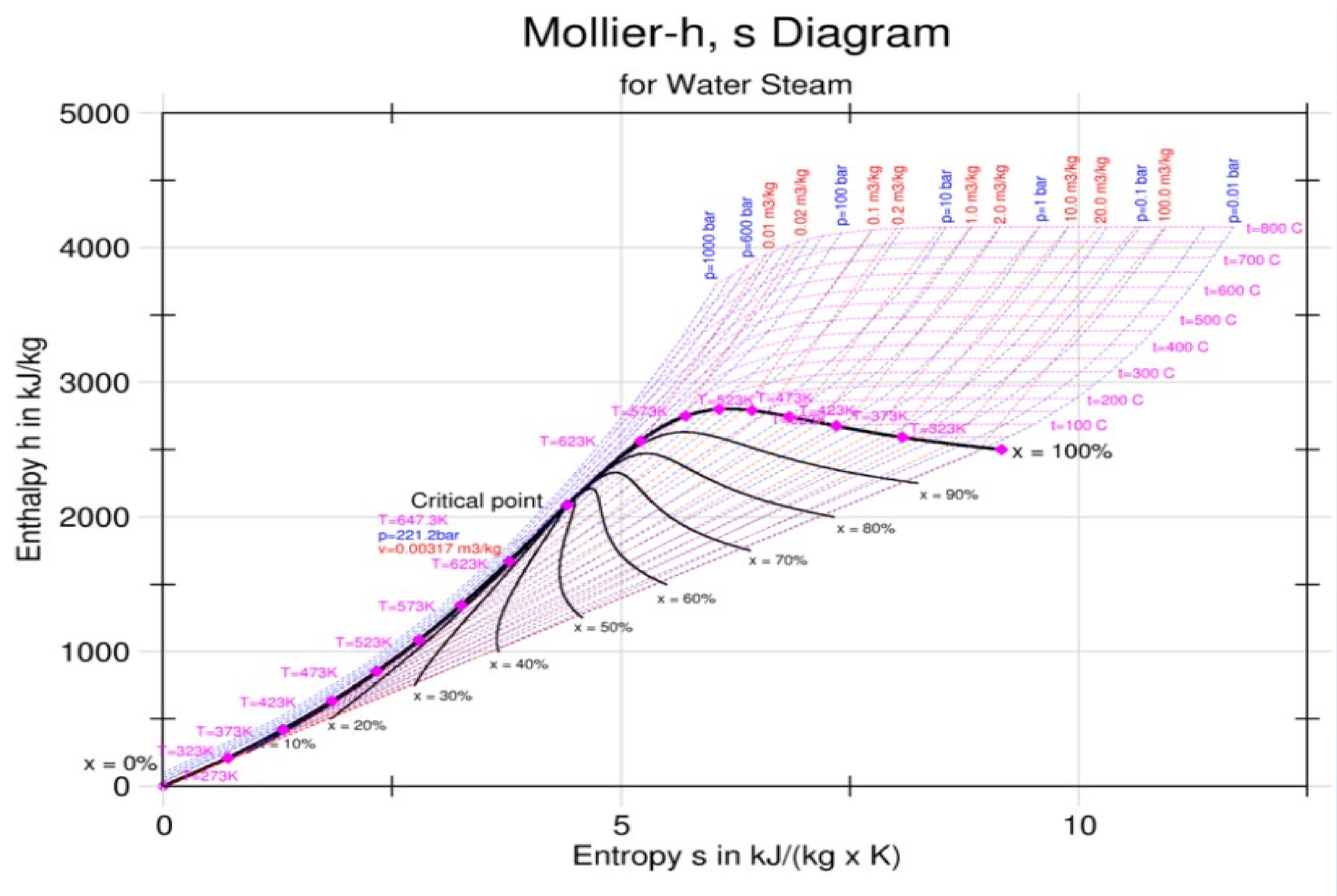

Process 3 – 4: Isentropic (reversible, adiabatic) expansion in the turbine

$$\dot{W}_T = \dot{m}\,(h_3 - h_4) \quad \text{positive}$$

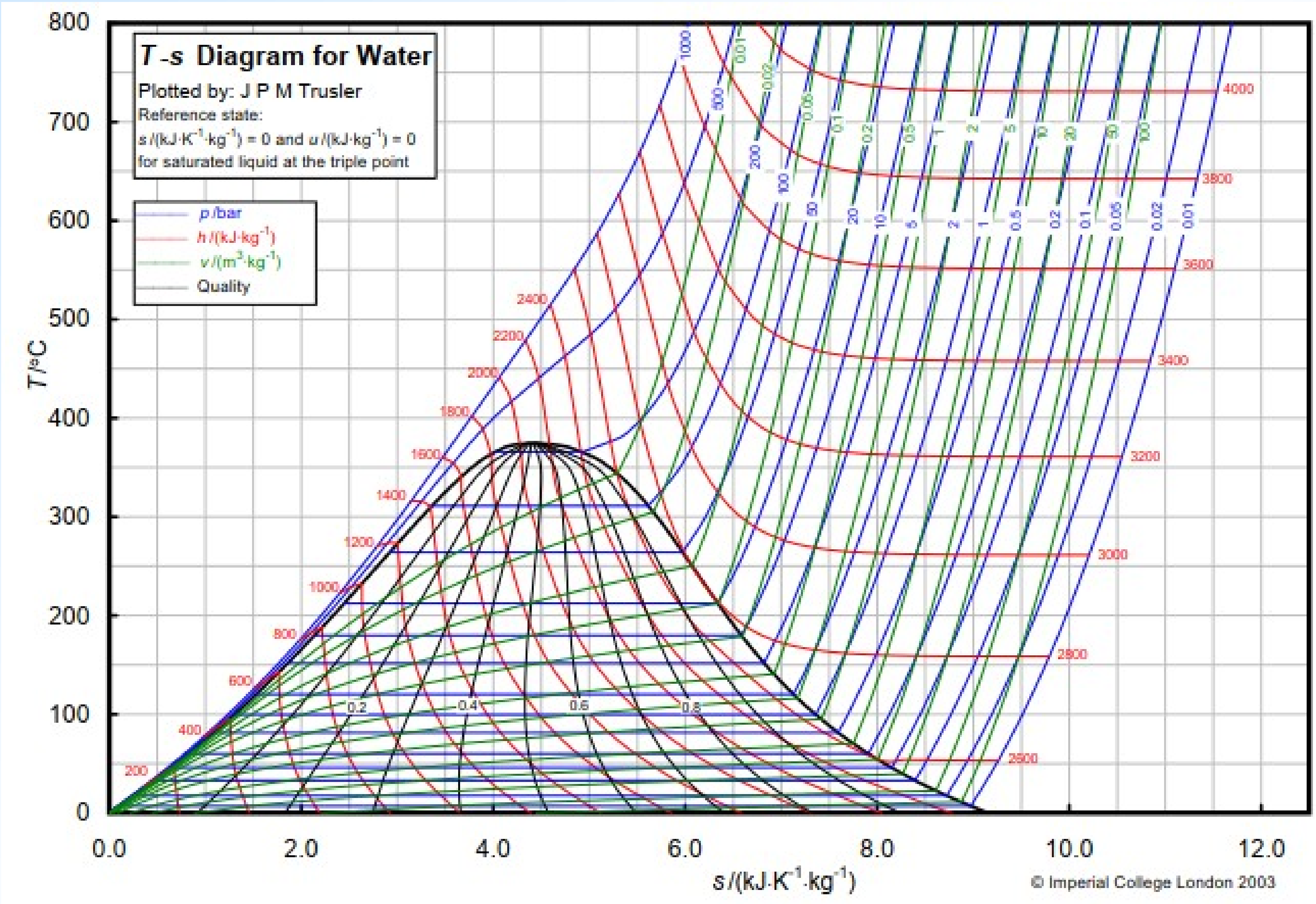



Process 1 – 2: Isentropic compression of liquid

$$\dot{W}_P = \dot{m}\,(h_1 - h_2) \quad \text{negative}$$



Mollier-h, s Diagram
for Water Steam
Enthalpy h in kJ/kg
Entropy s in kJ/(kg x K)
0
1000
2000
3000
4000
5000
5
10
Critical point
T=647.3K
p=221.2bar
v=0.00317 m3/kg
x = 0%
x = 100%
x = 10%
x = 20%
x = 30%
x = 40%
x = 50%
x = 60%
x = 70%
x = 80%
x = 90%
T=273K
T=323K
T=373K
T=423K
T=473K
T=523K
T=573K
T=623K
p=1000 bar
p=600 bar
0.01 m3/kg
0.02 m3/kg
p=100 bar
0.1 m3/kg
0.2 m3/kg
p=10 bar
1.0 m3/kg
2.0 m3/kg
p=1 bar
10.0 m3/kg
20.0 m3/kg
p=0.1 bar
100.0 m3/kg
p=0.01 bar
t=800 C
t=700 C
t=600 C
t=500 C
t=400 C
t=300 C
t=200 C
t=100 C


T-s Diagram for Water
Plotted by: J P M Trusler
Reference state:
s /(kJ·K⁻¹·kg⁻¹) = 0 and u /(kJ·kg⁻¹) = 0
for saturated liquid at the triple point
p /bar
h /(kJ·kg⁻¹)
v /(m³·kg⁻¹)
Quality
T/°C
s /(kJ·K⁻¹·kg⁻¹)
© Imperial College London 2003

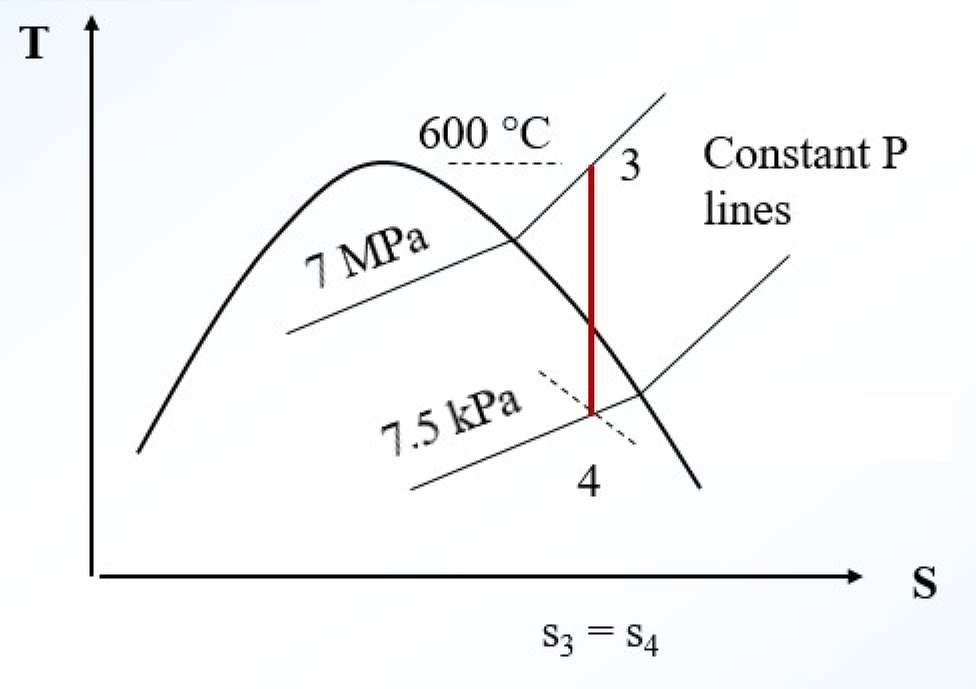
$$\dot{W}_{net} = \dot{W}_T + \dot{W}_P = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

$$\dot{W}_{net} = \dot{Q}_H + \dot{Q}_L = \dot{m}\left[(h_3 - h_2) - (h_4 - h_1)\right]$$

Thermal efficiency

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \left|\frac{\dot{Q}_L}{\dot{Q}_H}\right|$$

## Example




(a) Find the ideal work of expansion

(b) Find the pump work

$P_3$ = 7 MPa

$T_3$ = 600 °C

$P_4$ = 7.5 kPa

$s_3 = s_4$ isentropic expansion

(a) $\frac{\dot{W}}{\dot{m}} = \dot{w} = h_3 - h_4$

$P_3 = 7$ MPa, $T_3 = 600$ °C $\rightarrow$ $h_3 = 3650$ kJ/kg, $s_3 = 7.0894$ kJ/kg.K

$P_4 = 7.5$ kPa, $T_3 = 600$ °C $\rightarrow$

$h_f = 168.79$ kJ/kg $\quad s_f = 0.5764$ kJ/kg.K

$h_g = 2574.8$ kJ/kg $\quad s_g = 8.2515$ kJ/kg.K

$h_{fg} = 2406.0$ kJ/kg $\quad s_{fg} = 7.6750$ kJ/kg.K

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{7.0894 - 0.5764}{7.6750} = 0.849$$

$$h_4 = h_f + x_4\, h_{fg} = 168.79 + (0.849)\,(2406) = 2211 \text{ kJ/kg}$$

$$\frac{\dot{W}_T}{\dot{m}} = \dot{w}_T = h_3 - h_4 = 3650 - 2211 = 1439 \text{ kJ/kg}$$

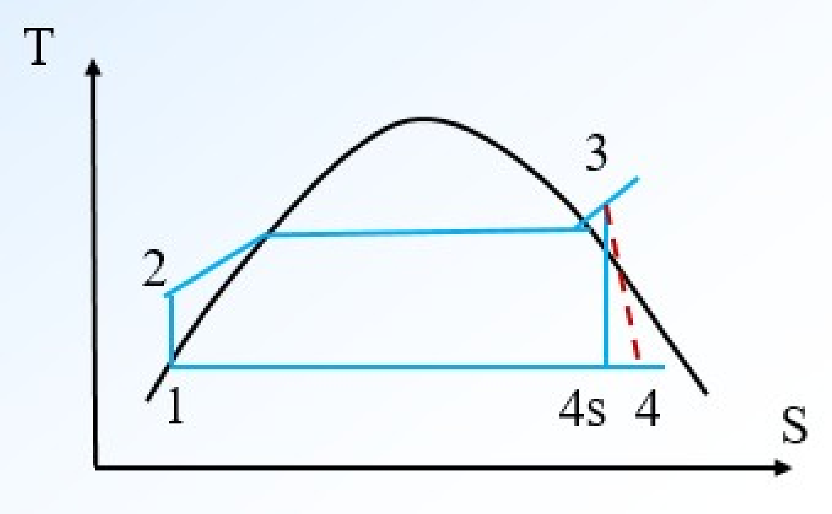
(b) $$\frac{\dot{W}_P}{\dot{m}} = \dot{w}_P = h_1 - h_2 = -\int_1^2 v\, dP \cong v_f \left(P_1 - P_2\right)$$

$$= (0.001)\ (-\ 7\ 106 + 7.5\ 103) = -\ 7\ 103\ \text{N.m/kg}$$

$$= -\ 7\ \text{kJ/kg}$$

Note that $w_P$ is only 0.5 % of $w_T$.

## 6.3 Isentropic Efficiency of a Turbine

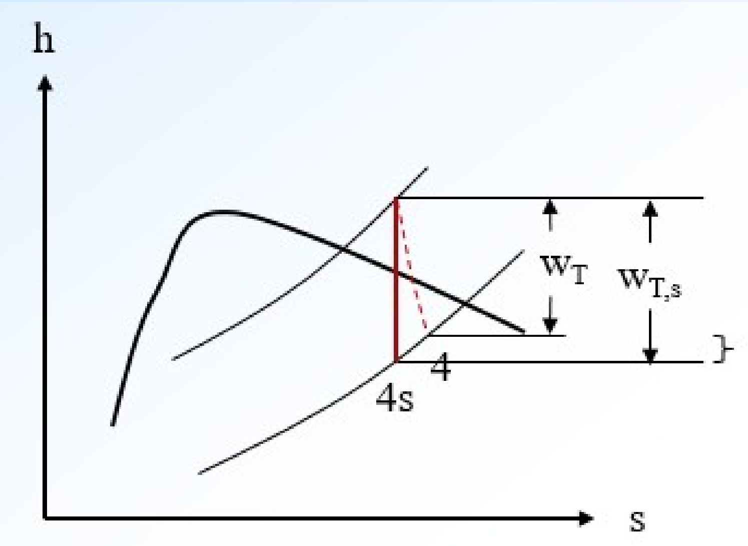



The ideal (isentropic – reversible and adiabatic) expansion process is: 3 – 4s. The real (irreversible) process is: 3 – 4.

The real process might be approximated to be adiabatic, but certainly not reversible. There is always friction.

Define isentropic turbine efficiency:

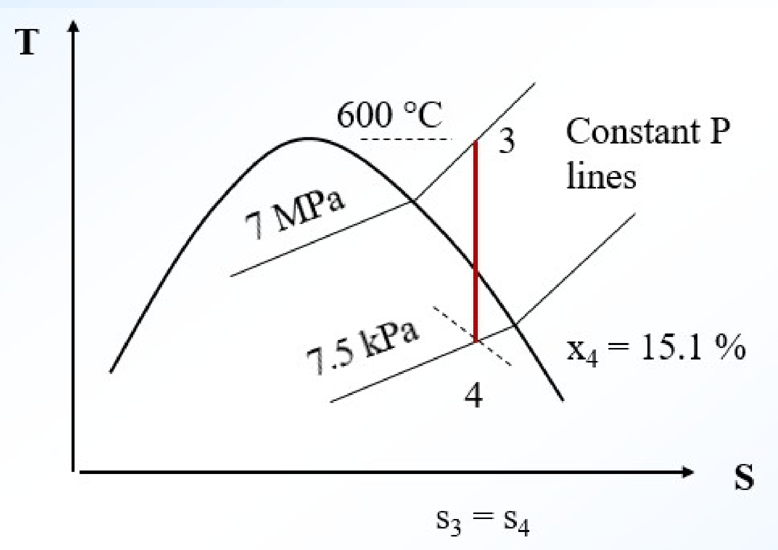
$$\eta_T = \frac{\text{Work from actual (real) expansion}}{\text{Work from isentropic expansion}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$




On Molier Chart

Lost work or
Reducion in available energy

Energy is really NOT lost. What happens to it?




See the previous example problem.

$$\frac{\dot{W}_{T,s}}{\dot{m}} = 1439 \text{ kJ/kg} \qquad \text{If } \eta_T = 0.85$$

$$\frac{\dot{W}_T}{\dot{m}} = \eta_T \frac{\dot{W}_{T,s}}{\dot{m}} = (0.85)(1439) = 1223 \text{ kJ/kg}$$

$$h_4 = h_3 - \frac{\dot{W}_T}{\dot{m}} = 3650 - 1223 = 2427 \text{ kJ/kg}$$

Actual enthalpy of the steam at State 4

$$x_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{2427 - 167.79}{2406} = 0.939$$

Higher quality

Increase in available energy

Or «Lost Work»

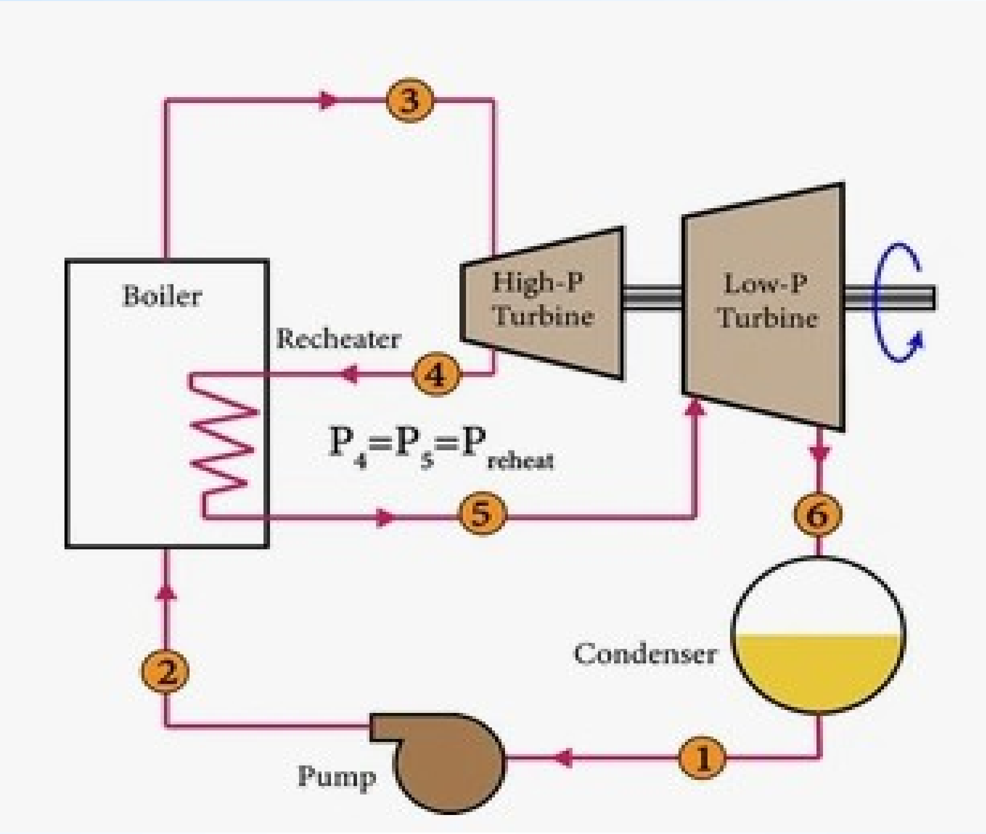
$$w_{T,s} - w_T = h_4 - h_{4,s} = 2427 - 2211 = 216 \text{ kJ/kg}$$

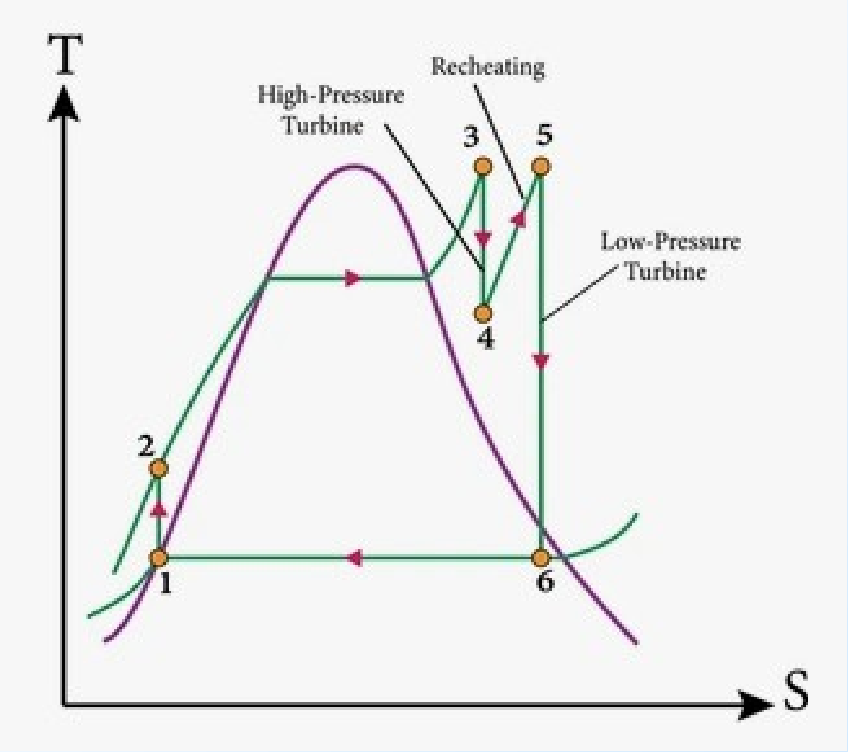
Same value

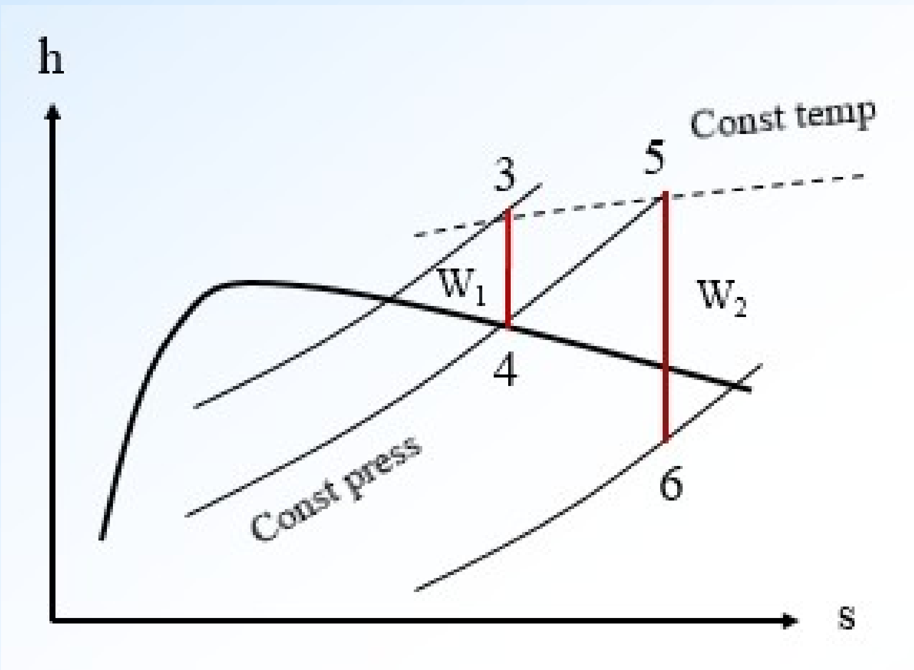
Find ($T_4$ Δs) as if heat is lost at $T_4$ from state 4 to state 4s

$$T_4 \,\Delta s = (40.29 + 273)\,(7.783 - 7.0894) = 216 \text{ kJ/kg}$$

## 6.4.1 Rankine Cycle with Reheat






$$\dot{W}_1 = \dot{m}\,(h_3 - h_4)$$

$$\dot{W}_2 = \dot{m}\,(h_5 - h_6)$$

$\dot{W}_P = \dot{m}\,(h_1 - h_2)$ □ $v_f\,(P_1 - P_2)$

negative

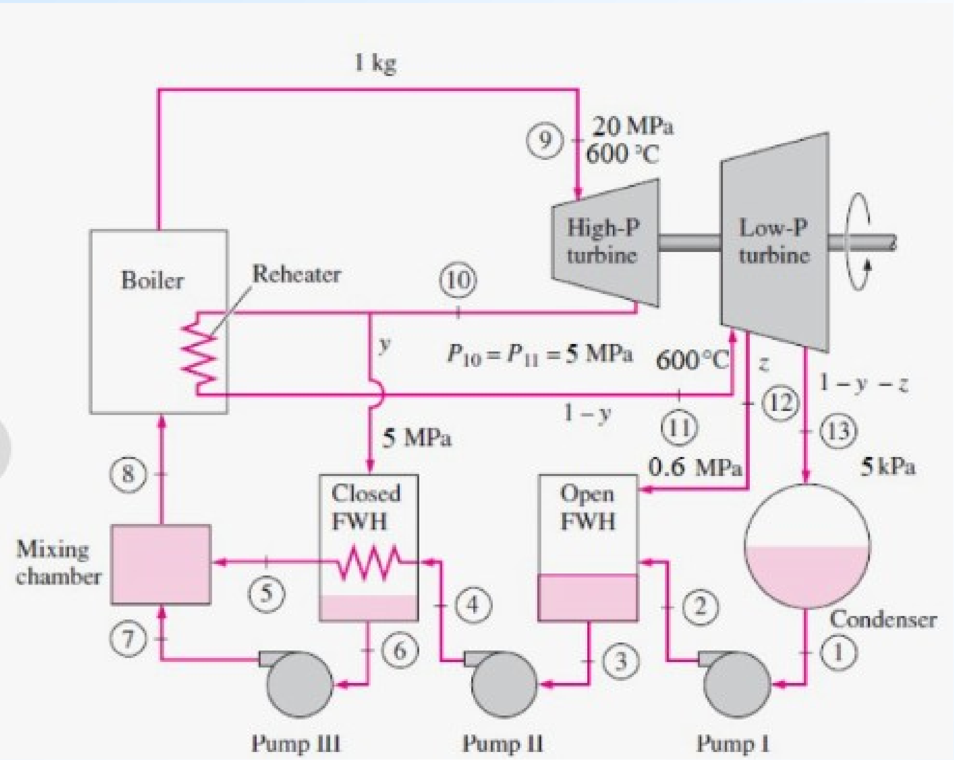
$$\dot{W}_{net} = \dot{W}_1 + \dot{W}_2 + \dot{W}_P$$

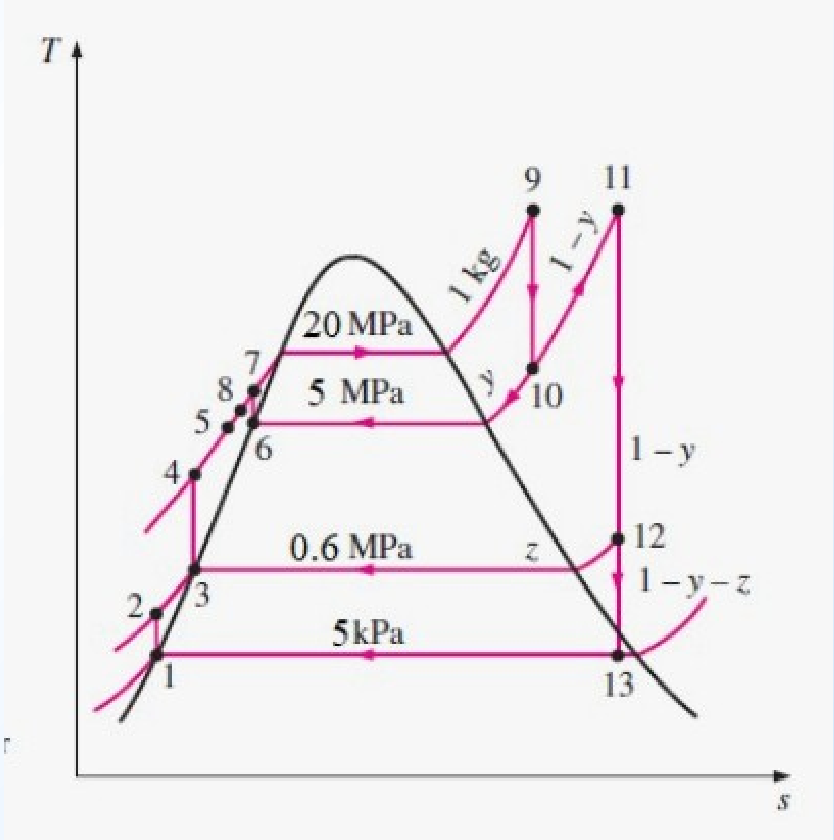
$$\dot{Q}_H = \dot{m}\,(h_3 - h_2) + \dot{m}\,(h_5 - h_4)$$

$$\eta_T = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

Thermal efficiency of Rankine cycle with reheat

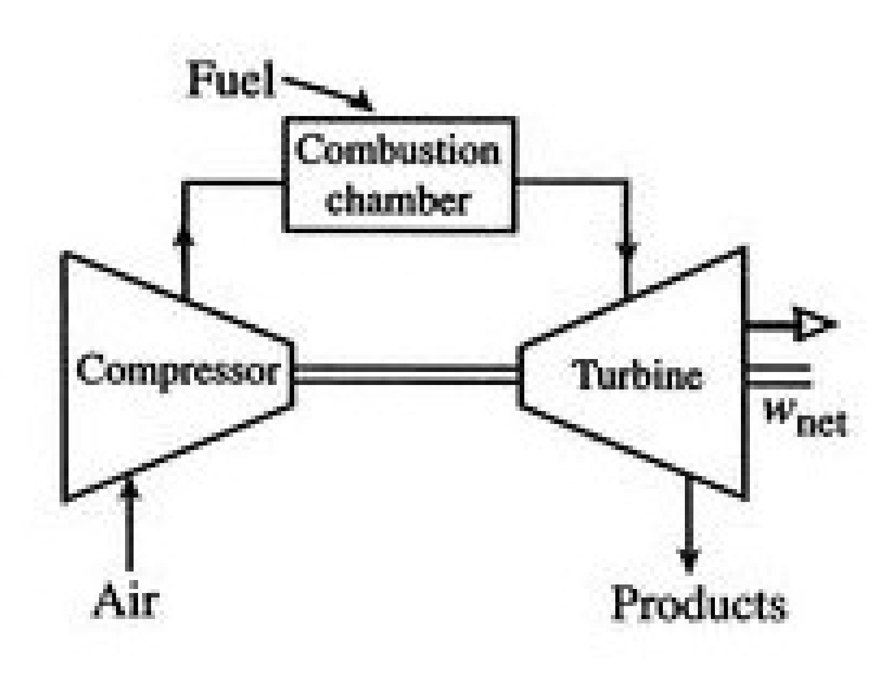
## 6.4.2 Rankine Cycle with Reheat and Regeneration

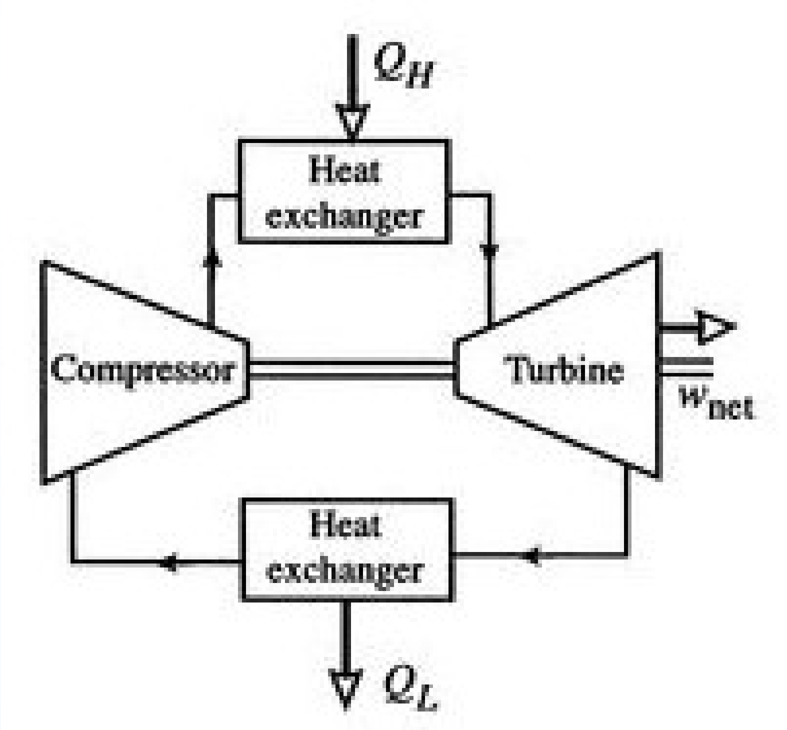


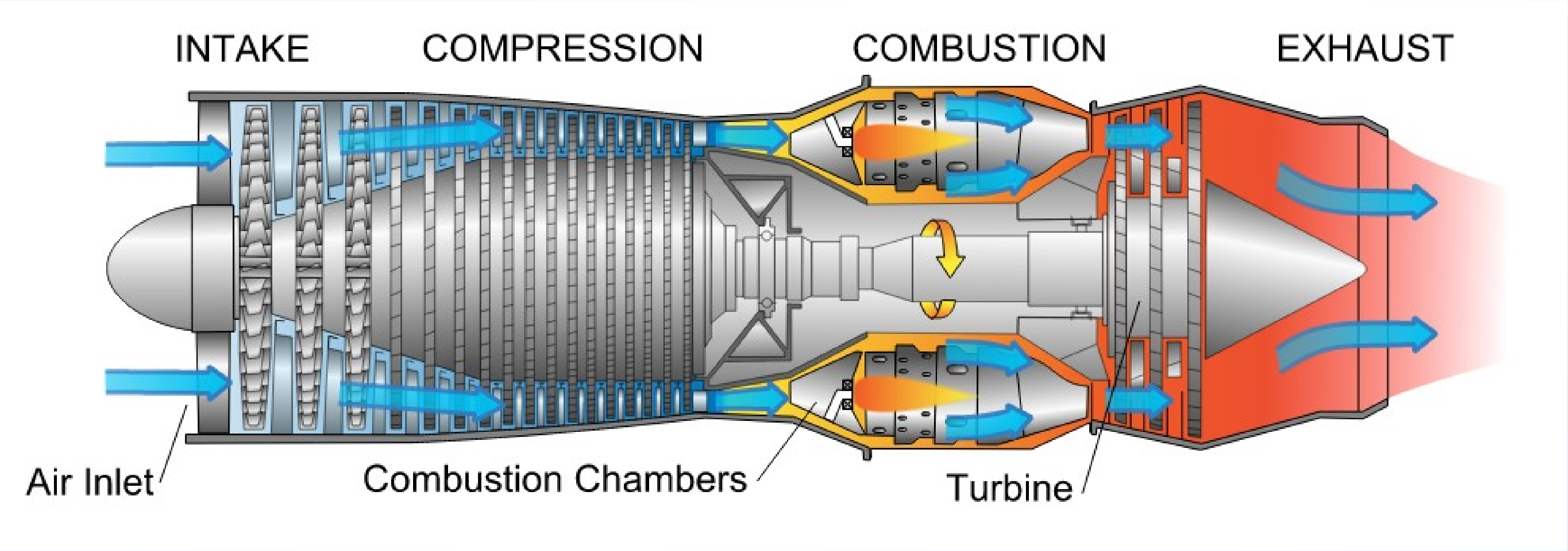



## 6.5 Gas Cycles

### Gas turbine engines - Brayton cycle (Joule cycle)

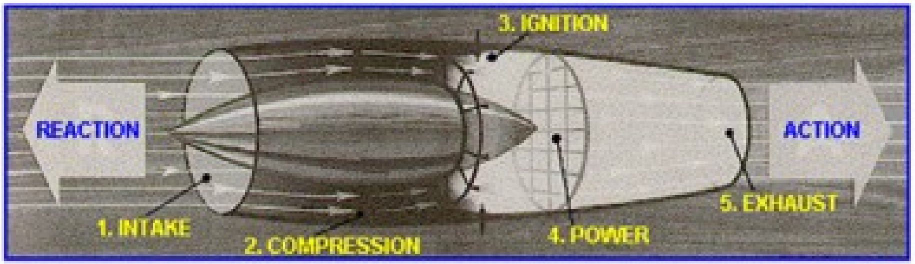



INTAKE
COMPRESSION
COMBUSTION
EXHAUST
Air Inlet
Combustion Chambers
Turbine

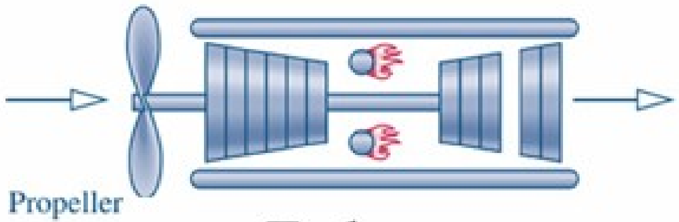
# Turbojet

# Basic Types of Jet Engines



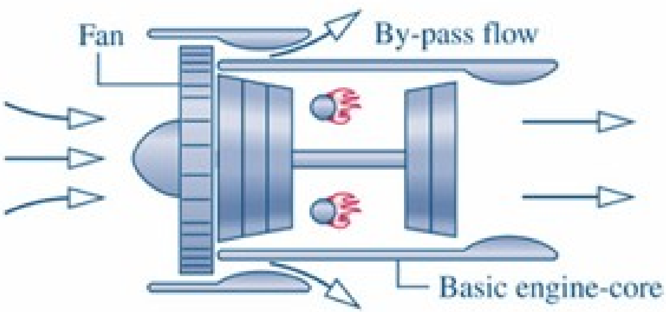

## Ramjet

High Speed, Supersonic Propulsion, Passive Compression/Expansion



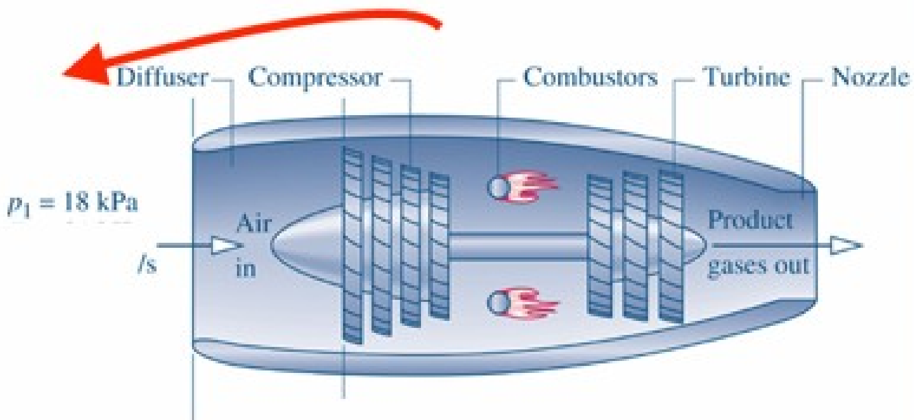

## Turboprop

Low to Intermediate Subsonic
Small Commuter Planes




## Turbofan

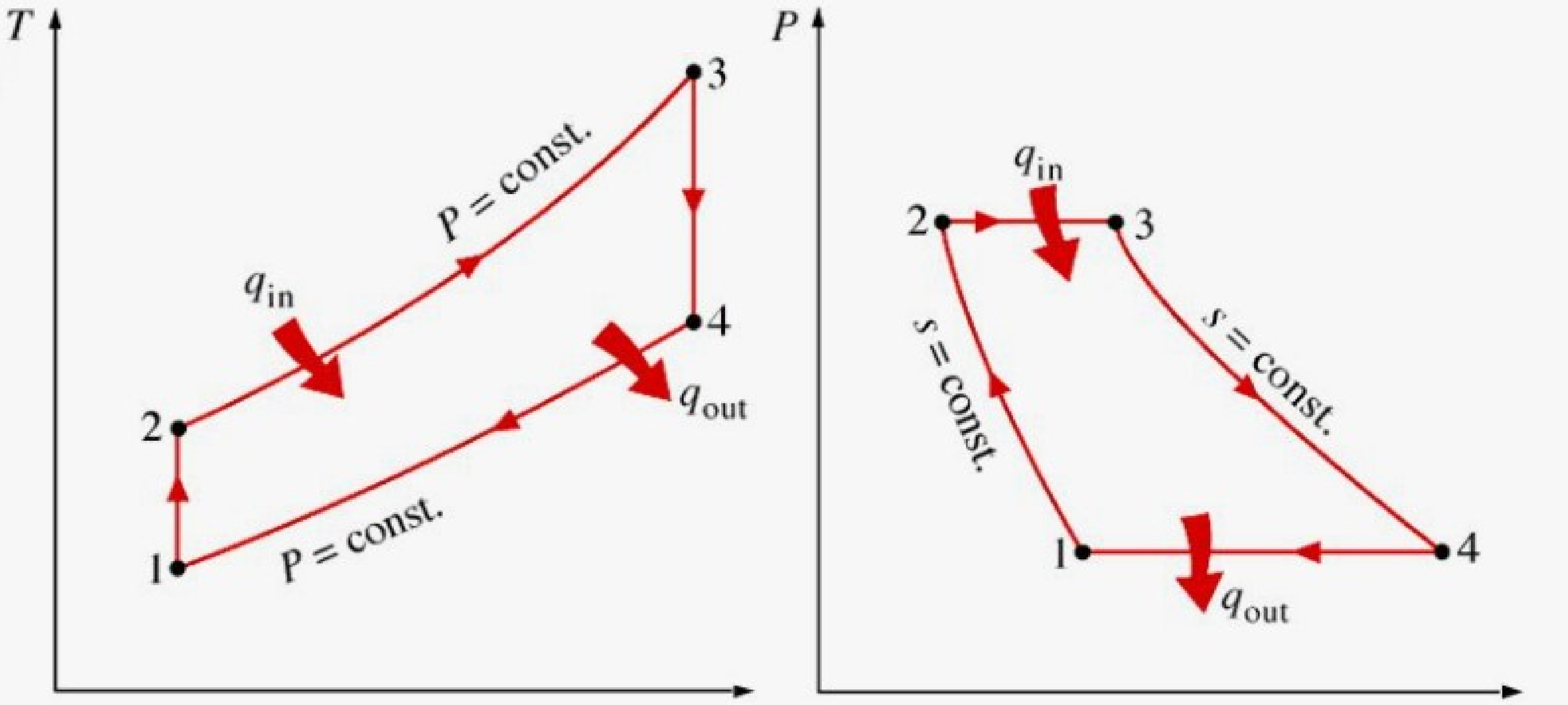
Larger Passenger Airliners
Intermediate Speeds, Subsonic Operation




## Turbojet

High Speeds Supersonic or Subsonic Operation

# Ideal Brayton Cycle




(a) *T*-*s* diagram

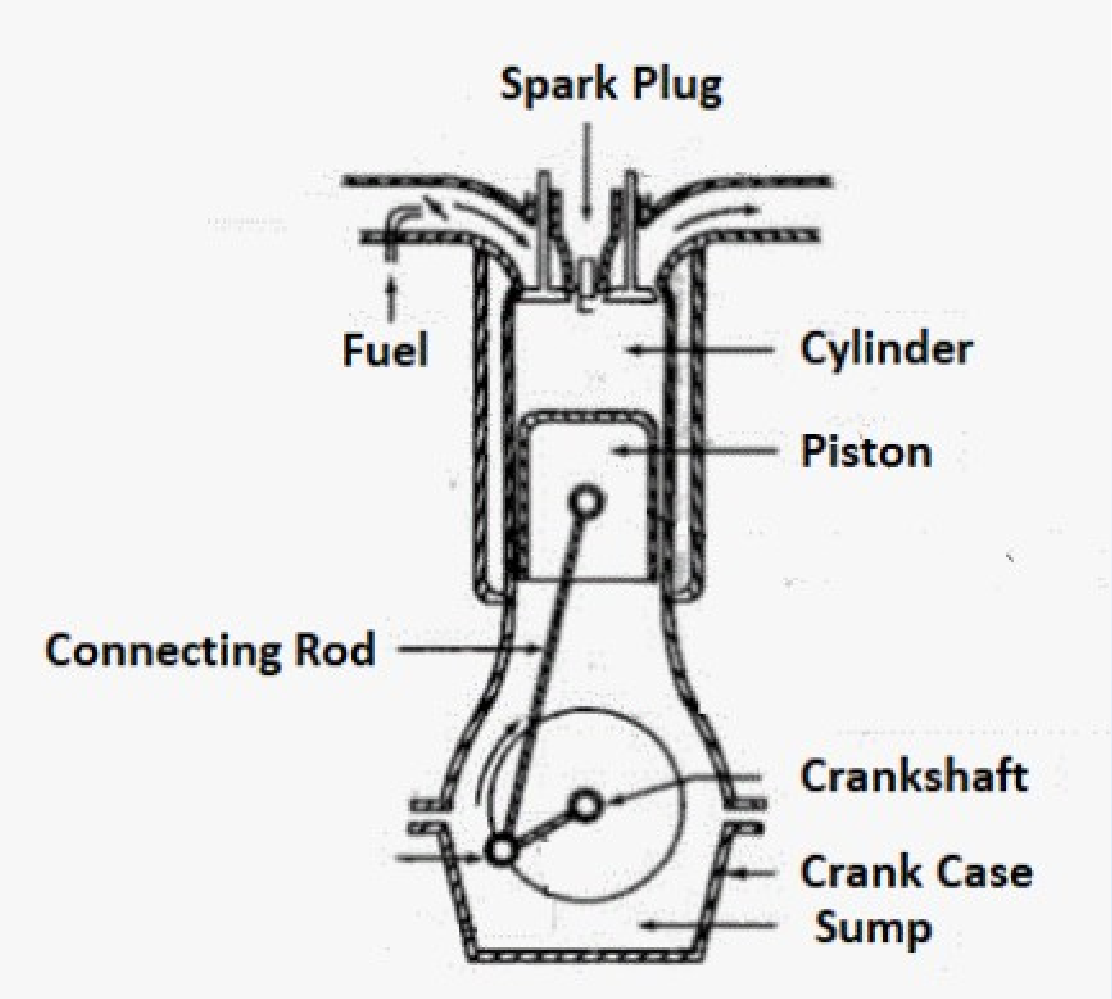
(b) *P*- υdiagram

**George Bailey Brayton**

American Mechanical Engineer

1830 - 1892

## Internal Combustion Engines – Otto Cycle (Gasoline Engine)

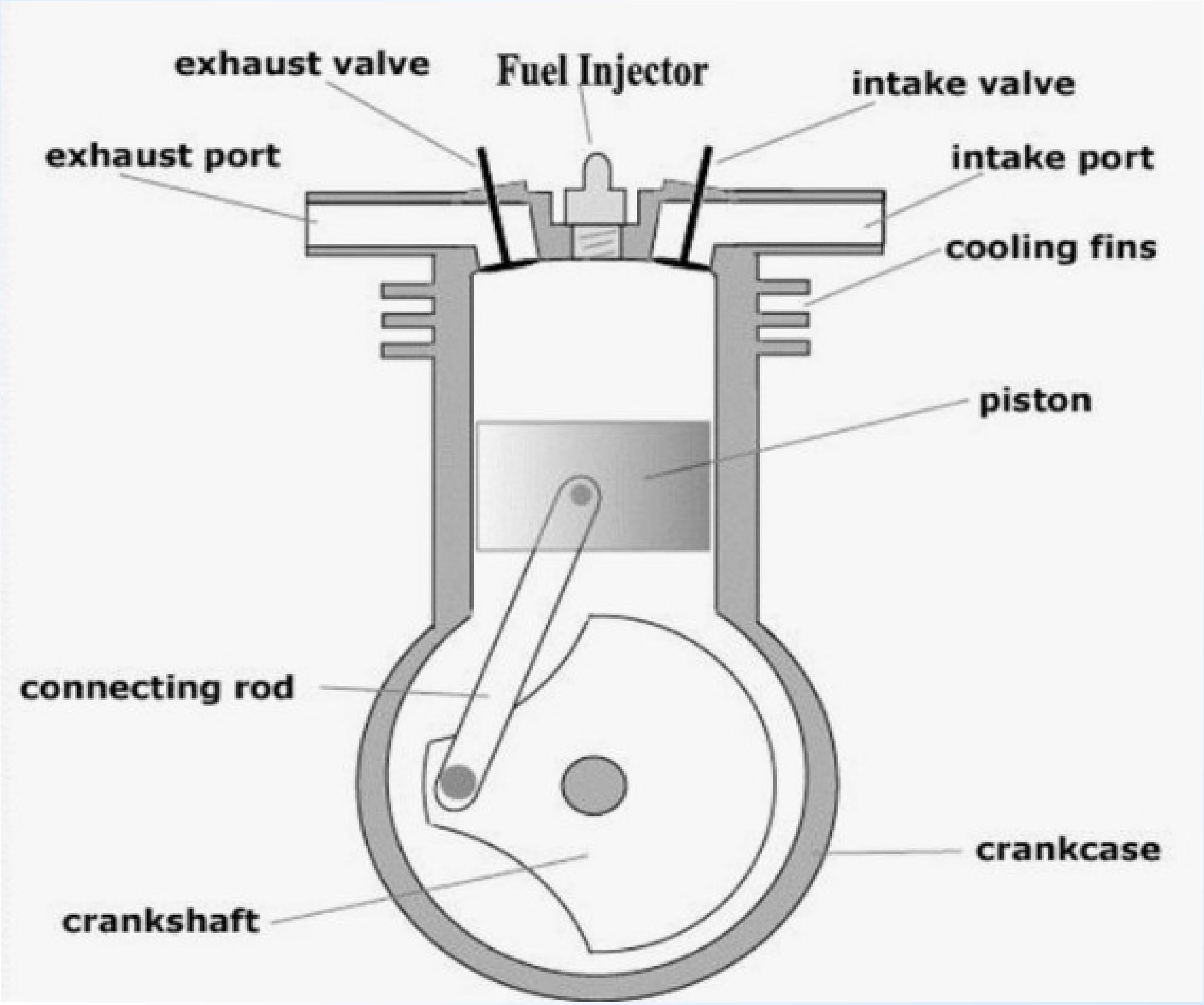


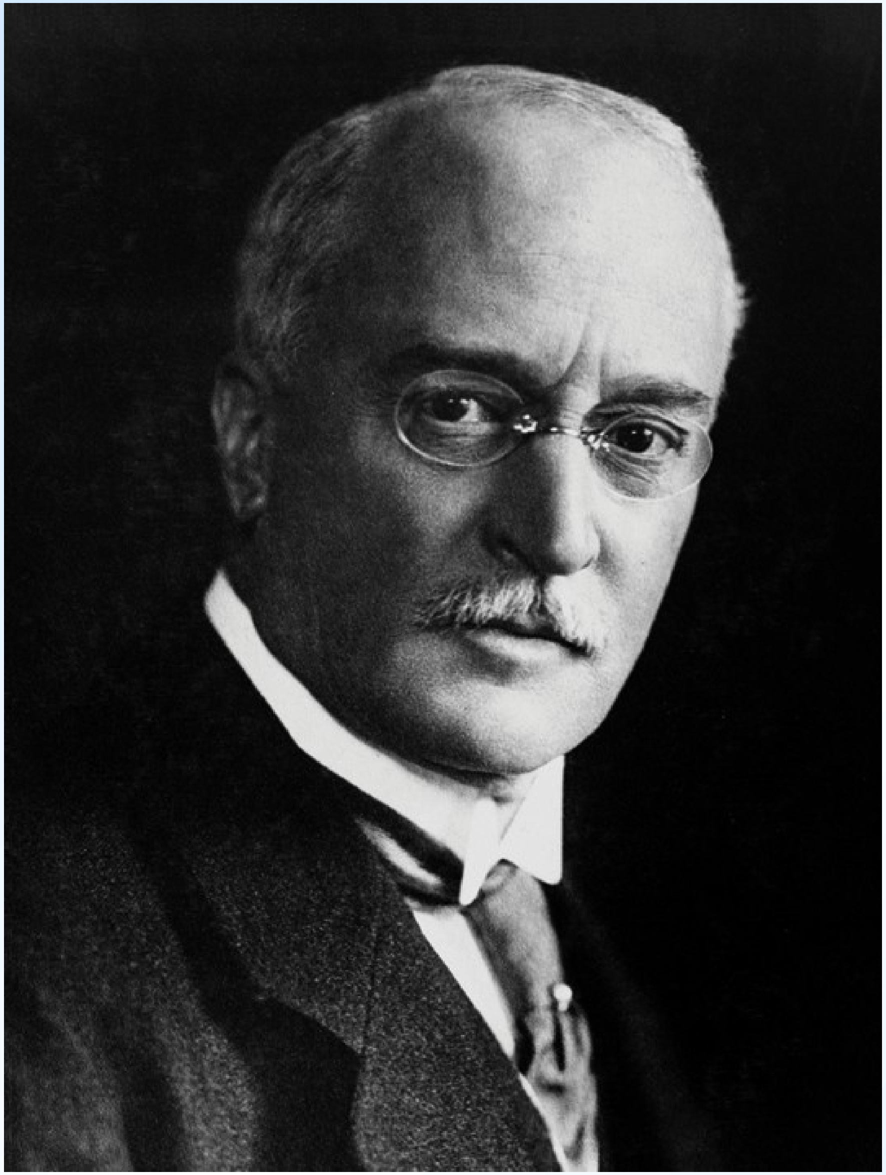
**Nicolaus August Otto**

German Engineer

1832 - 1891

## Internal Combustion Engines – Diesel Engine





**Rudolf Diesel**

German Engineer

1858 - 1913

**Ideal Gas** $P\,V = m\,R\,T$ or $P\,v = R\,T$

$P\,V = n\,R_0\,T$ $R_0$ = Universal gas constant

= 8.314 kJ/kmol.K

For an ideal gas, u, h, $c_p$,and $c_v$ are functions of temperature, but not pressure.

For a reversible and isothermal process: $P\,v = \text{Constant}$

For a reversible and adiabatic (isentropic) process: $P\,v^{\gamma} = \text{Constant}$

$\gamma$ is the isentropic index of expansion or compression

For an isentropic process:

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma/\gamma-1}$$ isentropic pressure ratio

$$\frac{v_1}{v_2} = \left(\frac{T_1}{T_2}\right)^{1/1-\gamma}$$ isentropic compression ratio

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\gamma-1/\gamma}$$

Definition of specific heat for an ideal gas: $R = c_p - c_v$ and $\gamma = \frac{c_p}{c_v}$

$$c_v = \frac{R}{\gamma - 1}$$ and $$c_p = \frac{\gamma}{\gamma - 1}$$

Assumptions for an ideal gas cycle:

- An ideal gas with the above relaltions is the only working fluid
- $c_p$ and $c_v$ are constants
- The working fluid is non-viscous (no firiction)

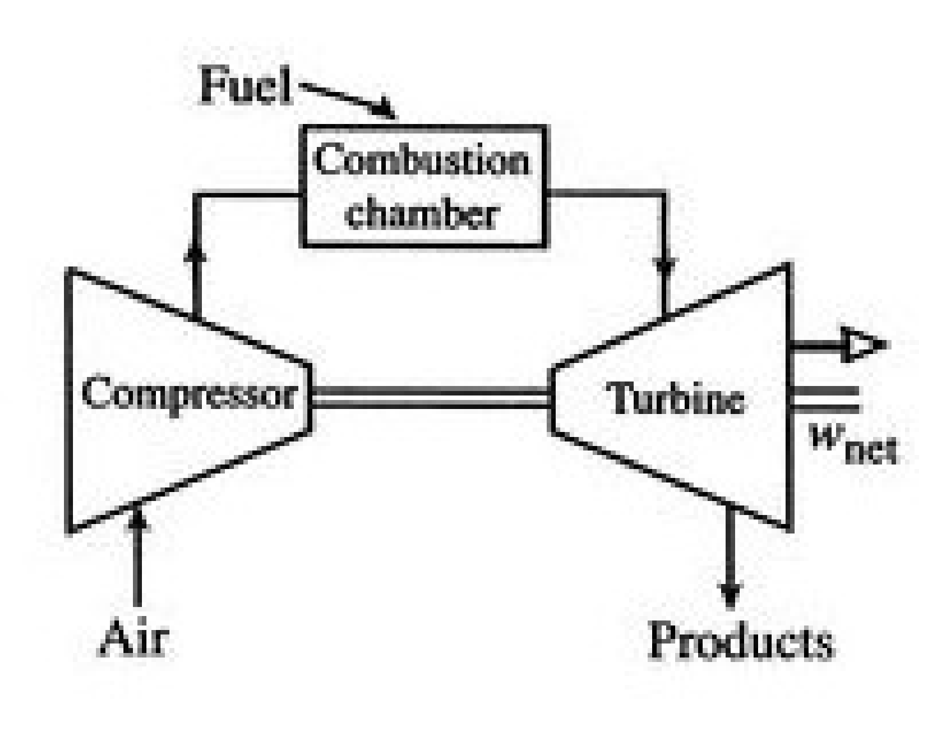
Real gas cycles are reasonable close to ideal cycles.

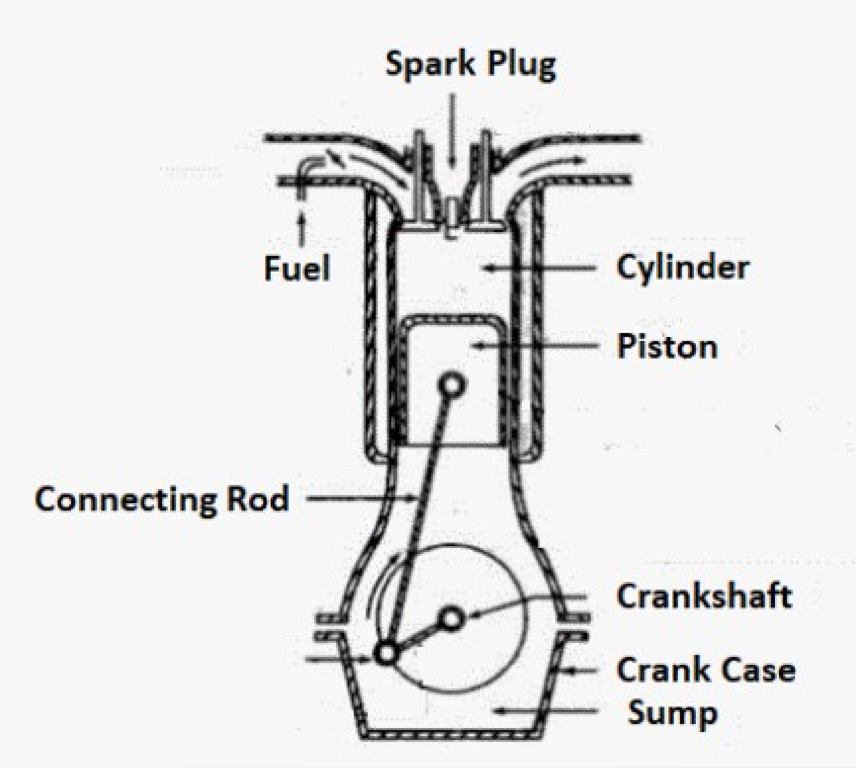
Only the efficiency of a real cycle is slightly less.

Distinguish between close cycle and open cycle.

In an open cycle (such as internal combustion engines and gas turbine-compressor combinations), the fluid is thrown out at some point and fresh fluid is taken in.

# Open Cycles


Fuel
Combustion chamber
Compressor
Turbine
$W_{net}$
Air
Products


Spark Plug
Fuel
Cylinder
Piston
Connecting Rod
Crankshaft
Crank Case
Sump

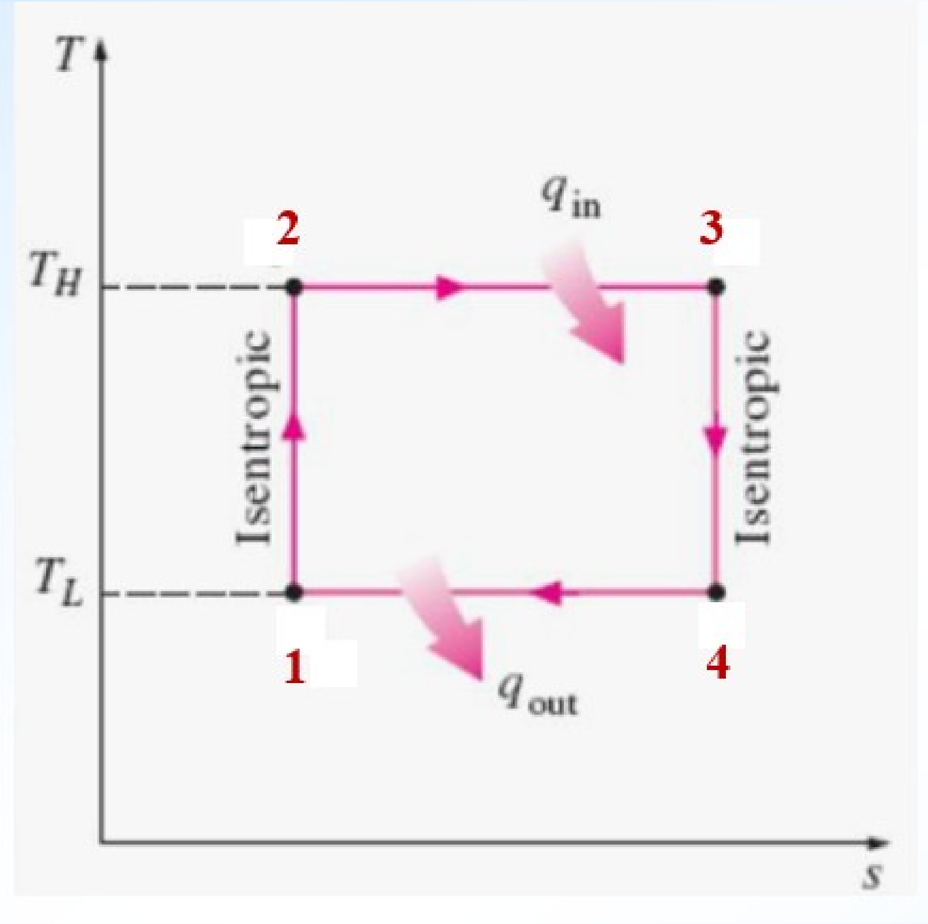
## Air Standard Cycle

In order to carry out a simplified analysis, the following are assumed:

- Air with $c_p$ = 1.0035 kJ/kg.K and $\gamma$ = 1.4 is the only working fluid that behaves like an ideal gas.
- It is called standard because the properties such as $c_p$ and $\gamma$ values for air are at standard conditions of T = 25 °C and P = 100 kPa.
- All the processes that make up the cycle are internally reversible.
- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.
- The combustion process is replaced by a heat-addition process from an external source.

The actual open-cycle efficiency may differ a great deal from an air-standard cycle.

## 6.5.1 Carnot Gas Cycle




1 – 2 Isentropic compression

pressure and temperatre increases

2 – 3 Isothermal heat addition

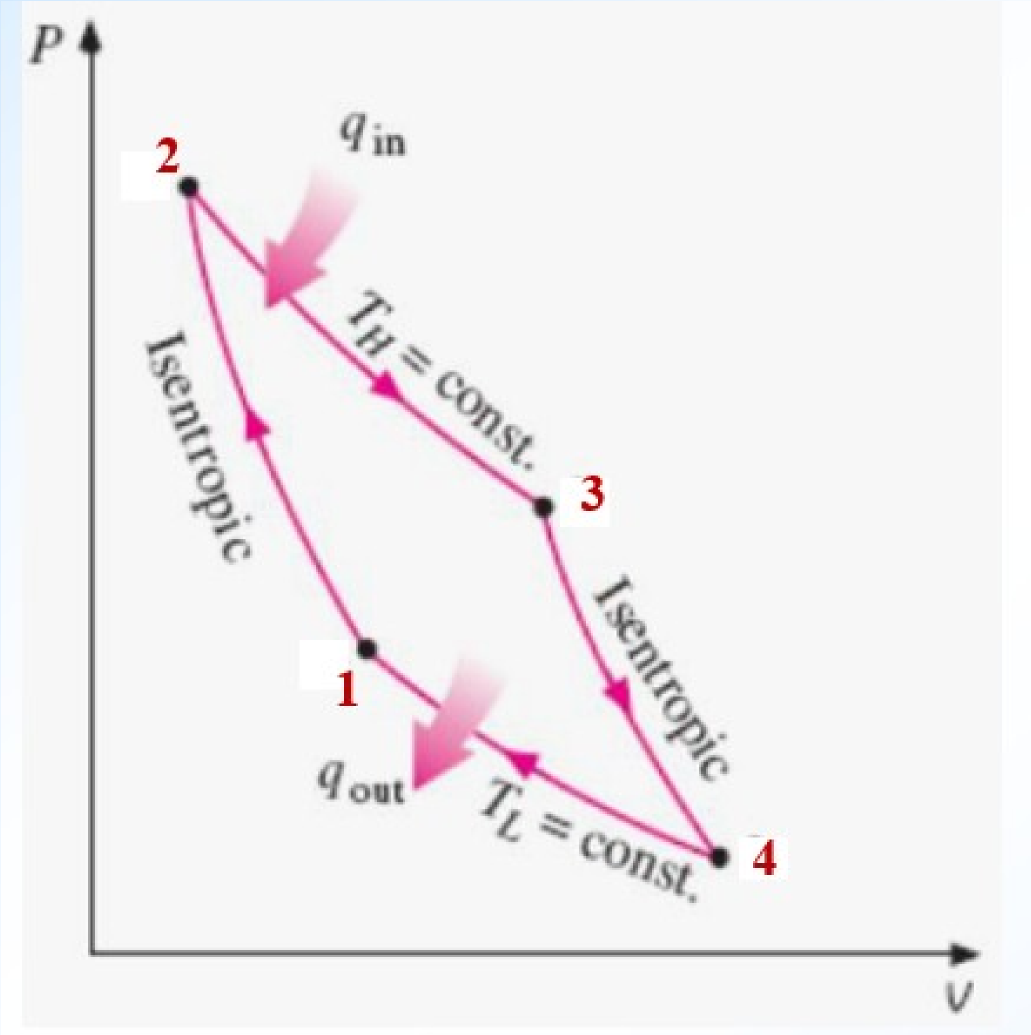
press. decrases, vol. increases

3 – 4 Isentropic expansion

temp and press. decrases, vol. incr.

4 – 1 Isothermal heat removal

Note that all these reversible (ideal) processes are in the gas region.




$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

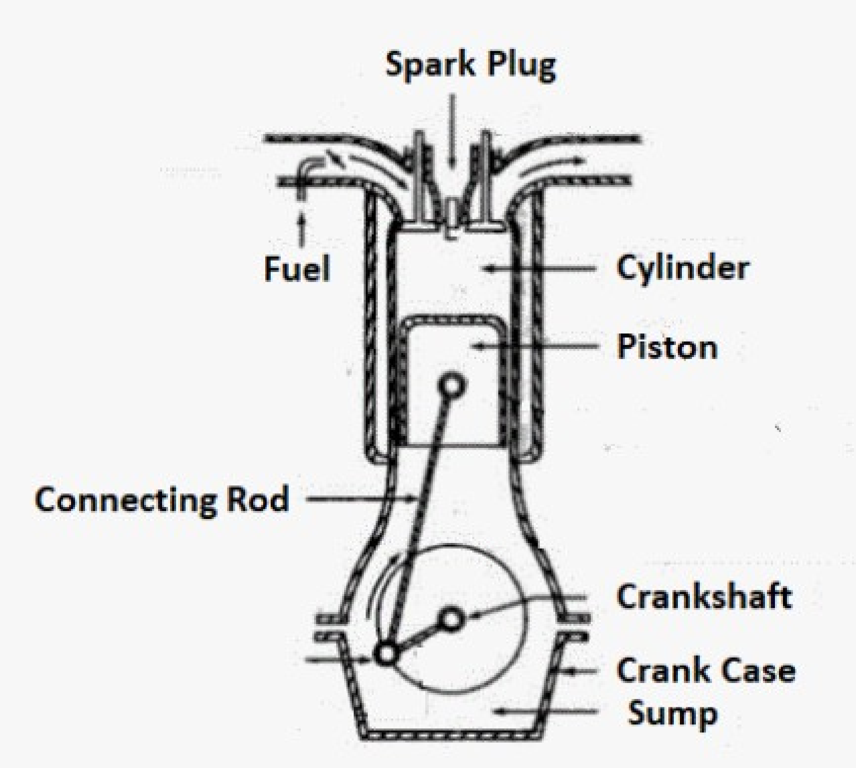
$$= 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1} = 1 - \left(\frac{v_3}{v_4}\right)^{\gamma-1}$$

$$\frac{v_1}{v_2} = \frac{v_4}{v_3} = r$$

Isentropic compression ratio

Obviously, neither the IC nor the gas-turbine cycles can work like this. At least, the addition and removal of heat cannot be isothermal.

## 6.5.2 Air Standard Otto Cycle

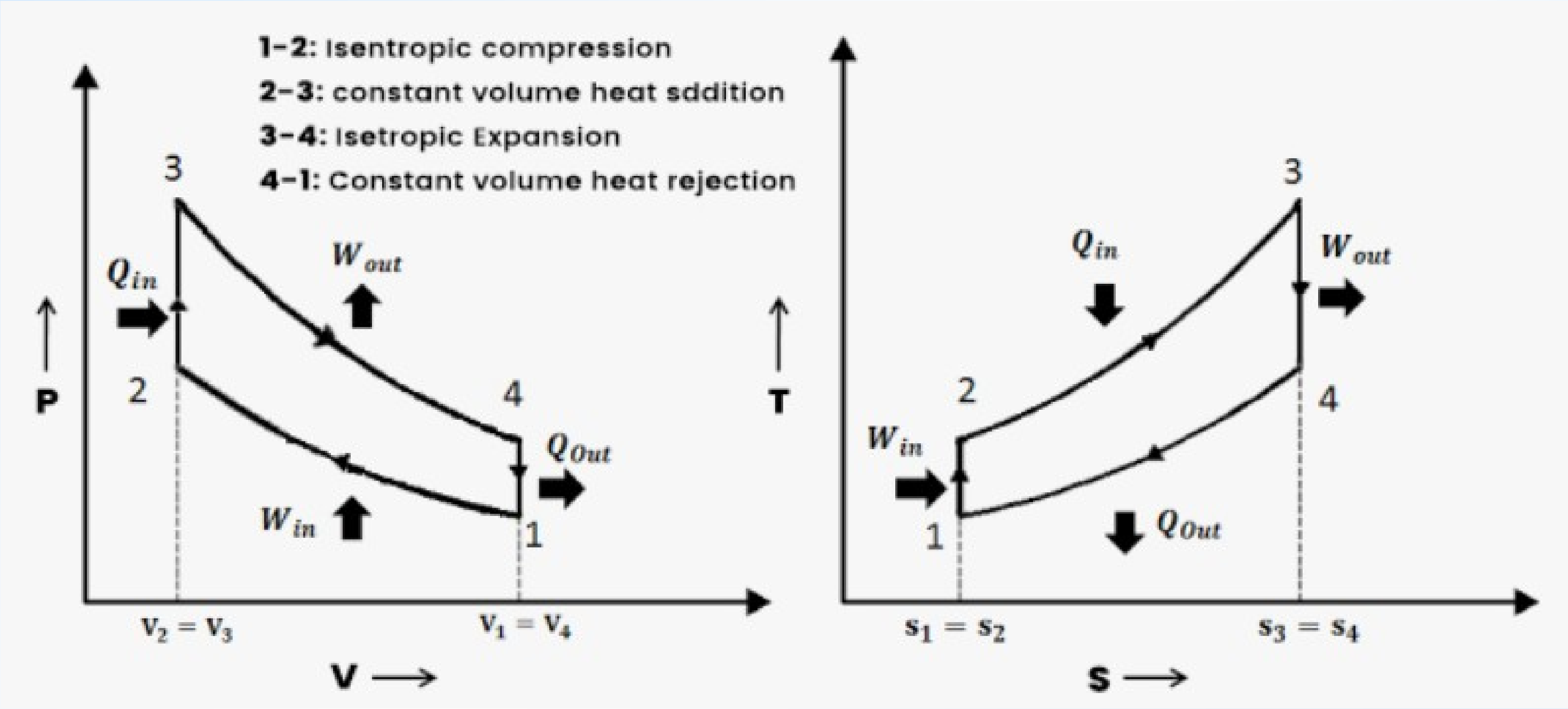



Simulate the operation of the engine with a closed cycle where air is the working fluid.

Operate under the same pressure and temperature states, and qualitatively determine the change in other parameters.

| Actual Operation | Ideal Operation |
|---|---|
| Air + Fuel mixture gets in during the intake (downward motion) of the piston | Pure air at standard state ($c_p$ = 1.0035 kJ/kg.K and $\gamma$ = 1.4) exists at State 1 |
| Piston moves up, compressing the air fuel mixture | Isentropic compression of air to State 2 |
| Spark ignites the air fuel mixture and combustion occurs | Heat is added externaly at constant volume to State 3 (not constant temp.) |
| Piston moves down in tge power stroke expanding the gases of combustion | Isentropic expansion of air to State 4 |
| Piston moves up and throws outthe exhaust gases | Heat is removed at constant volume to State 1 (not at constant temp.) |

# Ideal Otto Cycle




$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{\sum Q}{Q_{in}} = 1 - r^{1-\gamma}$$

$$ {}_2Q_3 = U_3 - U_2 = m\,c_v\,(T_3 - T_2) = Q_{in} $$

$$ {}_4Q_1 = U_1 - U_4 = -\,m\,c_v\,(T_4 - T_1) $$

$$ W_{net} = {}_2Q_3 - {}_4Q_1 = m\,c_v\left[(T_3 - T_2) - (T_4 - T_1)\right] $$

$$ \eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} $$

Define compression ratio $r = \dfrac{V_1}{V_2} = \dfrac{V_4}{V_3}$

Isentropic process in an ideal gas

$$ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} $$

$$ \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} $$

Therefore

$$ \frac{T_2}{T_1} = \frac{T_3}{T_4} $$

$$ \eta_{th} = 1 - \frac{1}{r^{\gamma-1}} = 1 - r^{1-\gamma} $$

Different from Carnot efficiency

## Example

An engine operates on air-standard Otto cycle. The conditions at the start of compression are $T_1$ = 27 °C = 300 K and $P_1$ = 100 kPa. Heat added is 1840 kJ/kg. The compression ratio is 8. Find the temparture and pressure at each state of the cycle, thermal efficiency $\eta_{th}$, and the mean effective pressure.

Process 1 -> 2: Isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} \quad => \quad T_2 = T_1\, r^{\gamma-1} = (300)\,(8)^{1.4-1} = 689.2 \text{ K}$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma} \quad => \quad P_2 = P_1\, r^{\gamma} = (100)\,(8)^{1.4} = 1837.9 \text{ kPa}$$

Process 2 -> 3: Constant volume heat addition

$$\frac{Q_{in}}{m} = u_3 - u_2 = c_v \left(T_3 - T_2\right) = 1840 \text{ kJ/kg}$$

$$T_3 = \frac{Q_{in}/m}{c_v} + T_2 = \frac{1840}{0.7176} + 689.2 = 3253.5 \text{ K}$$

$$P_3 = P_2 \frac{T_3}{T_2} + T_2 = (1837.9) \frac{3253.5}{689.2} = 8676.1 \text{ kPa}$$

Process 3 -> 4: Isentropic expansion

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma\text{-}1} = r^{1-\gamma} \quad => \quad T_4 = T_3\, r^{1-\gamma} = (3253.5)\,(8)^{1\text{-}1.4} = 1416.2 \text{ K}$$

$$\frac{P_4}{P_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} = r^{-\gamma} \quad => \quad P_4 = P_1\, r^{-\gamma} = (8676.1)\,(8)^{-1.4} = 472.1 \text{ kPa}$$

Thermal efficiency $\eta_{th} = 1 - r^{1-\gamma} = 1 - 8^{0.4} = 0.565$ 56.5 % quite high

Mean Effective Pressure, mep (used in conjuction with reciprocating engines)

It is defined as the pressure that, if acted on the piston during the entire power stroke, would do an an amount of work equal to that actually done on the piston.The work done for one cycle is found by multiplying this pressure by the area of the piston (minus the area of the rod on the crank end) and by the stroke.

$$v_1 = \frac{R\,T_1}{P_1} = \frac{(0.287)\,(300)}{100} = 0.861\ m^3/kg$$

$$v_2 = \frac{v_1}{r} = \frac{0.861}{8} = 0.1076\ m^3/kg$$

$$\frac{W_{net}}{m} = \eta_{th}\,\frac{Q_{in}}{m} = (0.565)\,(1840) = 1039.6\ kJ/kg \quad \text{per cycle}$$

mep $$P_m = \frac{W_{net}}{\text{Displacement volume}} = \frac{W_{net}/m}{v_1 - v_2} = \frac{1039.6}{0.861 - 0.1076} = 1379.9 \text{ kPa}$$

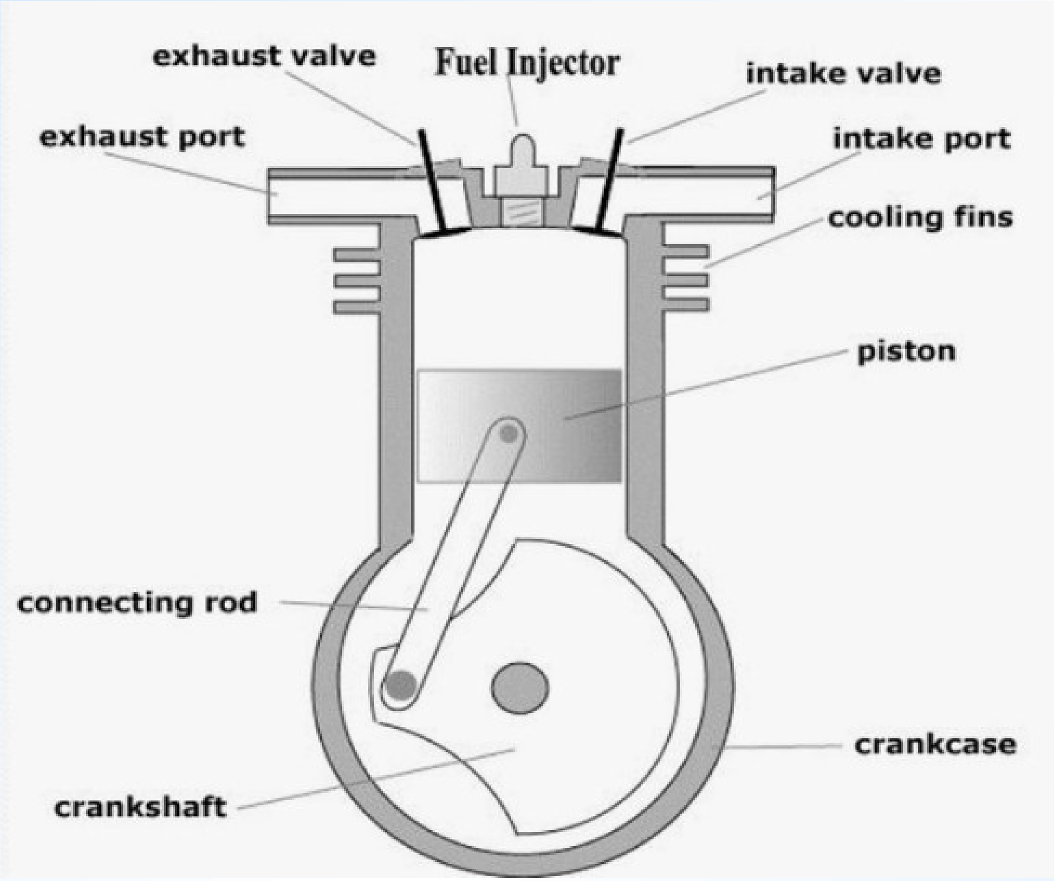
$P_m$ is a useful measure in comparing engines operating on different cyles. The greater the $P_m$, the smaller may be the engine for a given work output.

For Carnot cycle, $P_m$ is quite low, seldom exceeding 138 kPa.

The maximum temperature and pressure are quite high. So is $\eta_{th}$. Note that the variation in cv is neglected, there is no dissociation, and heat is added at constant volume. In an actual engine, these are not true.

In the solution of the above problem, alternatively air table can be used instead of the ideal gas relations. Note that, in this case, $\eta \neq 1 - r^{1-\gamma}$ since $c_v$ and γ are no longer assumed to be constant.

## 6.5.3 Air Standard Diesel Cycle



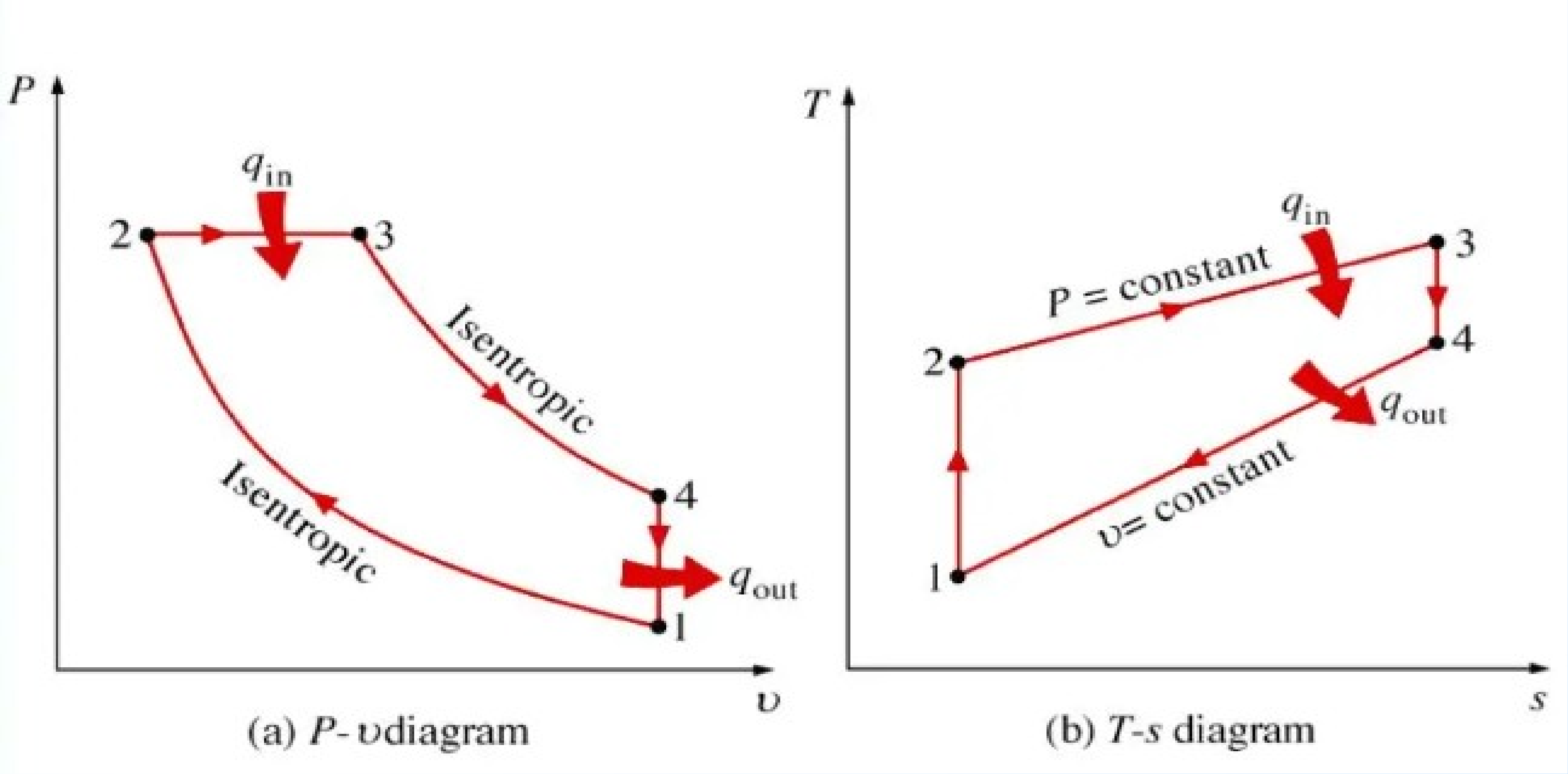

No spark plug

Air only is compressed to high pressure and temperature. Then, the fuel is injected. As the fuel mass is entering the chamber, it burns, small mass at a time while the piston is moving down. So the volume and temperature increases. This process is approximated as constant pressyre heat addition.

| Actual Operation | Ideal Operation |
|---|---|
| Air is taken in during intake (downward) motion of the piston | Pure air at standard conditions exists at State 1 |
| Air is compressed to high pressure and temperature as the piston moves up | Isentropic compression of air to State 2 |
| Fuel is injected. It burns as it enters the combustion chamber while the piston is moving down | Heat is added externaly at constant pressure to State 3 |
| Piston completes its downward motion expanding the gases of combustion | Isentropic expansion of air to State 4 |
| Piston moves up and throws out the exhaust gases | Heat is removed at constant volume back to State 1 |

## Ideal Diesel Cycle

1 - 2 Isentropic compression
2 - 3 Constant pressure heat addition
3 – 4 Isentropic expansion
4 – 1 Constant volume heat rejection



$$ {}_2Q_3 = H_3 - H_2 = m\,c_p\,(T_3 - T_2) = Q_{in} $$

$$ {}_4Q_1 = U_1 - U_4 = -\,m\,c_v\,(T_4 - T_1) $$

$$ W_{net} = {}_2Q_3 - {}_4Q_1 = m\,c_p\,(T_3 - T_2) - m\,c_v\,(T_4 - T_1) $$

$$ \eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} $$

Compression ratio $r = \dfrac{V_1}{V_2}$ Note that $\dfrac{V_1}{V_2} \neq \dfrac{V_4}{V_3}$

For the same $Q_{in}$, $\eta_{th}$ is slightly less than that of the Otto cycle.

## Example

An engine operates on air-standard Diesel cycle with inlet conditions of 300 K and 100 kPa. The compression ratio is 16, and heat added is 1400 kJ/kg. Determine the maximum temperature and pressure and $\eta_{th}$.

$$r = \frac{v_1}{v_2} = 16 \quad \Rightarrow \quad v_2 = \frac{v_1}{16}$$

Ideal gas: $P\,v = R\,T$

Isentropic compression $P\,v^{1.4} = \text{Constant}$

Process 1 – 2: Isentropic compression

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma} \quad \Rightarrow \quad P_2 = P_1\,r^{\gamma} = (100)\,(16)^{1.4} = 4850.3 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r^{\gamma-1} \quad \Rightarrow \quad T_2 = T_1\,r^{\gamma-1} = (300)\,(16)^{0.4} = 909.4 \text{ K}$$

## Process 2 – 3: Heat addition at constant pressure

$$P_3 = P_2 = 4850.3 \text{ kPa} \qquad \text{maximum pressure}$$

$$\frac{Q_{in}}{m} = \frac{{}_2Q_3}{m} = h_3 - h_2 = c_p \left(T_3 - T_2\right) = 1400 \text{ kJ/kg}$$

$$T_3 = \frac{1400}{1.0035} + 909.4 = 2304.5 \text{ K}$$

## Process 3 – 4: Isentropic expansion

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} \neq r^{\gamma-1} \qquad r = \frac{v_1}{v_2} \qquad r \neq \frac{v_4}{v_3}$$

## Process 4 – 1: Heat rejection at constant volume

$$v_1 = \frac{R_1 T_1}{P_1} = v_4$$

Ideal gas equation: $v_3 = \frac{R_1\ T_3}{P_3}$

$$v_4 = v_1 = \frac{R_1\ T_1}{P_1}$$

$$\frac{v_3}{v_4} = \frac{R\ T_3\ /\ P_3}{R\ T_1\ /\ P_1}$$

$$\frac{v_3}{v_4} = \frac{T_3}{T_1}\ \frac{P_1}{P_2}$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \left(\frac{T_3}{T_1}\ \frac{P_1}{P_2}\right)^{\gamma-1} \quad => \quad T_4 = T_3 \left(\frac{T_3}{T_1}\ \frac{P_1}{P_2}\right)^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{T_3}{T_1}\ \frac{P_1}{P_2}\right)^{\gamma-1} = (2304.5) \left(\frac{2304.5}{300}\ \frac{100}{4850.3}\right)^{0.4} = 1102.7\ K$$

$$\frac{P_4}{P_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} \quad => \quad P_4 = P_3 \left(\frac{T_3}{T_1}\ \frac{P_1}{P_2}\right)^{\gamma}$$

$$= (4850.3) \left(\frac{2304.5}{300}\ \frac{100}{4850.3}\right)^{1.4} = 367.6\ kPa$$

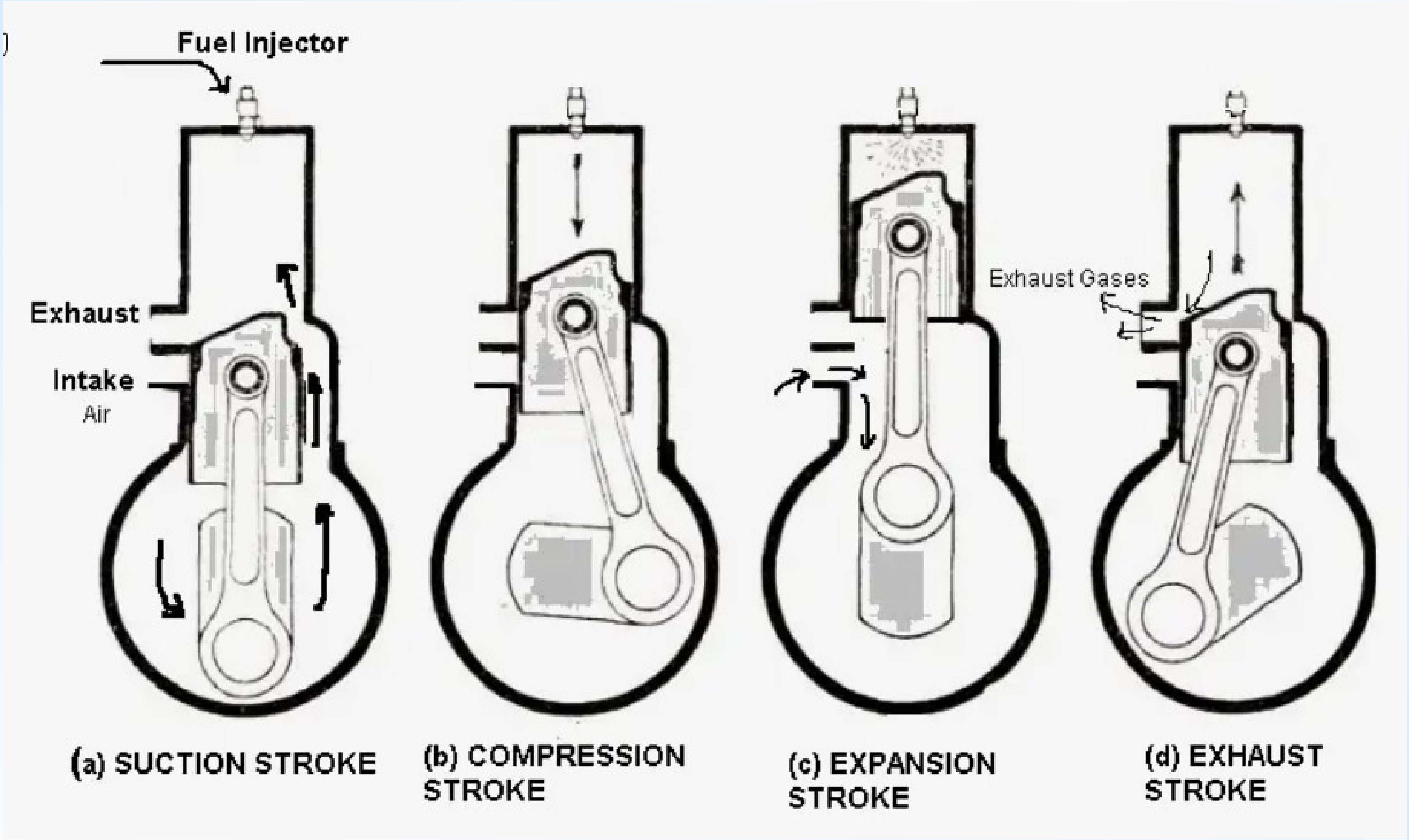
Heat addition: $\frac{{}_2Q_3}{m} = 1400 \text{ kJ/kg}$

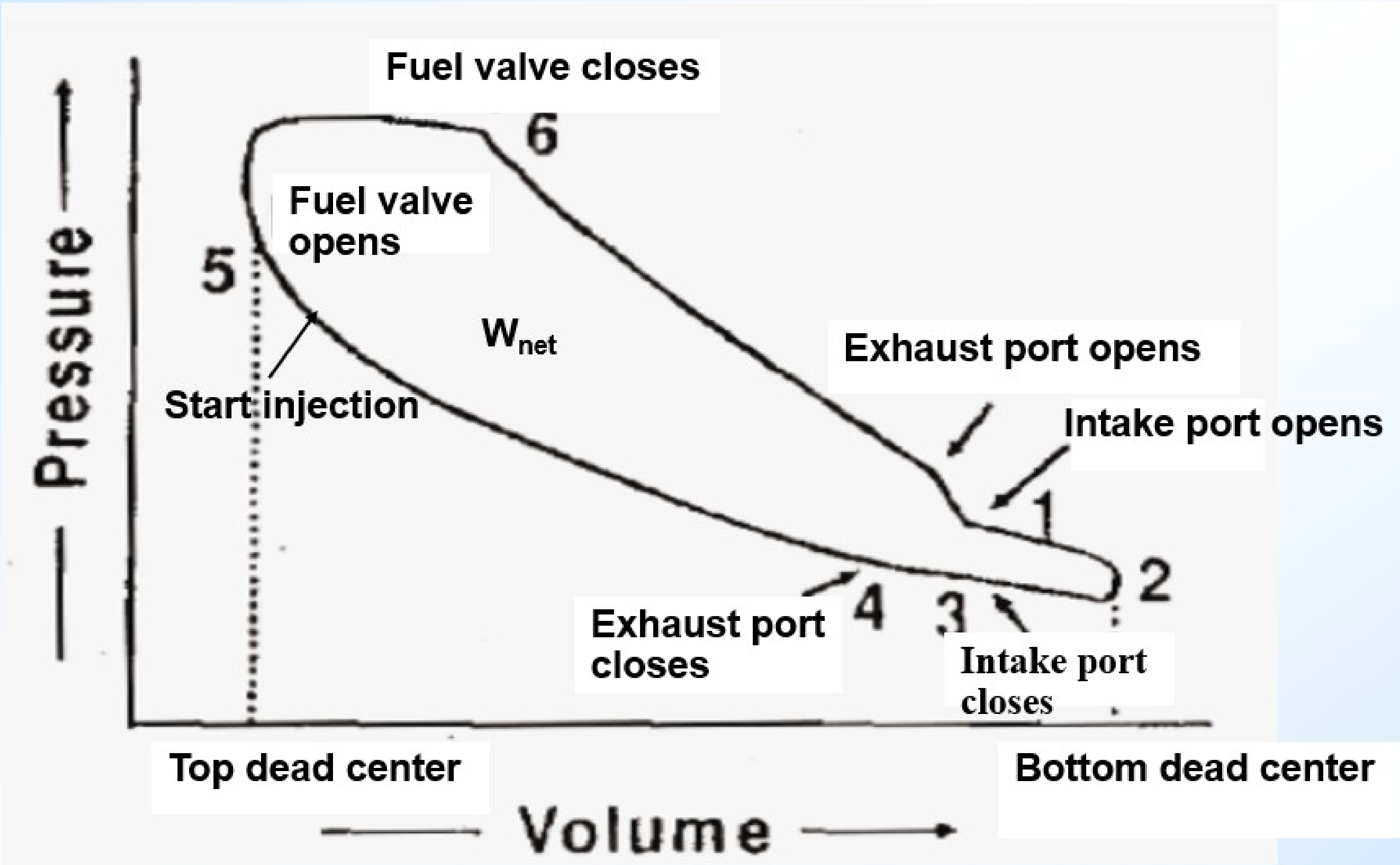
Heat rejection: $\frac{{}_4Q_1}{m} = -c_v \left(T_4 - T_1\right) = -(0.7176)\left(1102.7 - 300\right) = -576 \text{ kJ/kg}$

$$\frac{W_{net}}{m} = 1400 - 576 = 824 \text{ kJ/kg}$$

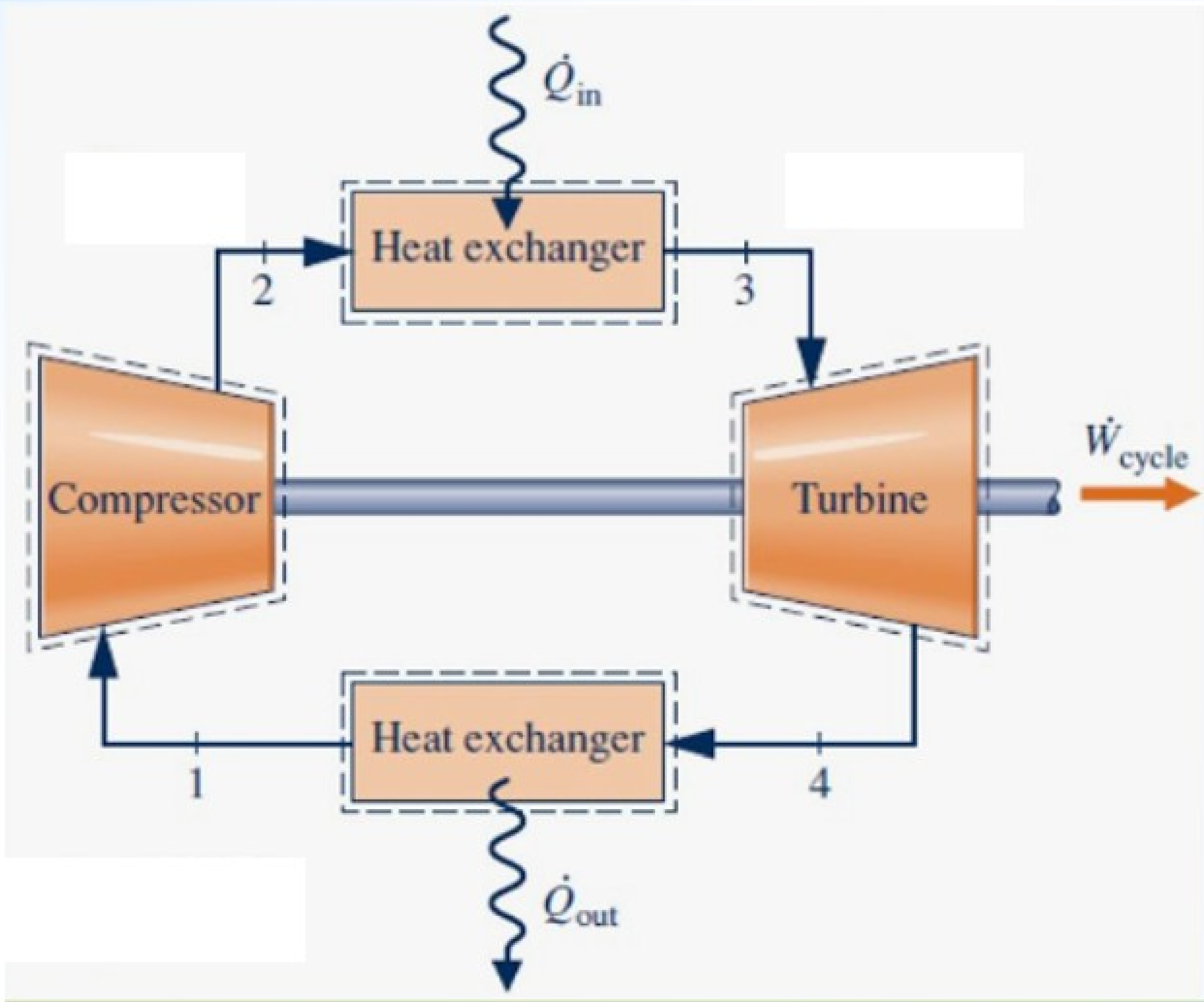
$$\eta_{th} = \frac{824}{1400} = 0.589 \quad \Rightarrow \quad 58.9\ \%$$

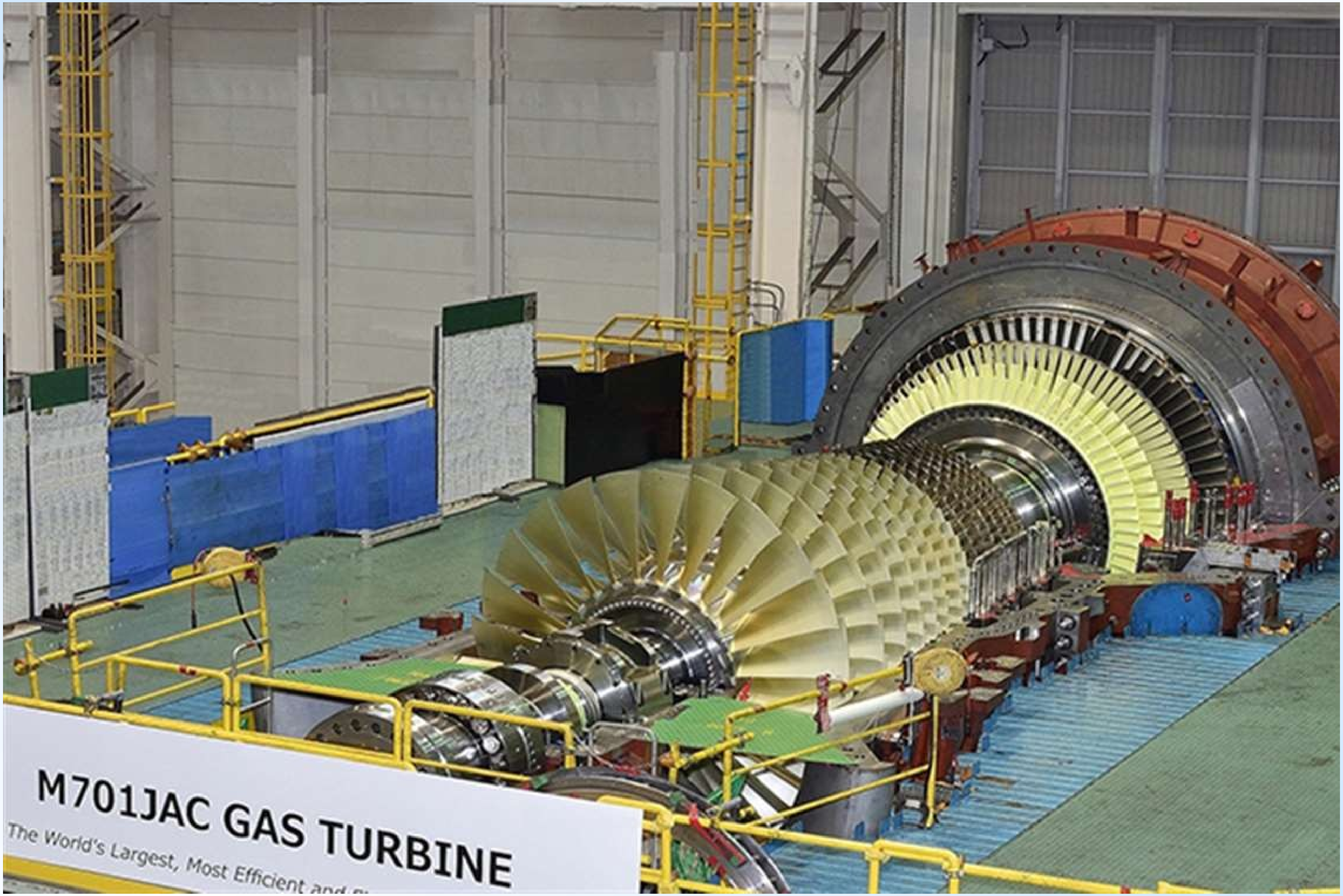
## Two-Stroke Diesel Engine

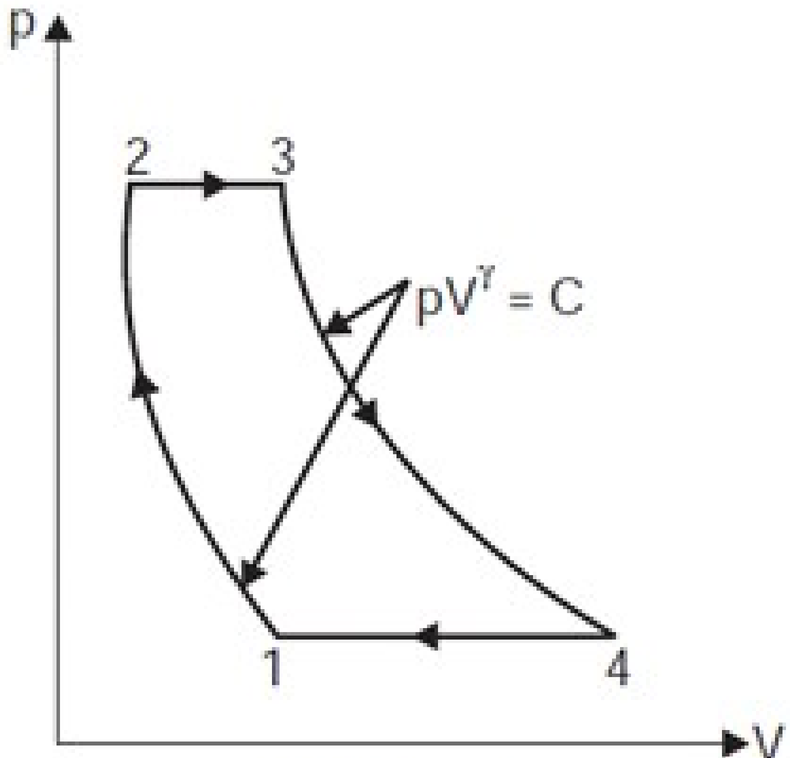



Fuel valve closes
6
Fuel valve opens
5
Pressure
Wnet
Start injection
Exhaust port opens
Intake port opens
1
2
4
3
Exhaust port closes
Intake port closes
Top dead center
Bottom dead center
Volume

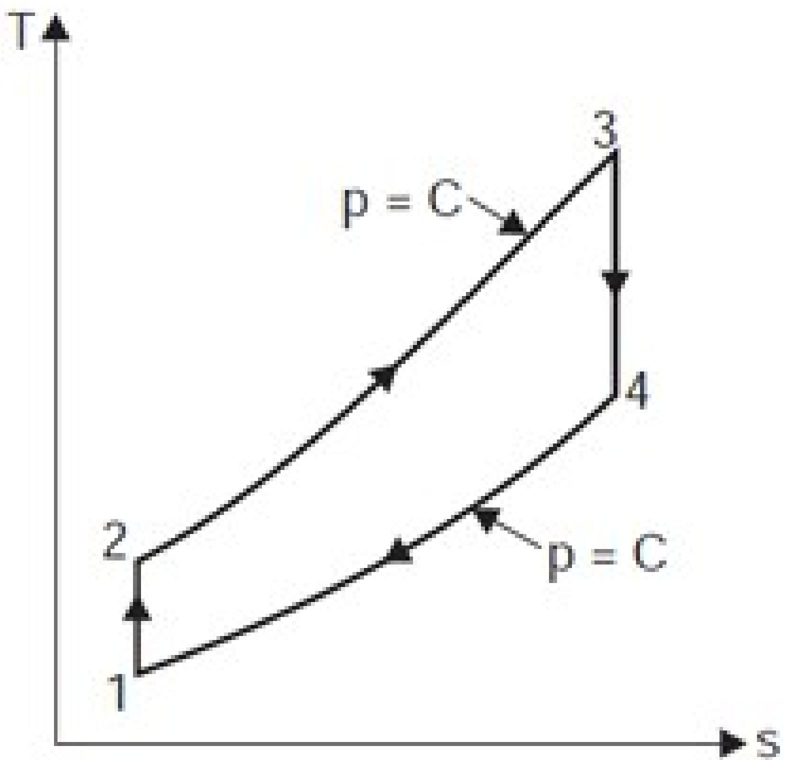
## 6.5.3 Air Standard Brayton Cycle




M701JAC GAS TURBINE
The World's Largest, Most Efficient and




*p-V* diagram




*T-s* diagram.

Thermal efficiency
$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{\sum Q}{Q_{in}} = \frac{(h_3 - h_2) - (h_4 - h_1)}{(h_3 - h_2)}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$ for constant $c_p$

Heat addition and rejection occur at constant pressure
$$\left.\begin{array}{l} P_1 = P_4 \\ P_2 = P_3 \end{array}\right\} \quad \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

Define Pressure Ratio
$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\eta_{th} = 1 - \frac{1}{r_p^{\gamma-1/\gamma}} = 1 - r_p^{1-\gamma/\gamma}$$

$\eta_{th}$ depends on $r_p$.

The maximum temperature $T_3$ does not have an effect on the optimum performance.

Let $T_1$ and $T_3$ be fixed (upper and lower tempeatures in the cycle).

Let's find $T_2$ and $T_4$ for maximum work:

$$W_{net} = m\,c_p\,(T_3 - T_2) - m\,c_p\,(T_4 - T_1)$$

Substitute $T_4 = \dfrac{T_3\,T_1}{T_2}$

$$W_{net} = m\,c_p\left[(T_3 - T_2) - \frac{T_3\,T_1}{T_2} + T_1\right]$$

$T_1$ and $T_3$ are fixed. The only variable is $T_2$.

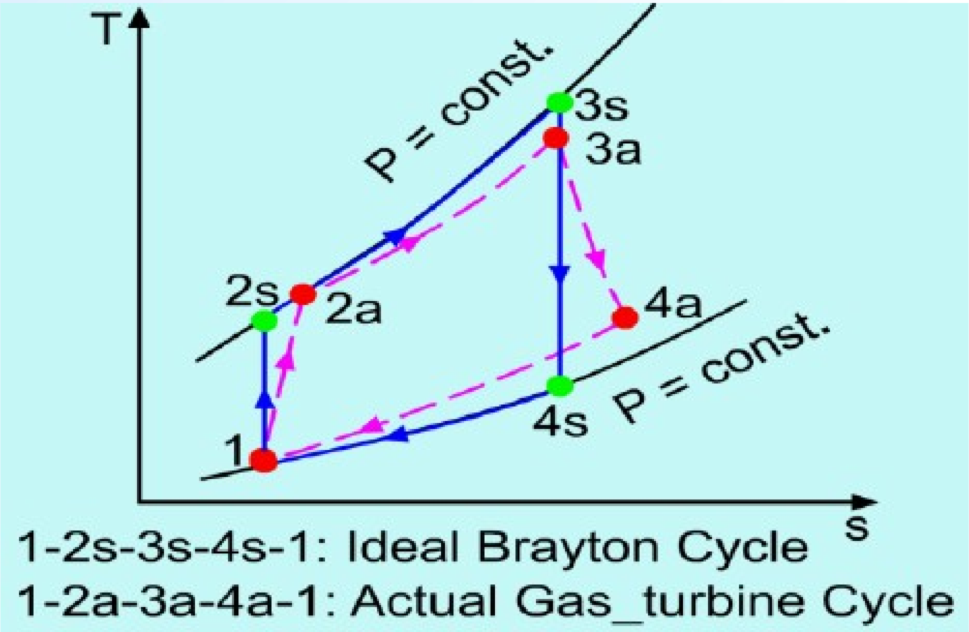
For $W_{net_{max}}$ => $\dfrac{dW_{net}}{dT_2} = 0$ => $T_2 = \sqrt{T_1\,T_3} = T_4$ For maximum work output

For industrial use $\dfrac{T_3}{T_1}$ □ 3.5 □ 4

For aircraft engines with air-cooled blades $\dfrac{T_3}{T_1}$ □ 5 □ 5.5

What makes the Brayton cycle ideal is the assumption that compression and expansion are isentropic.

One may define, separately, compressor and turbine efficiencies (similar to a steam turbine).

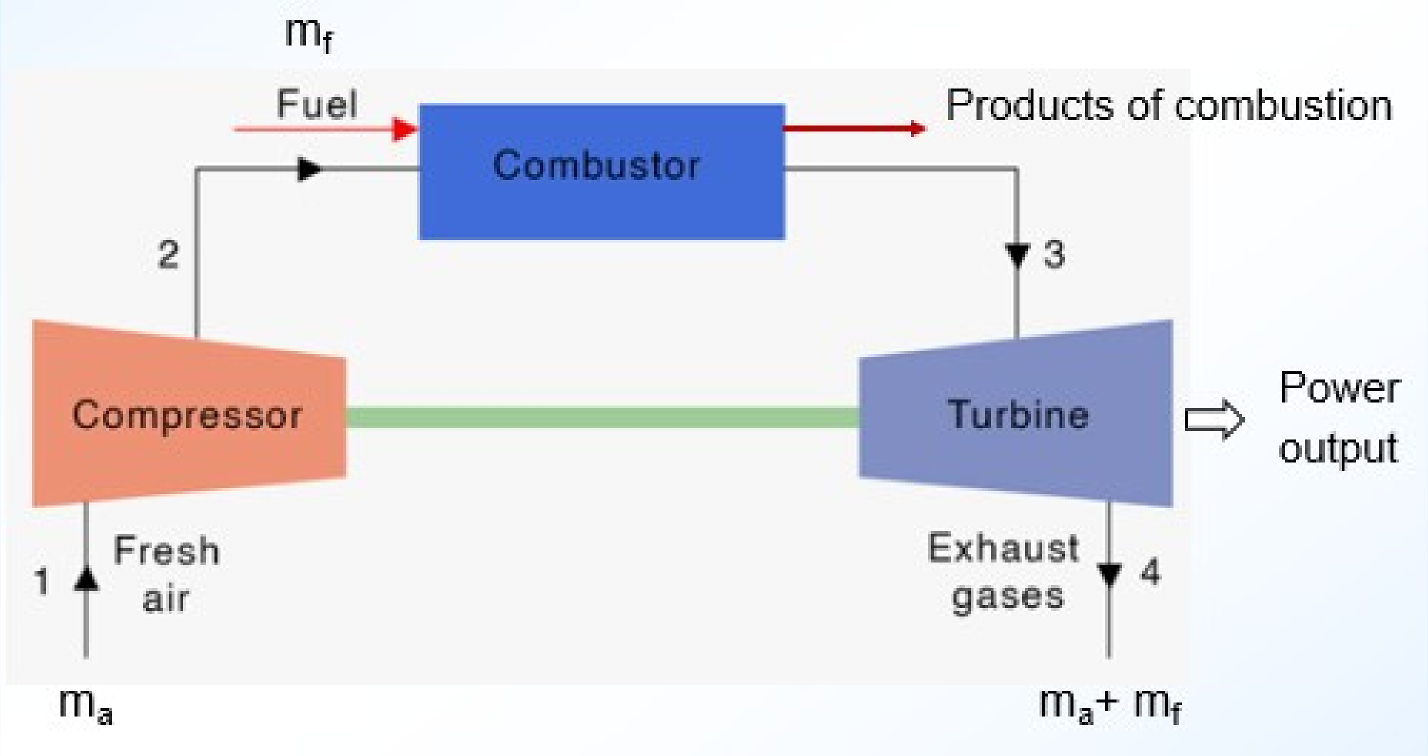



$$\eta_C = \frac{W_{ideal}}{W_{actual}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\eta_T = \frac{W_{actual}}{W_{ideal}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

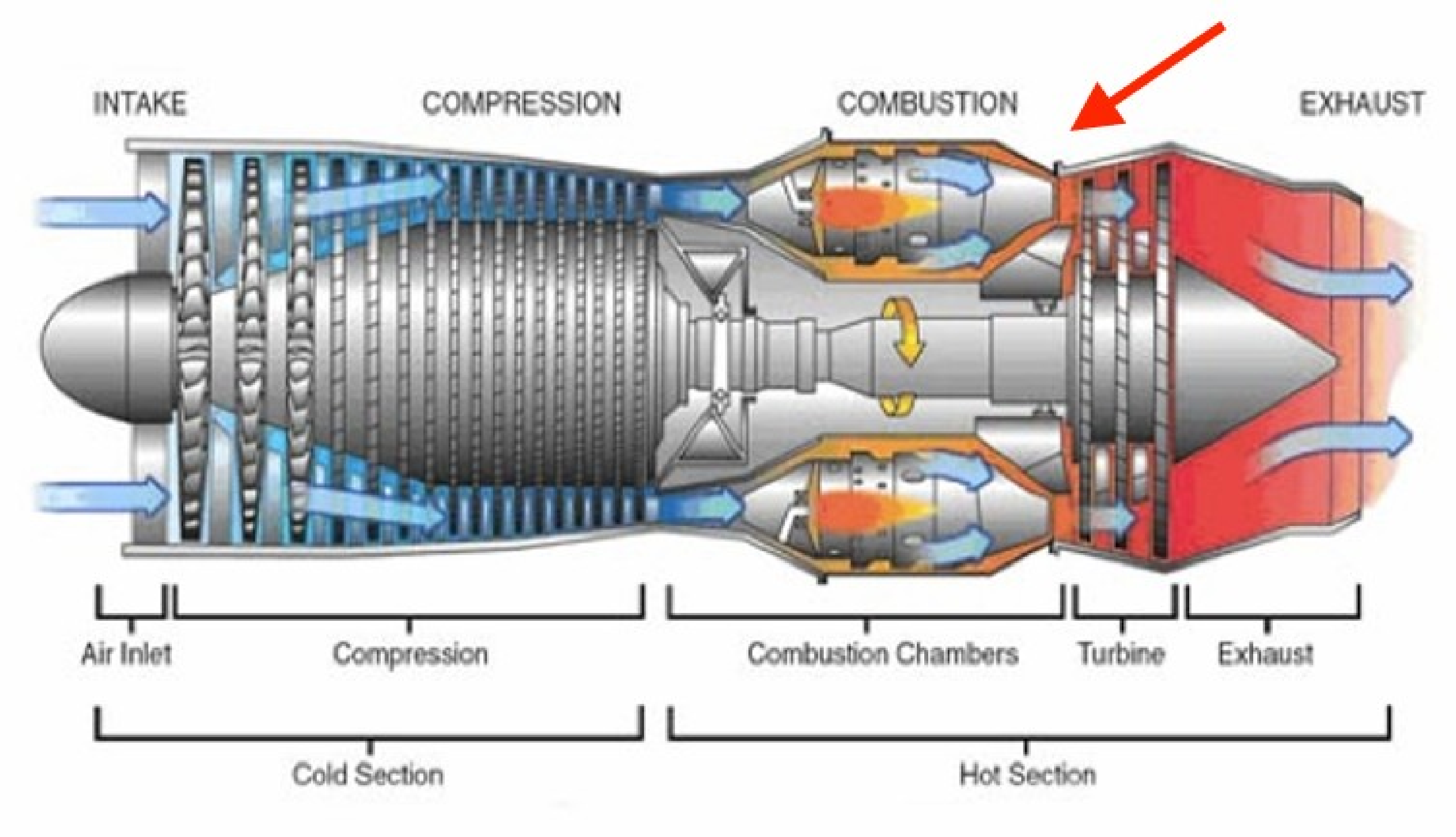
Note the differences in the definitions

## Actual (real) Open Brayton Cycle




In general, $\eta_{th}$ of such a system is rather low. The temperature of the exhaust is rather high, so a lot of unused available energy is thrown out.

# Aircraft Jet Propulsion Unit



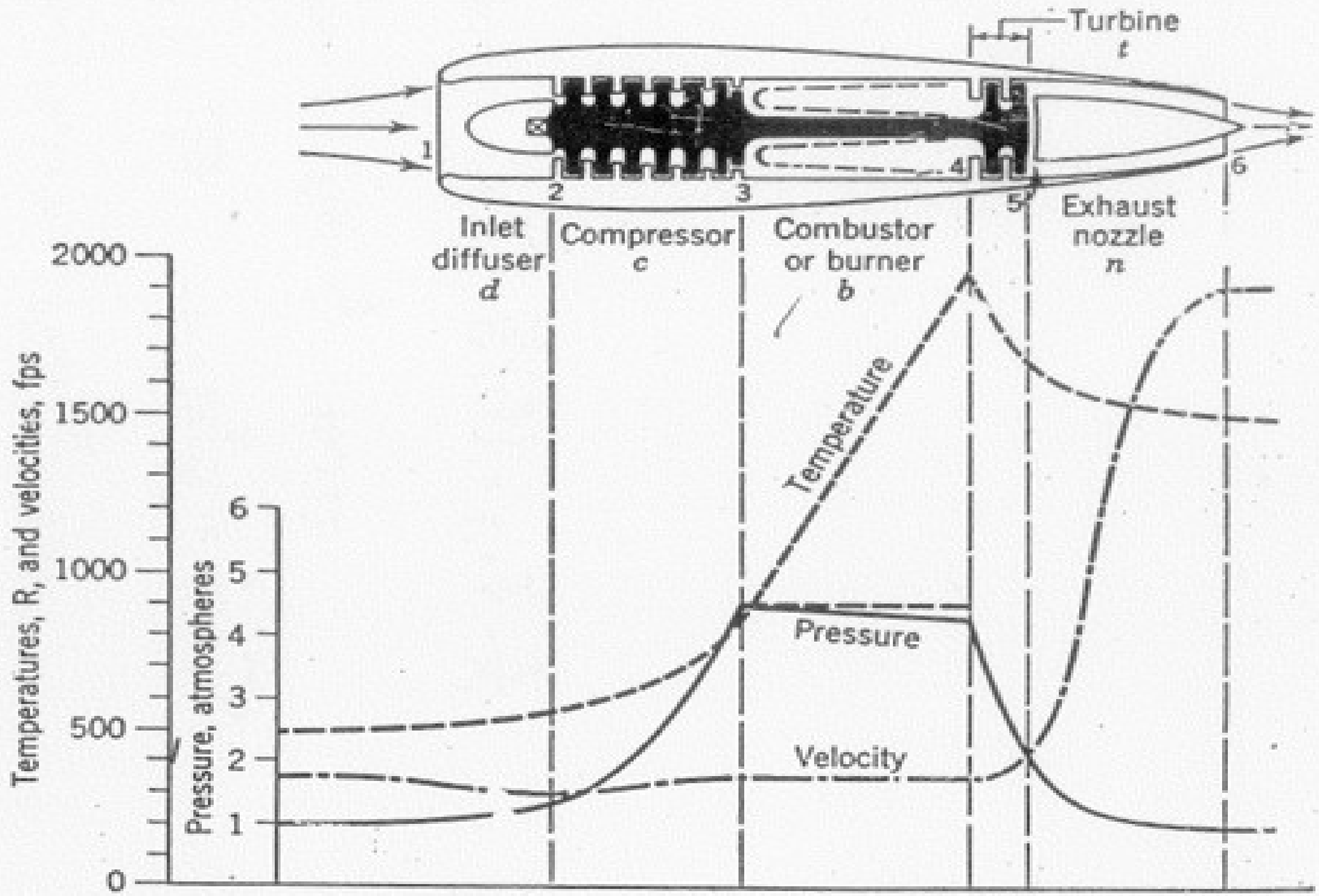
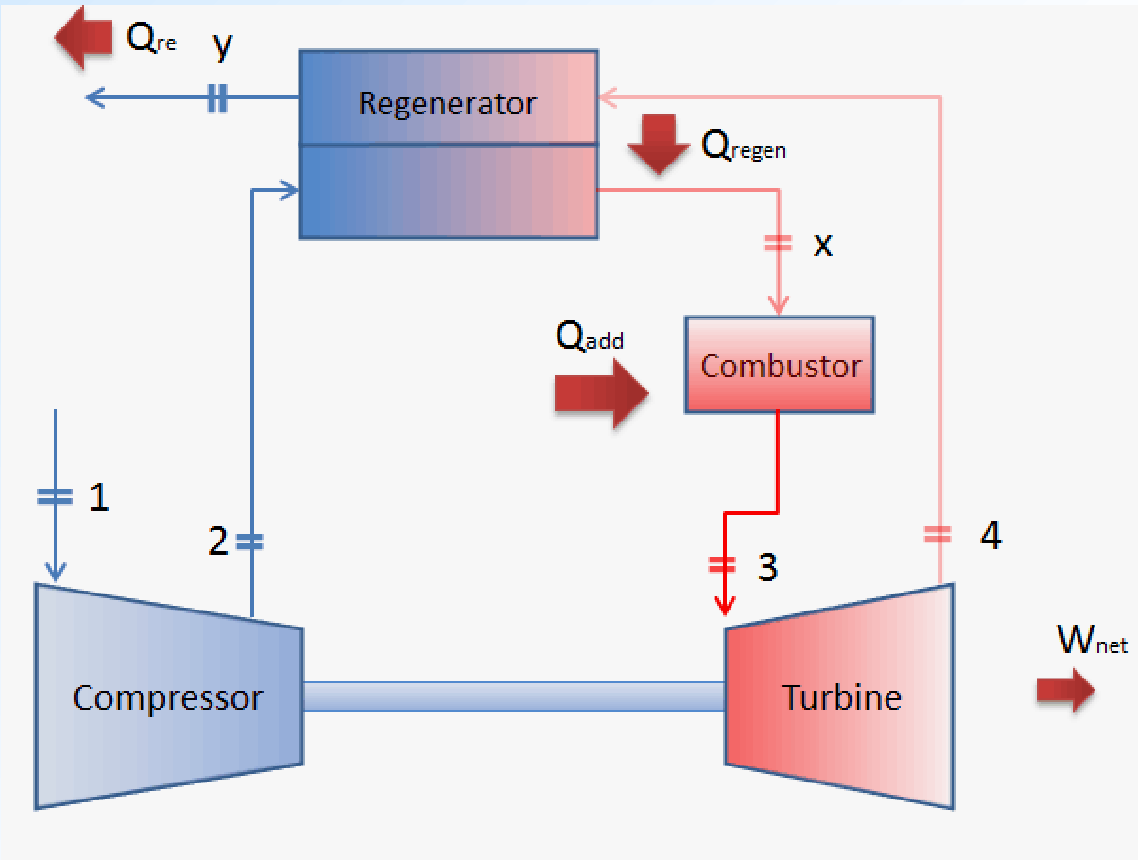


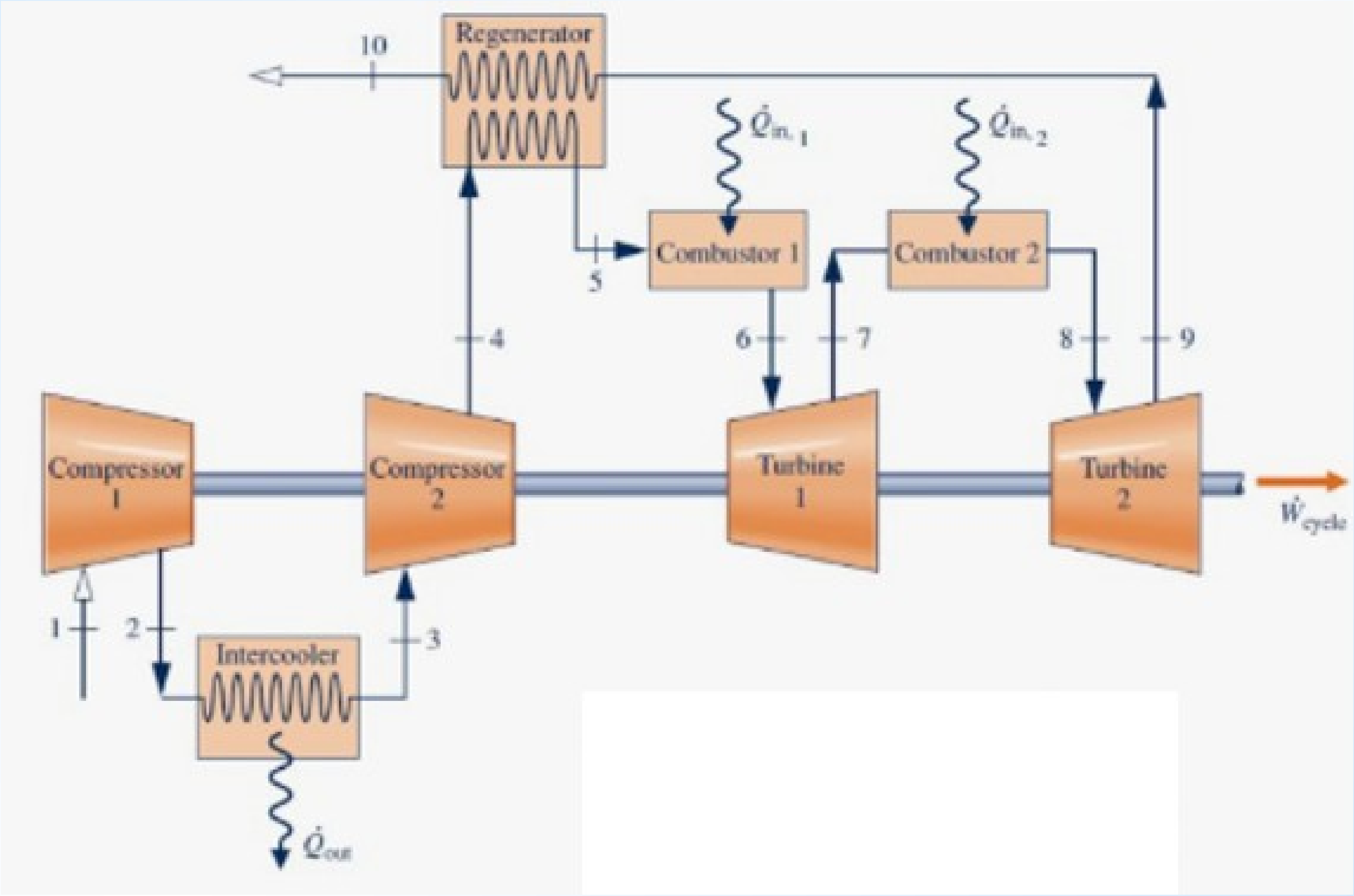

Fig. 10.35. Pressure, temperature, and velocity changes in the aircraft jet-propulsion unit.

## 6.5.5 Regeneration

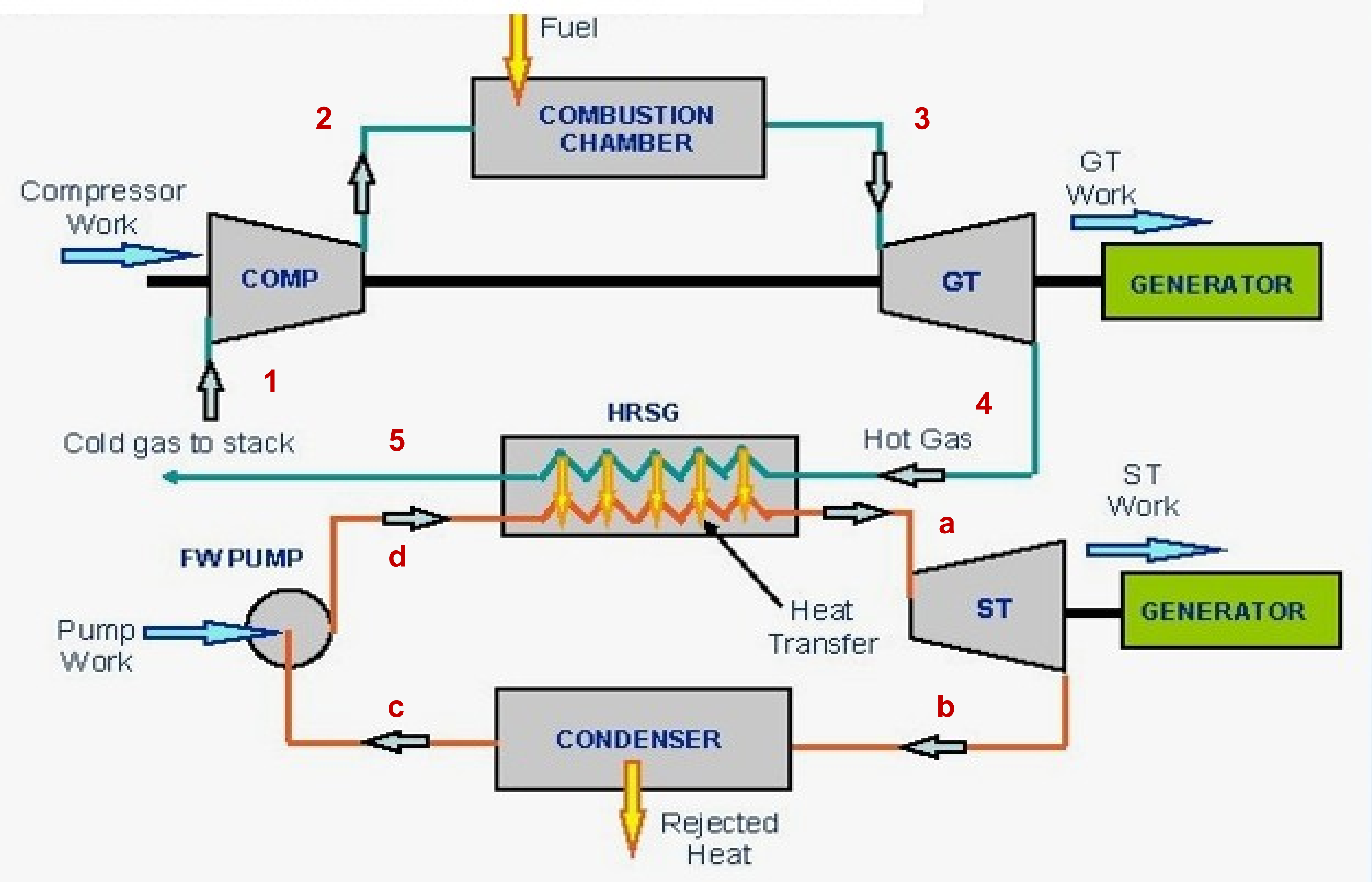



Such a system will increase $\eta_{th}$ by decreasing the fuel ($m_f$) requirement. $W_{T,net}$ stays the same or even decreases a little due to pressure effects.

## Bryton Cycle with Regeneration and Intercooling



## 6.5.6 Combined Cycle Power Plant



## Example

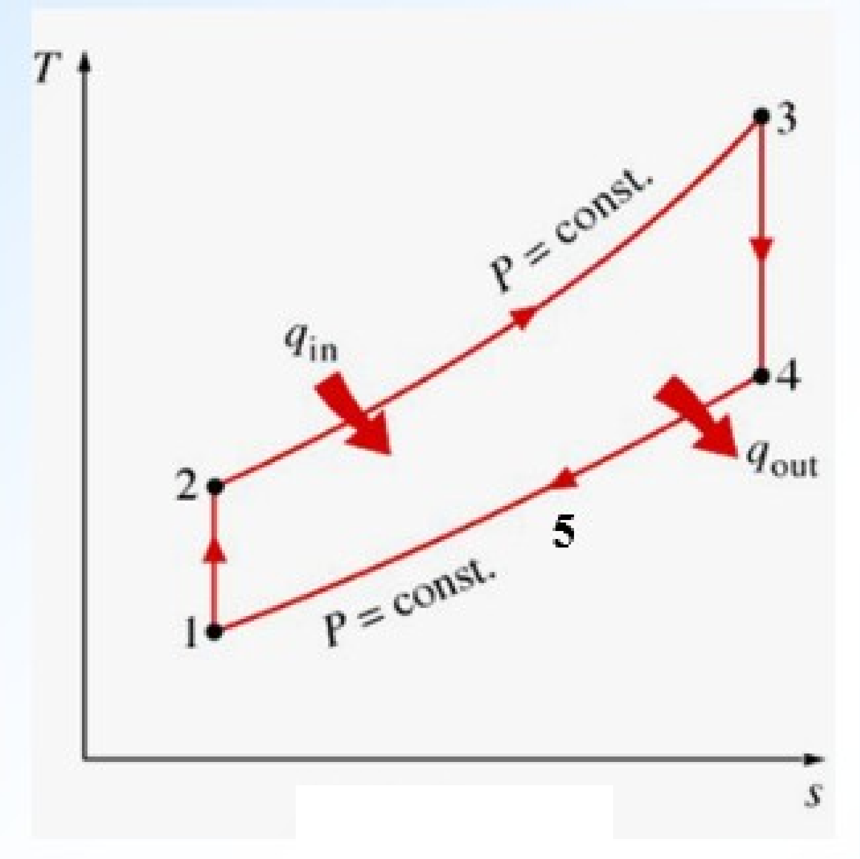
A combined-cycle gas-turbine / steam power plant produces a total of 37300 kW. The exhaust from the gas turbine leaves the steam generator at 147 °C. Staem leaves the generator at 6 MPa and 400 °C. Air enters the gas turbine unit at 290 K and 100 kPa. Pressure ratio of the compressor is 10. The entarance temperature to gas turbine is 1400 K. Steam condensor pressure is 13 kPa. Find the flow rates in each cycle and the total thermal efficiency.

$P_a = 6$ MPa  $P_b = 13$ kPa  $T_a = 400\ ^\circ C$  $P_1 = 100$ kPa  $T_1 = 290\ ^\circ C$

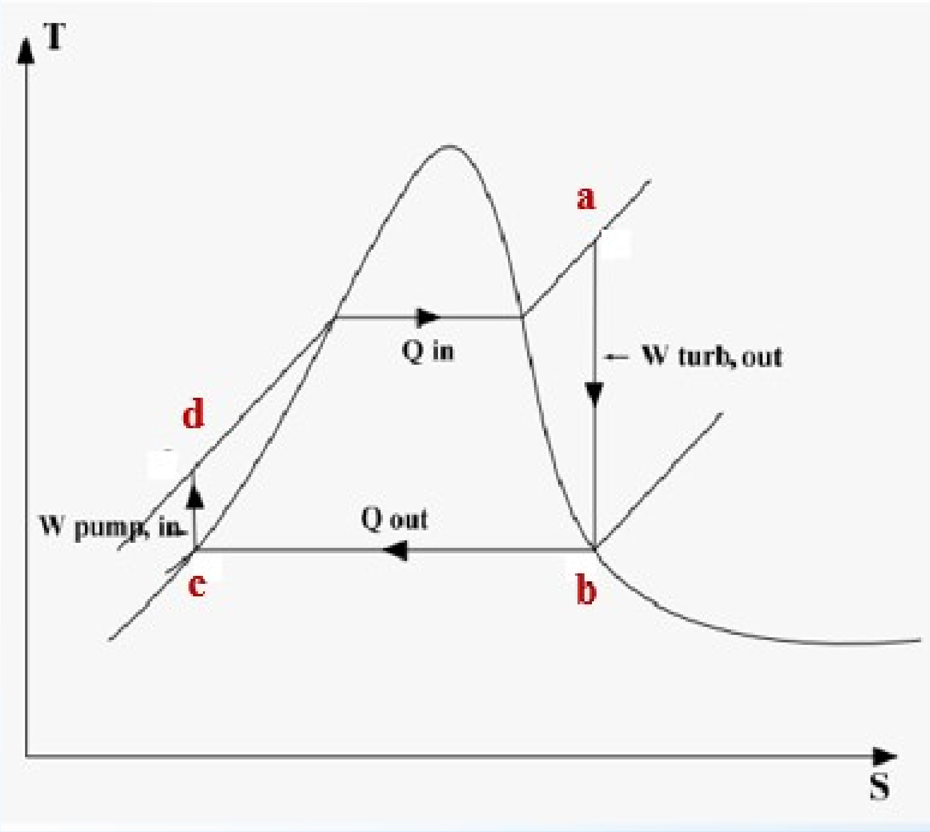
$T_5 = 147\ ^\circ C = 420$ K  $r_p = 10$  $W_{net\ total} = 37300\ kW = W_{net\ air} + W_{net\ steam}$

Find $\dot{m}_a$, $\dot{m}_s$, $\eta_{th}$

Gas cycle




Steam cycle



Process 1 – 2: Isentropic compression with r = 10

$P_1 = 100$ kPa   $T_1 = 290\ ^\circ C$

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^{\gamma} = r^{-\gamma} \quad => \quad P_2 = P_1\, r^{\gamma} = (100)\,(10)^{1.4} = 2511.9 \text{ kPa}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{1-\gamma} = r^{1-\gamma} \quad => \quad T_2 = T_1\, r^{\gamma-1} = (290)\,(10)^{0.4} = 728.4 \text{ K}$$

Process 2 – 3: Heat addition at constant pressure

$P_2 = P_2 = 2511.9$ kPa   Maximum pressure in the cycle

$$\frac{Q_{in}}{\dot{m}_a} = c_p\,(T_3 - T_2) = (1.0035)\,(1400 - 728.4) = 673.9 \text{ kJ/kg air}$$

$Q_{in} = 673.9\ \dot{m}_a$   $\dot{m}_a$ is in kg air/s

Process 3 – 4: Isentropic expansion such that $P_4 = P_1 = 100$ kPa

$$\frac{T_4}{T_3} = \frac{P_4\, v_4}{P_3\, v_3}$$ also $$P_4\, v_4^{\gamma} = P_3\, v_3^{\gamma}$$

or $$\frac{P_4}{P_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} \Rightarrow \frac{v_3}{v_4} = \left(\frac{P_4}{P_3}\right)^{1/\gamma}$$

$$\frac{T_4}{T_3} = \frac{P_4}{P_3}\left(\frac{P_3}{P_4}\right)^{1/\gamma} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = (1400)\left(\frac{100}{2511.9}\right)^{\frac{0.4}{1.4}} = 557.35\ \text{K}$$

## Process 4 – 5: Heat rejection (to steam) at constant pressure

$$P_5 = P_4 = P_1 = 100 \text{ kPa} \qquad T_5 = 147\ ^\circ\text{C} = 420 \text{ K}$$

$$Q_{\substack{out \\ air}} = \dot{m}_a\ c_p\ (T_5 - T_4) = \dot{m}_a\ (1.0035)\ (-537.35 + 420) = -137.8\ \dot{m}_a \text{ kW}$$

$$Q_{\substack{out \\ air}} = -Q_{\substack{in \\ steam}} = -137.8\ \dot{m}_a \text{ kW}$$

## Work done in the gas compressor (process 1 – 2)

$$\dot{w}_C = \int_1^2 P\, dV = P_1\, v_1^{\gamma} \int_1^2 \frac{dv}{v^{\gamma}} = \frac{P_1\, v_1^{\gamma}}{1-\gamma}\left(v_2^{1-\gamma} - v_1^{1-\gamma}\right) = \frac{P_2\, v_2 - P_1\, v_1}{1-\gamma}$$

$$\dot{W}_C = \frac{\dot{m}_a\ R\ (T_2 - T_1)}{1-\gamma} = \dot{m}_a\ \frac{(0.287)\ (728.4 - 290)}{1 - 1.4} = -314.6\ \dot{m}_a \text{ kW}$$

Work done in the gas turbine (process 3 – 4)

$$\dot{W}_T = \frac{\dot{m}_a \, R \, (T_4 - T_3)}{1 - \gamma} = \dot{m}_a \, \frac{(0.287)\,(557.35 - 1400)}{1 - 1.4} = 604.6 \, \dot{m}_a \; \text{kW}$$

Net work in the gas cycle $\dot{W}_{\substack{net \\ air}} = \dot{W}_T + \dot{W}_C = (604.6 - 314.6)\,\dot{m}_a = 290 \, \dot{m}_a$ kW

Calculations for the steam cycle

$P_a$ = 6 MPa, $T_a$ = 400 °C } $h_a$ = 3177.2 kJ/kg steam, $s_a$ = 6.5408 kJ/kg.K

Process a – b: Isentropic expansion

$P_b$ = 13 kJ/kg, $s_a$ = 6.5408 kJ/kg.K } $h_b$ = 2101.8 kJ/kg steam, $T_b$ = 51 °C, $x_b$ = 0.7932

Process b – c: Heat removal at constant temperature

$$T_c = T_b = 51\ ^{\circ}C = T_{sat}$$

$$h_f = h_c = 213.67 \text{ kJ(kg steam}$$

Process c – d: Isentropic compression

$$h_d = h_c + \int_c^d v\, dP \cong h_c + v_{fc}\ \Delta P$$

$$= 213.67 + (0.001013)\ (6000 - 13) = 219.73 \text{ kJ/kg steam}$$

Work done in the steam turbine

$$\frac{\dot{W}_T}{\dot{m}_s} = h_a - h_b = 3177.2 - 2101.8 = 1075.4 \text{ kJ/kg steam}$$

Work done in the pump

$$\frac{\dot{W}_P}{\dot{m}_s} = h_c - h_d = 213.67 - 219.73 = - 6.06 \text{ kJ/kg steam}$$

Net work in the steam cycle

$$\frac{\dot{W}_{net}}{\dot{m}_s} = \frac{\dot{W}_T}{\dot{m}_s} + \frac{\dot{W}_P}{\dot{m}_s} = 1075.4 - 6.06 = 1069.34 \text{ kJ/kg steam}$$

Heat added to the steam in the steam generator

$$\dot{Q}_{\substack{in\\steam}} = \dot{m}_s \left(h_a - h_d\right) = \dot{m}_s \left(3177.2 - 219.73\right)$$

$$= 2957.47\,\dot{m}_s$$

$$\dot{Q}_{\substack{out\\air}} = -137\ 8\ \dot{m}_a$$

$$\frac{\dot{m}_s}{\dot{m}_a} = 0.0466\ \frac{\text{kg steam}}{\text{kg air}}$$

Solve simultaneously

Total net work

$$\dot{W}_{\substack{net\\total}} = \dot{W}_{\substack{net\\air}} + \dot{W}_{\substack{net\\steam}} = 290\,\dot{m}_a + 1069.34\,\dot{m}_s = 37300 \text{ kW}$$

Solve simultaneously

$$\dot{m}_a = 109.8 \text{ kg air/s}$$

$$\dot{m}_s = 5.1 \text{ kg steam/s}$$

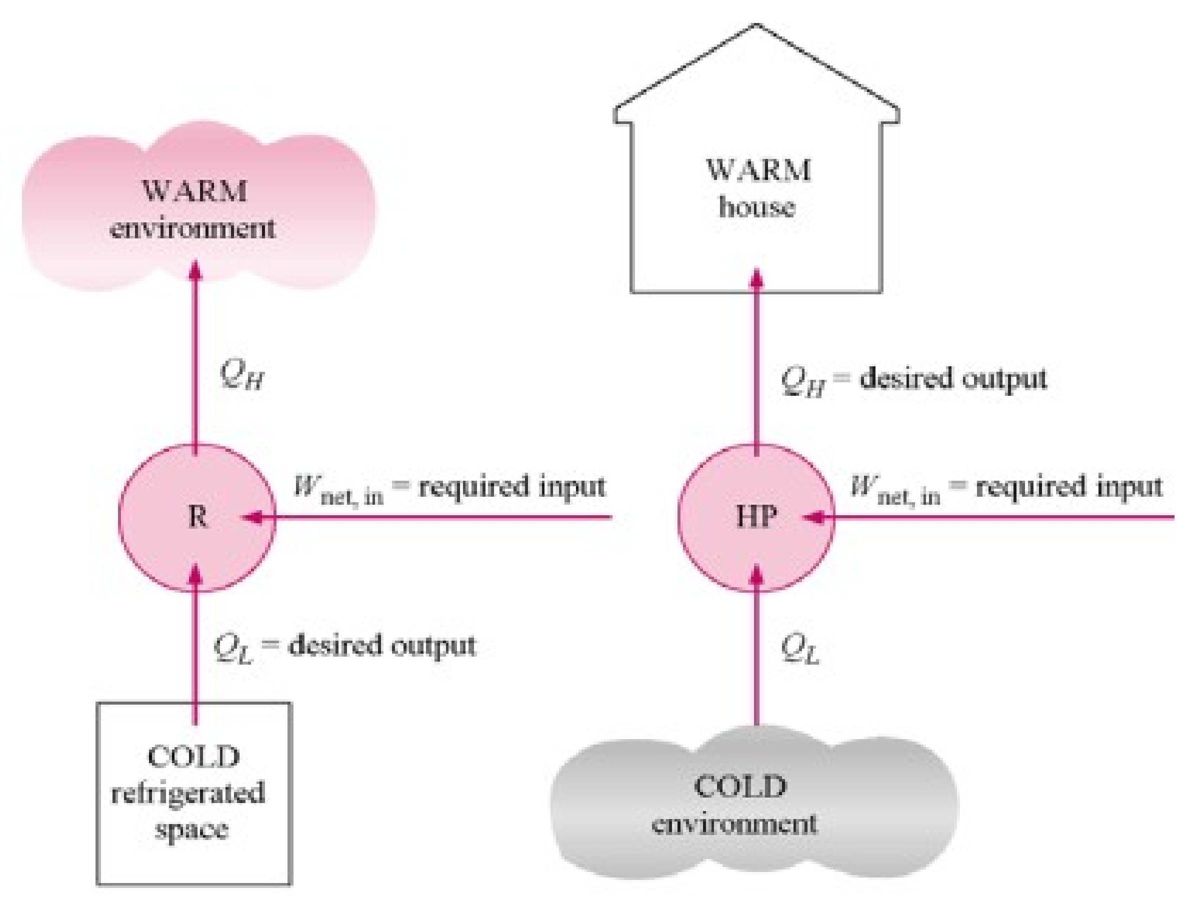
Thermal efficiency

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{37300}{\dot{Q}_{in,air}} = \frac{37300}{(109.8)\,(673.9)} = 0{,}504$$

$$\eta_{th} = 50.4\ \%$$

## 6.6 Refrigeration Cycles

Refrigeration: Maintaining a system at a temperature less than the surroundings.




(a) Refrigerator (b) Heat pump

The purpose of a refrigerator is the removal of heat, called the cooling load, form a low temperature medium.

The purpose of a heat pump is the transfer of heat to a high temperature medium, called the heating load.

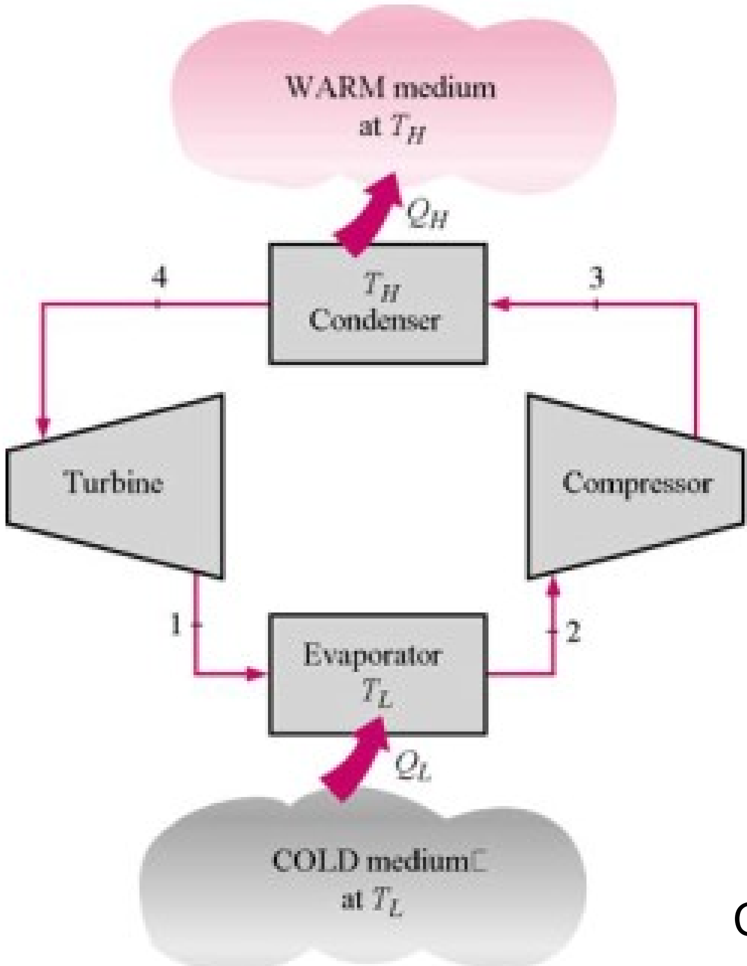
The performance of refrigerators and heat pumps is expressed in terms of Coefficient of Performance, COP.

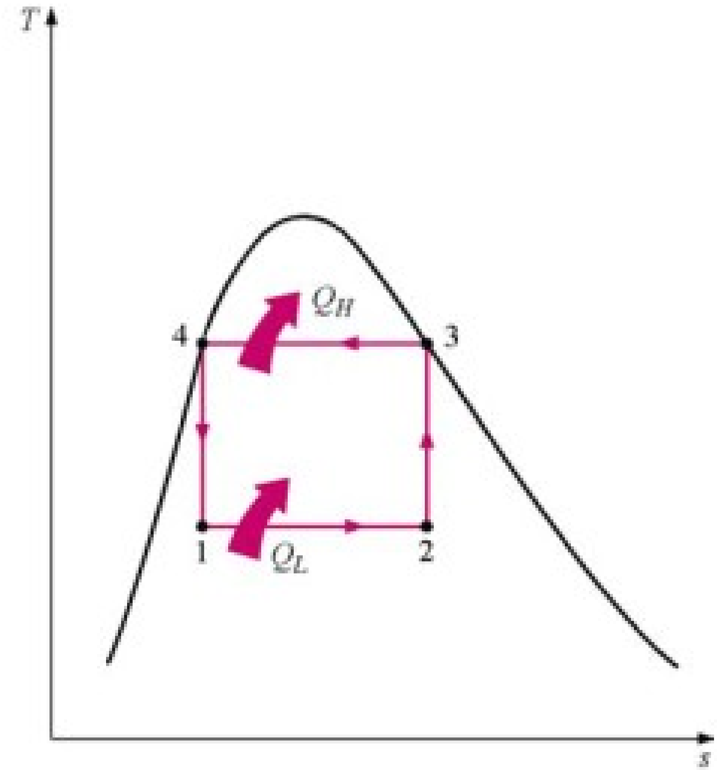
$$COP_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{\dot{Q}_L}{\dot{W}_{net,in}}$$

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{\dot{Q}_H}{\dot{W}_{net,in}}$$

$$COP_{HP} = COP_R + 1$$

# 6.6.1 Reversed Carnot Cycle



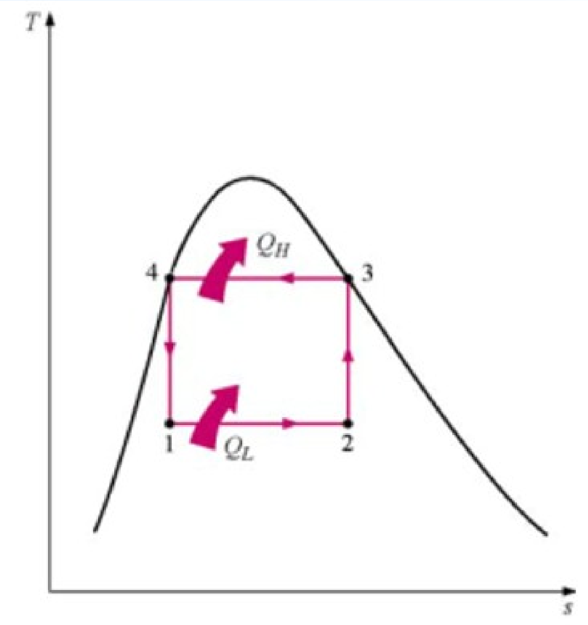




$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{T_L \ \Delta S}{(T_H - T_L)\ \Delta S} = \frac{T_L}{T_H - T_L}$$

The compressor intake pressure (lowest in the cycle, $P_2$) is kept higher than the atmospheric pressure so that air cannot leak in.

A refrigerant whose boiling temperature is less than the surrounding temperature at the atmospheric pressure is used.

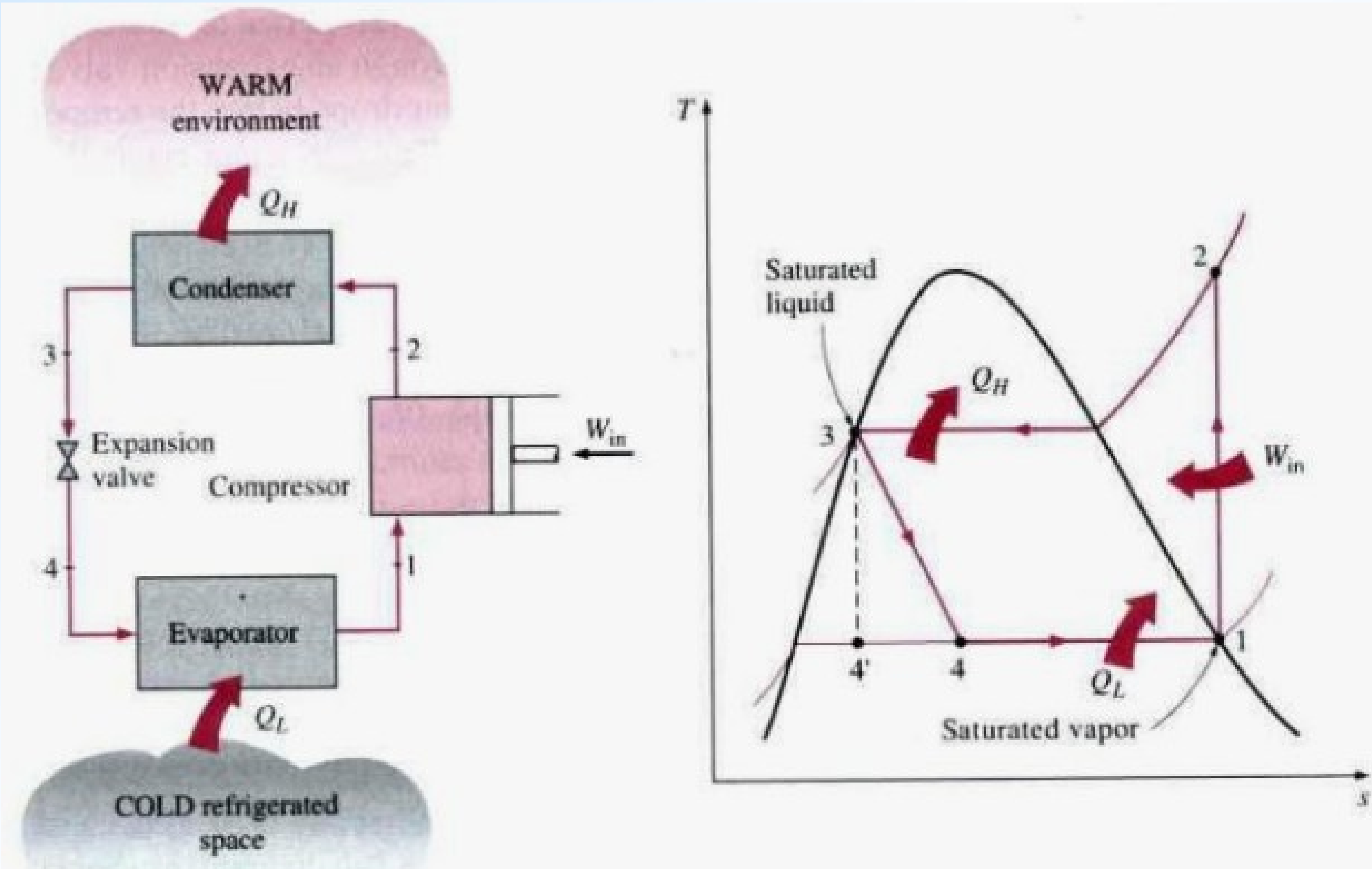
Freon-12 has a boiling point of - 29.7 °C at 1 atm.

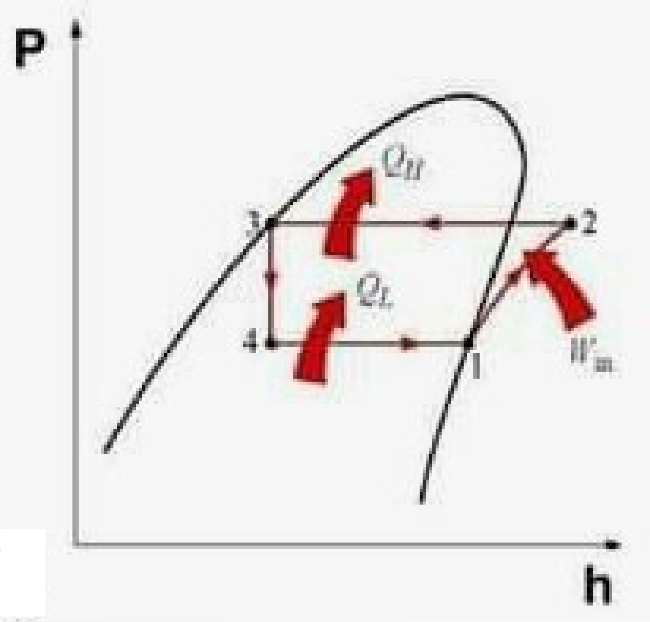



Difficulties with the Carnot cycle:

- Reciprocating compressurs should not operate in the 2-phase region as lubricating oil may be washed away during the compression process.
- The work performed by the expander is very small compared to the compressore work, and h cost of an such an expander would be unnecessarily high.

## 6.6.1 Vapor Compression Cycle






In refrigeration, a P-h diagram is often used rather than a T-s diagram.

- Refrigerant receives heat until it is saturated vapor at State 1.
- Expansion (3 – 4) is an irreversible throttling process for which only an expansion valve is necessary.

A **ton of refrigeration** is a unit of power used in some countries (especially in North America) to describe the heat-extraction rate of refrigeration and air conditioning equipment. It was originally defined as the rate of heat transfer that results in the freezing or melting of 1 short ton (2,000 lb; 907.17 kg) of pure ice at 0 °C (32 °F) in 24 hours. It is equivalent to 3.516 kW of heat removal rate.

2000 lb of ice = 1 short ton = 907.17 kg

Latent heat of ice = 334.9 kJ/kg

Energy necessary to melt the ice = (907.17) (334.9) = 3.0384 $10^5$ kJ

Rate of energy to melt the ice in 24 hours = $\dfrac{3.0384\ 10^5}{(24)\ (3600)} = 3.516$ kW

**Example**

A standar vapor compression system produces 20 tons of refrigeration using Freon-12 as the refrigerant while operating between a condensor temperature of 41.6 °C and an evaporator temperature of - 25 °C. Determine (a) refrigeration effect in kJ/kg; (b) the ciculation rate in kg/s; (c) power supplied; (d) COP; and (e) heat rejected in kW.

(a) $\dfrac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$ refrigeration effect in kJ/kg

Using Freon-12 table (Dichlorodifluoromethane)

$T_1 = -25\ °C$, $P_1 = 123.7$ kPa, Saturated vapor $\Rightarrow$ $h_1 = h_g = -176.352$ kJ/kg, $s_1 = 0.7121$ kJ/kg.K, $v_1 = 0.131166\ m^3/kg$

Using Freon-12 table

$T_3 = 41.6\ °C$ (saturated liquid) $\Rightarrow$ $h_3 = h_f = 76.17$ kJ/kg, $P_3 = 1$ MPa

During the throttling process, enthalpy does not change. So, $h_4 = h_3 = h_f = 76.17$ kJ/kg

$$\frac{\dot{Q}_{in}}{\dot{m}_{in}} = h_1 - h_4 = 176.352 - 76.17 = 100.18 \text{ kJ/kg}$$

(b) Given $Q_{in}$ = 20 tons of refrigeration = (20) (3.516) = 70.32 kW

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{70.32}{100.18} = 0.7019 \text{ kg/s}$$

(c) $\dot{W}_C = \dot{m}\left(h_2 - h_1\right)$

Isentropic compression => $s_2 = s_1 = 0.7121$ kJ/kg.K

$s_2 = 0.7121$ kJ/kg.K

$P_2 = P_3 = 1$ MPa

Superheated table

} $h_2 = 213.46$ kJ/kg

$$\dot{W}_C = \dot{m}\left(h_2 - h_1\right) = (0.7019)\,(213.46 - 176.35) = (0.7019)\,(37.11) = 26.05 \text{ kW}$$

(d)
$$COP = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_C/\dot{m}} = \frac{100.18}{37.11} = 2.7$$

(e) $\dot{Q}_{out} = \dot{m}\left(h_3 - h_2\right) = (0.7019)\left(76.17 - 213.46\right) = -96.36 \text{ kW}$

The size of the compressor depends on the volumetric flow rate that the compressor must handle at the outlet conditions.

$$\dot{V} = \dot{m}\, v_1 = (0.7019)\,(0.131166) = 0.092 \text{ m}^3/\text{s}$$
$$= 92 \text{ liters/s}$$