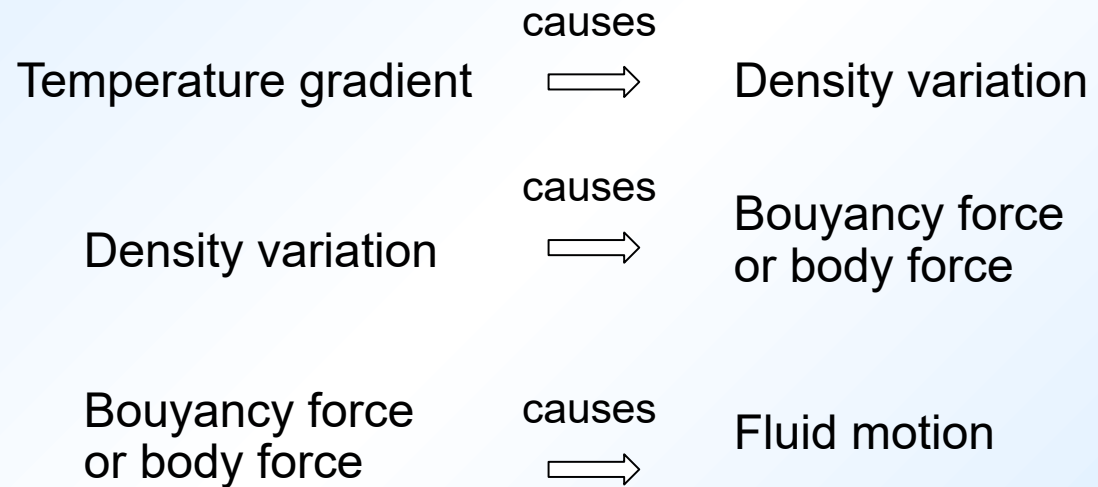
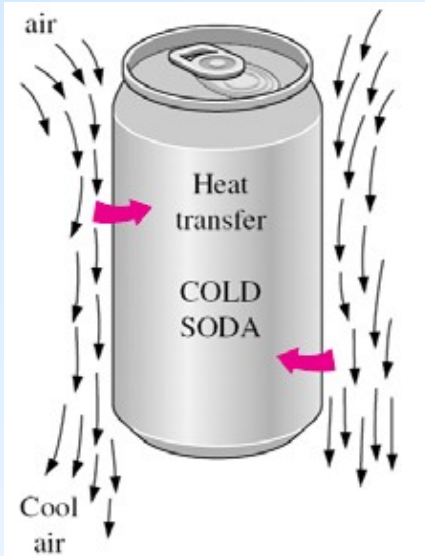
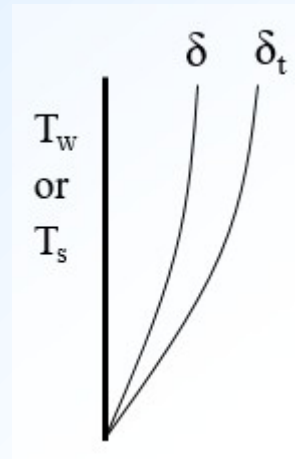
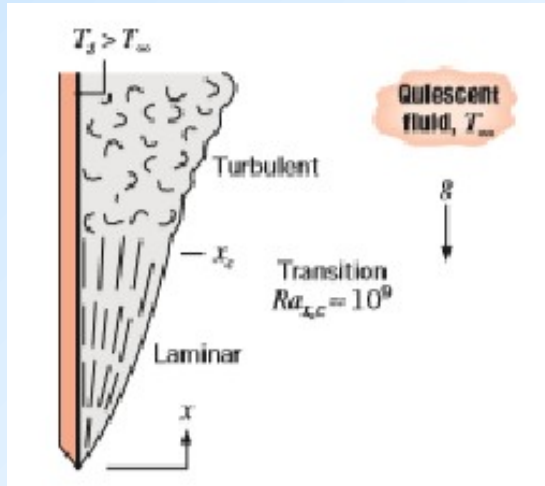


10 - Natural (Free) Convection



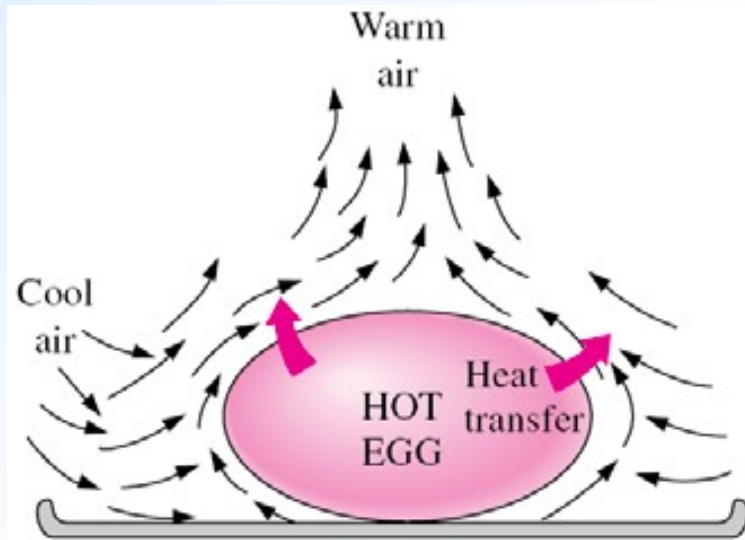
This is called **free** or **natural** convection. The solution is quite complicated.

When $T_w - T_{\text{fluid}}$ is not so large, we may use equations for constant property, incompressible fluid with bouyancy forces when the density change is included in the body force term in the momentum equation.



Buoyancy forces are responsible for the fluid motion in natural convection.

Viscous forces oppose the fluid motion.



Buoyancy forces are expressed in terms of fluid temperature differences through the **volume expansion coefficient**

$$\beta = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P = \frac{-1}{\rho} \left. \frac{\partial \rho}{\partial T} \right|_P$$

Volume Expansion Coefficient:

$$\beta = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P = \frac{-1}{\rho} \left. \frac{\partial \rho}{\partial T} \right|_P$$

The volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx \frac{-1}{\rho} \frac{\Delta \rho}{\Delta T} = \frac{-1}{\rho} \left(\frac{\rho_{\infty} - \rho}{T_{\infty} - T} \right) \quad \text{at constant } P$$

$$\rho_{\infty} - \rho = \rho \beta (T_{\infty} - T) \quad \text{at constant } P$$

For an Ideal gas, ($P V = m R T$)

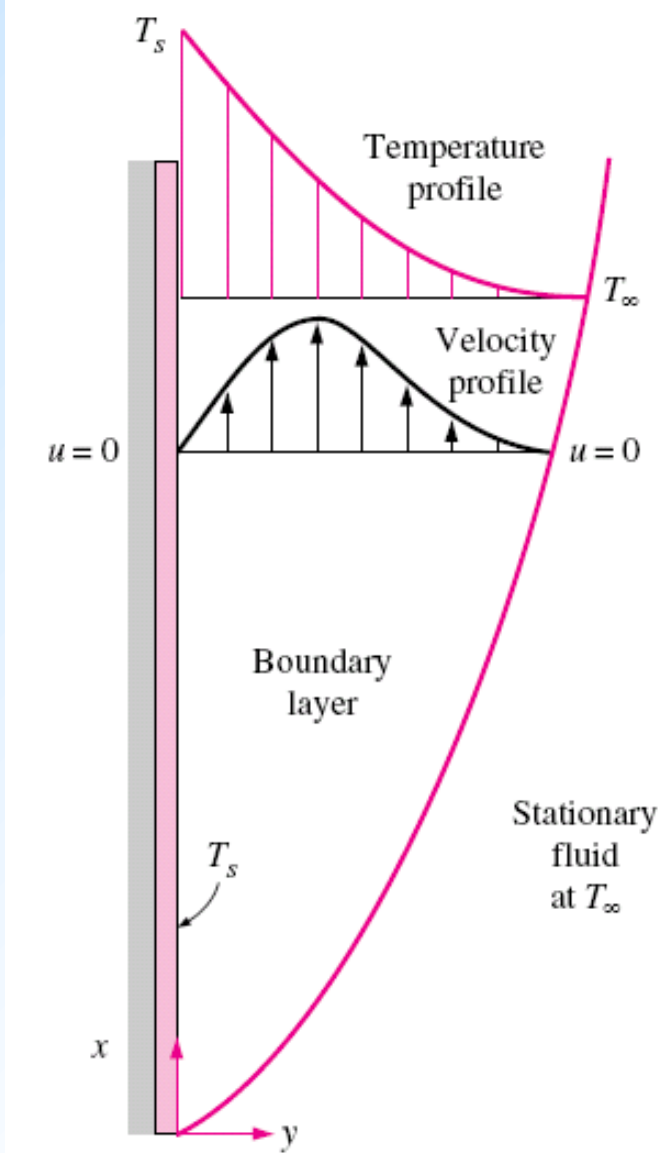
$$\beta_{ideal \text{ gas}} = \frac{1}{T} \quad \text{in } K^{-1}$$

10.1 Boundary Layer Equations

Consider a vertical hot flat plate immersed in a quiescent fluid body.

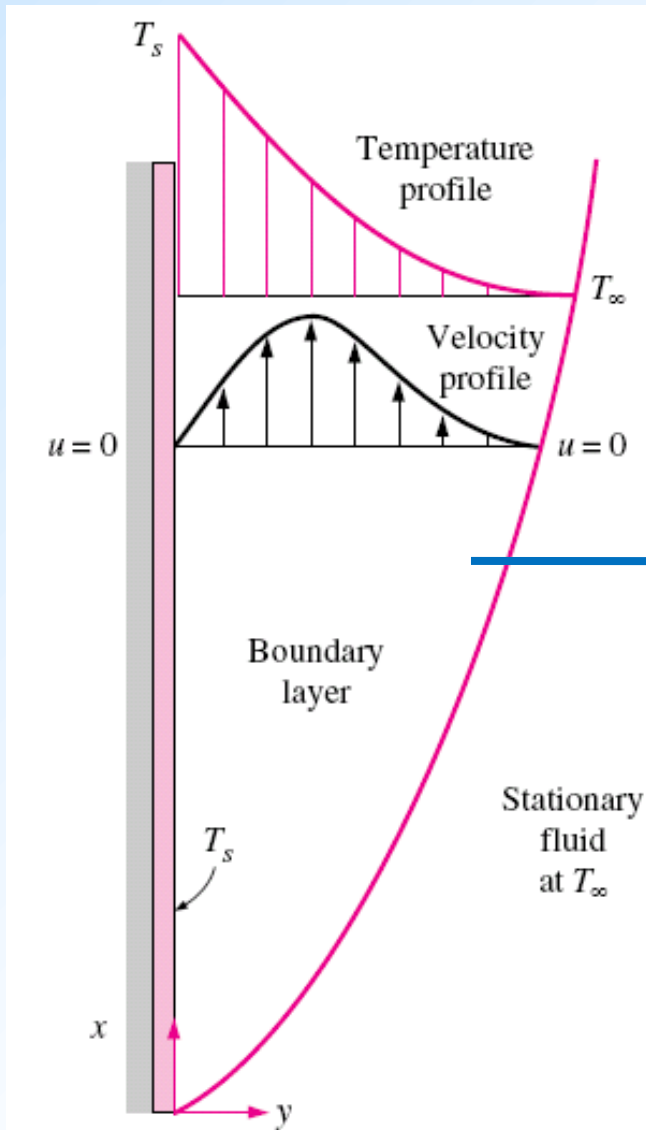
Assumptions:

- steady,
- laminar,
- two-dimensional,
- Newtonian fluid, and
- constant properties, except the density difference ($\rho - \rho_\infty$) (Boussinesq approximation).

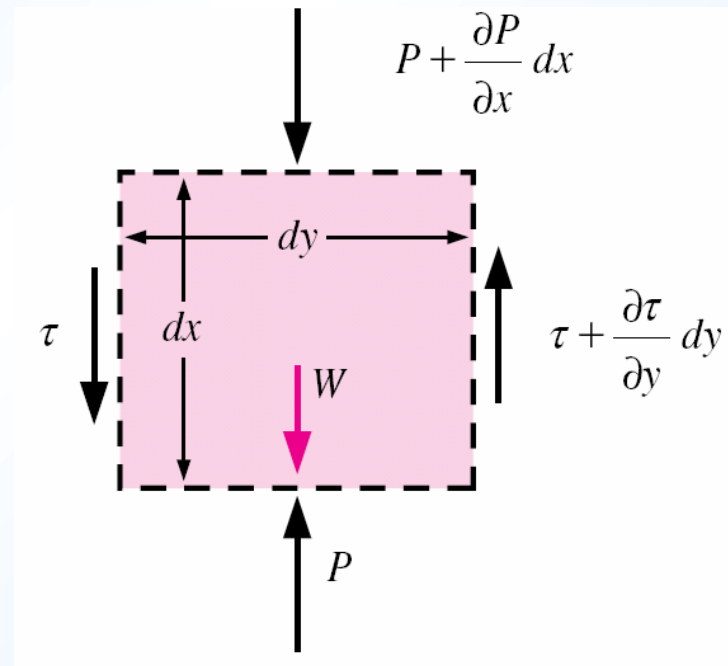




Joseph Valentin Boussinesq
French mathematician and physicist
1842 – 1929



Consider a differential volume element



Apply Newton's second law of motion to the volume element: $F = \text{mass} \times \text{Acc.}$

Newton's second law of motion in the x direction: $F_x = \delta m a_x$

Net viscous force

Net pressure force

Gravitational force

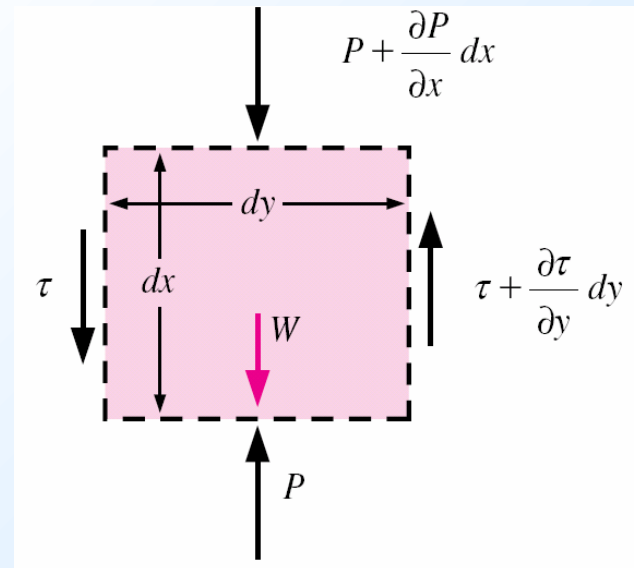
$$F_x = \left(\frac{\partial \tau}{\partial y} dy \right) (dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx \right) (dy \cdot 1) - \rho g (dx \cdot dy \cdot 1)$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \right) (dx \cdot dy \cdot 1)$$

$$\delta m = \rho (dx \cdot dy \cdot 1)$$

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$



Conservation of momentum
in the x direction
in the boundary layer

$$\left. \begin{array}{l} \text{Conservation of momentum} \\ \text{in the x direction} \\ \text{in the boundary layer} \end{array} \right\} \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

?

x momentum equation in the quiescent
fluid outside the boundary layer ($u = 0$)

$$\left. \begin{array}{l} \text{x momentum equation in the quiescent} \\ \text{fluid outside the boundary layer (} u = 0 \text{)} \end{array} \right\} \frac{\partial P}{\partial x} = -\rho_{\infty} g$$

In the boundary layer, $v \ll u$ and

$$\frac{\partial v}{\partial x} \cong \frac{\partial v}{\partial y} \cong 0$$

Force balance in the y direction:

$$\frac{\partial P}{\partial y} = 0 \quad \Rightarrow \quad \frac{\partial P}{\partial x} = \frac{\partial P_{\infty}}{\partial x} = -\rho_{\infty} g$$

Substitute:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_{\infty} - \rho) g$$

x momentum equation :

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho) g$$

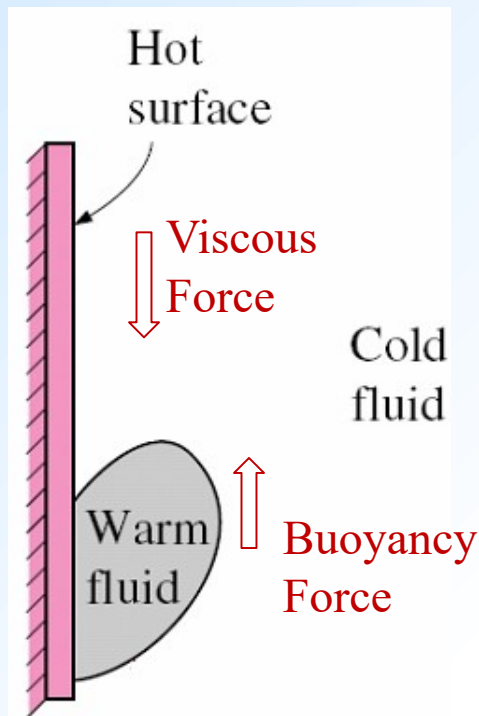
Substitute: $\rho_\infty - \rho = \rho \beta (T_\infty - T)$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty)$$

x momentum equation in non-dimensional form, when: $T^+ = \frac{T - T_\infty}{T_s - T_\infty}$

$$\left(u^+ \frac{\partial u^+}{\partial x^+} + v^+ \frac{\partial u^+}{\partial y^+} \right) = \frac{1}{\text{Re}_L} \frac{\partial^2 u^+}{\partial (y^+)^2} + \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} \frac{T^+}{\text{Re}_L^2}$$

The new non-dimensional parameter represents the natural convection effects, and is called the Grashof number, Gr.



$$Gr_L = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2}$$

$$Gr_L = \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

The flow regime in natural convection is governed by the *Grashof number*

$$Gr_L > 10^9 \quad \text{Flow is turbulent}$$



Franz Grashof
German Engineer
1826 – 1893

$$\frac{Gr_L}{Re^2} \quad \text{Bouyancy force / Inertia force}$$

$$\text{If } \frac{Gr_L}{Re^2} \ll 1 \quad \text{No free convection, no buoyancy}$$

$$\text{If } \frac{Gr_L}{Re^2} \gg 1 \quad \text{Buoyancy dominates}$$

Natural Convection over Surfaces

In free (or natural) convection, buoyancy forces are the only driving force that generates the flow field because there is no external field. Therefore, Re cannot be an independent parameter. Instead, Gr is.

$$Nu = f(Gr, Pr)$$

Define **Rayleigh number**:
$$Ra = Gr Pr = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu \alpha}$$

Sometimes, Ra is used in the relations: $Nu = f(Ra)$

See Table 12-3 for the correlations over various surfaces.



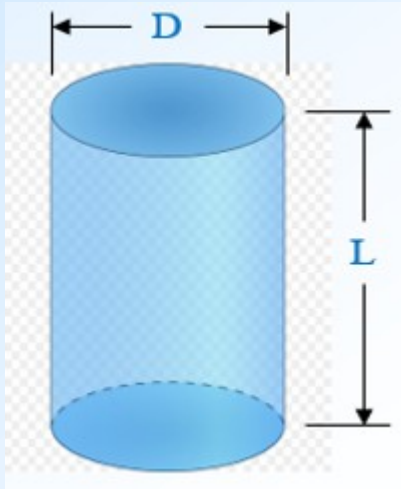
John William Strutt

3rd Baron Rayleigh

British Mathematician and Physicist

1842 – 1919

Example 1



Cylinder in water

$$D = 2.5 \text{ cm}$$

$$L = 0.5 \text{ cm}$$

$$T_{\infty} = 20 \text{ }^{\circ}\text{C}$$

$$T_w = 55 \text{ }^{\circ}\text{C}$$

$$h = ?$$

$$Q = ?$$

Evaluate properties of water
at the mean temperature

$$\begin{aligned}\bar{T} &= \frac{T_w + T_{\infty}}{2} \\ &= \frac{55 + 20}{2} = 37.5 \text{ }^{\circ}\text{C}\end{aligned}$$

Properties of water
at 37.5 °C

$$c_p = 4.174 \text{ kJ/kg.K}$$

$$\rho = 993 \text{ kg/m}^3$$

$$\mu = 6.82 \cdot 10^{-4} \text{ kg/m.s}$$

$$k = 0.63 \text{ W/m.K}$$

$$\text{Pr} = 4.53$$

$$\frac{g \beta \rho^2 c_p}{\mu k} = 3.3 \cdot 10^{10} \text{ m}^{-3} \cdot \text{K}^{-1}$$

$$Gr_L Pr = Ra_L = \frac{g \beta \rho^2 c_p}{\mu k} L^3 (T_w - T_\infty)$$

$$Ra_L = (3.3 \cdot 10^{10}) (0.5)^3 (55 - 20) = 1.03 \cdot 10^{11} > 10^9 \Rightarrow \text{Turbulent}$$

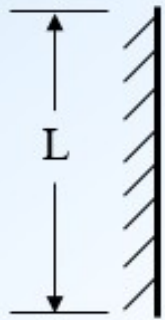
$$\left. \begin{array}{l} \frac{g \beta \rho^2 c_p}{\mu k} = 3.3 \cdot 10^{10} \text{ m}^{-3} \cdot \text{K}^{-1} \\ g = 9.8 \text{ m/s}^2 \end{array} \right\} \beta = 3.5 \cdot 10^{-4} \text{ K}^{-1} \text{ or } ^\circ\text{C}^{-1}$$

$$Nu_{\text{mean}} = \frac{h_{\text{mean}} L}{k} = C (Gr_L Pr)^m \quad \text{Use Table} \Rightarrow C = 0.1 \text{ and } m = 1/3$$

$$Nu_m = (0.1) (1.03 \cdot 10^{11})^{1/3} = 46.875 \quad h_m = \frac{0.63}{0.5} (46.875) = 59.06 \text{ W/m}^2 \cdot \text{K}$$

$$Q = h_m A (T_w - T_\infty) = (59.06) \pi (0.025) (0.5) (55 - 20) = 58.51 \text{ W}$$

Example 2



Air
 $P = 1 \text{ atm}$
 $T_w = 65 \text{ }^\circ\text{C}$
 $T_\infty = 10 \text{ }^\circ\text{C}$
 $L = 0.3 \text{ m}$

$h = ?$

$Q = ?$

Evaluate properties of air
 at the mean temperature

$$\bar{T} = \frac{T_w + T_\infty}{2} = \frac{65 + 10}{2} = 37.5 \text{ }^\circ\text{C} = 310.5 \text{ K}$$

Properties of air
 at 310.5 K

$$c_p = 1.006 \text{ kJ/kg.K}$$

$$k = 0.027 \text{ W/m.K}$$

$$\rho = 1.18 \text{ kg/m}^3$$

$$\text{Pr} = 0.707$$

$$\mu = 2.0 \cdot 10^{-5} \text{ kg/m.s}$$

$$\alpha = 0.23 \cdot 10^{-4} \text{ m}^2/\text{s}$$

$$\nu = 15.7 \cdot 10^{-6} \text{ m}^2/\text{s}$$

$$Gr_L = \frac{g \beta (T_w - T_\infty) L^3}{\nu^2} = \frac{(9.8) \frac{1}{283} (65 - 10) (0.3)^3}{(15.7 \cdot 10^{-6})^2} = 2.086 \cdot 10^8$$

$$Gr_L Pr = (2.086 \cdot 10^8) (0.707) = 1.475 \cdot 10^8 < 10^9 \quad \text{Flow is Laminar}$$

$$\left. \begin{array}{l} \text{Use Table} \\ \text{for } Ra \approx 1.5 \cdot 10^8 \end{array} \right\} \left. \begin{array}{l} C = 0.59 \\ m = 1/4 \end{array} \right\} \bar{Nu} = \frac{\bar{h} L}{k} = (0.59) (Ra)^{1/4} \\ = (0.59) (1.5 \cdot 10^8)^{1/4} = 65.02$$

$$\bar{h} = \frac{k}{L} \bar{Nu} = \frac{0.027}{0.3} (65.02) = 5.85 \text{ W/m}^2 \cdot \text{K}$$

Alternatively, we may use Table 7.2 to find h .



11 – Boiling and Condensation

