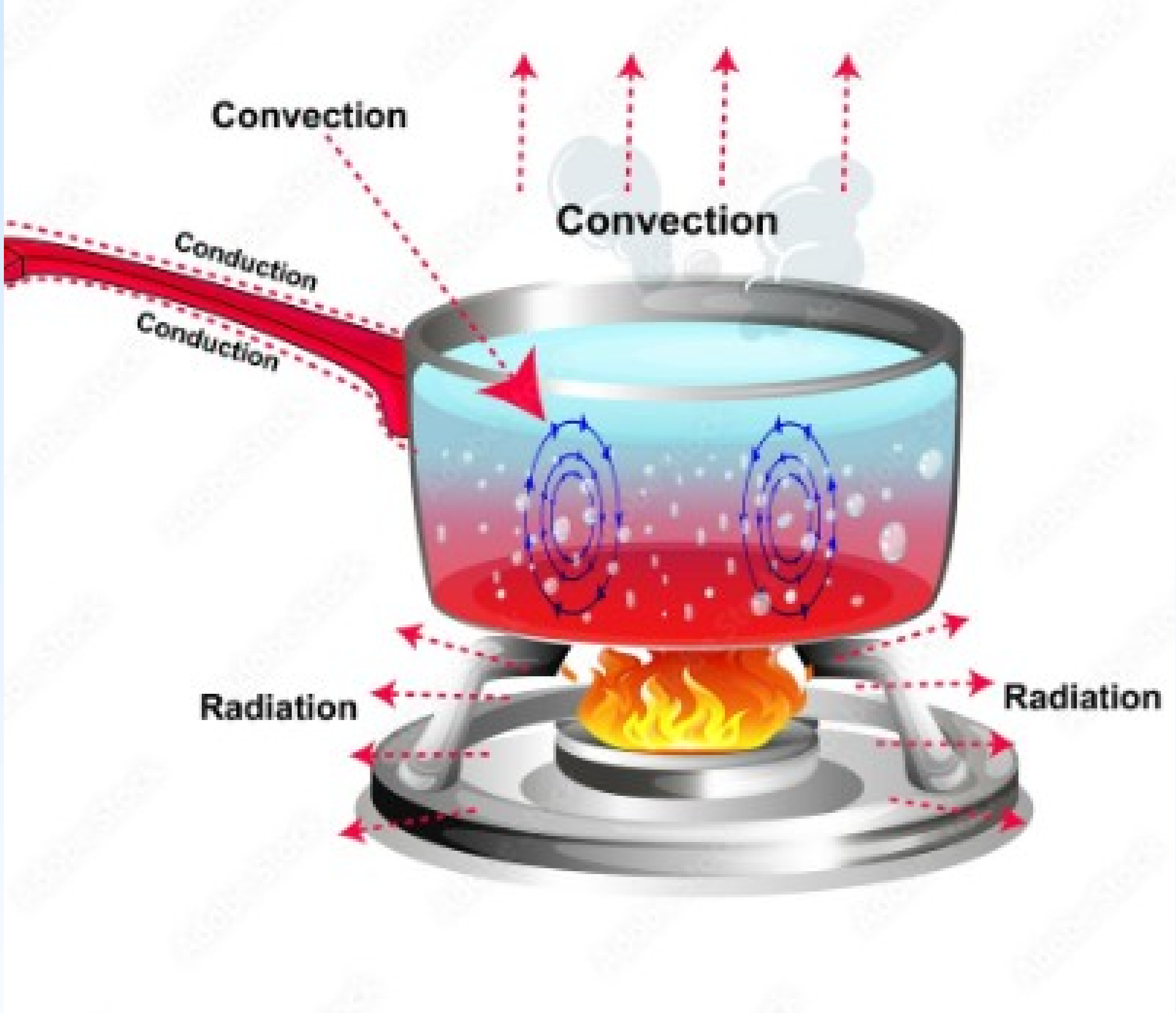
CONVECTION HEAT TRANSFER



Distingush between these two concepts:

- Convection within a fluid (not close to any solid boundary surface).
- Convection between a solid surface and a fluid adjacent to it.

7. Convection Heat Transfer - Basic Concepts and Equations

The mechanism of heat transfer between a fluid and a solid surface at different temperatures and in relative motion is typically called convection.

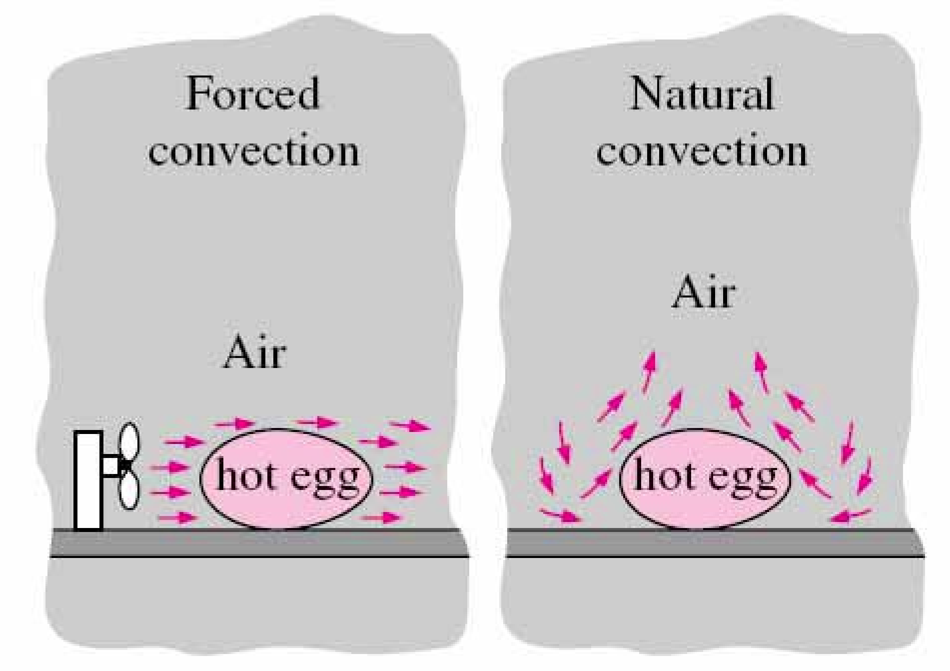
Convection = Conduction + Advection (fluid motion)

Convection is commonly classified into three sub-modes:

– Forced convection,

– Natural (or free) convection,

– Change of phase (liquid/vapor, solid/liquid, etc.)



Few comments:

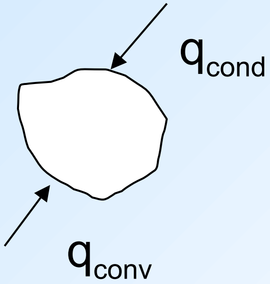
Conduction and convection are similar in that both mechanisms require the presence of a material medium (not true for radiation).

But, they are different in that convection requires the presence of a fluid in motion.

Heat transfer through a liquid or gas can be by conduction or convection, depending on the presence of any bulk fluid motion.

The fluid motion plays an important part, enhancing heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid.

Look at a chunk of fluid in motion (away from a wall):



Heat flux by conduction: $\dot{q}_{cond} = - k_{fluid} \left.\frac{\partial T_{fluid}}{\partial x}\right|_{\substack{pure \\ conduction}}$

Heat flux by convection: $\dot{q}_{conv} = \frac{\dot{m}}{A} c_p \Delta T_{fluid}$

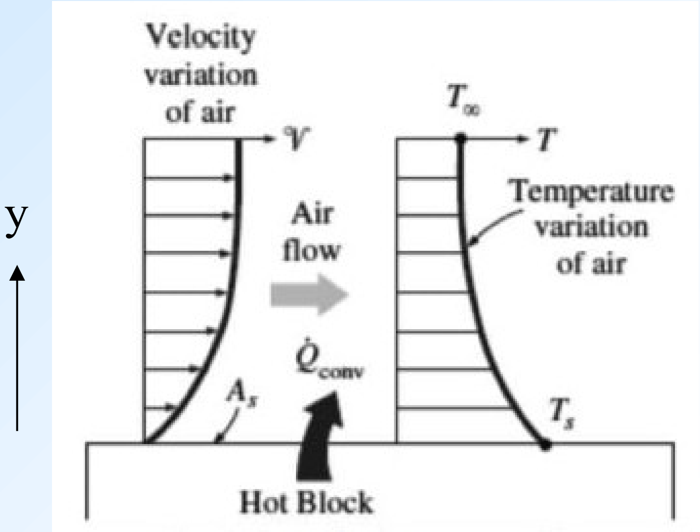
The total is: $\dot{q} = \dot{q}_{cond} + \dot{q}_{conv}$

m is the mass flow rate and $\frac{\dot{m}}{A} = \rho V$ where V is the velocity of the fluid

Therefore, the heat flow rate is always dependent on fluid velocity (and other properties of the fluid).

In order to find the temperature field (or distribution) in a fluid, the motion of the fluid that is the velocity field (or distribution) should be known.

The rate of convection heat transfer is observed to be proportional to the temperature difference and is expressed by **Newton's Law of Cooling** as



Heat transfer by convection

$$\dot{q} = h\left(T_s - T_f\right) = -k_{fluid}\left.\frac{\partial T_{fluid}}{\partial y}\right|_{y=0}$$

$\dot{q}$: Heat flux at the wall, in W

T_s: Surface temperature of the wall, in ˚C

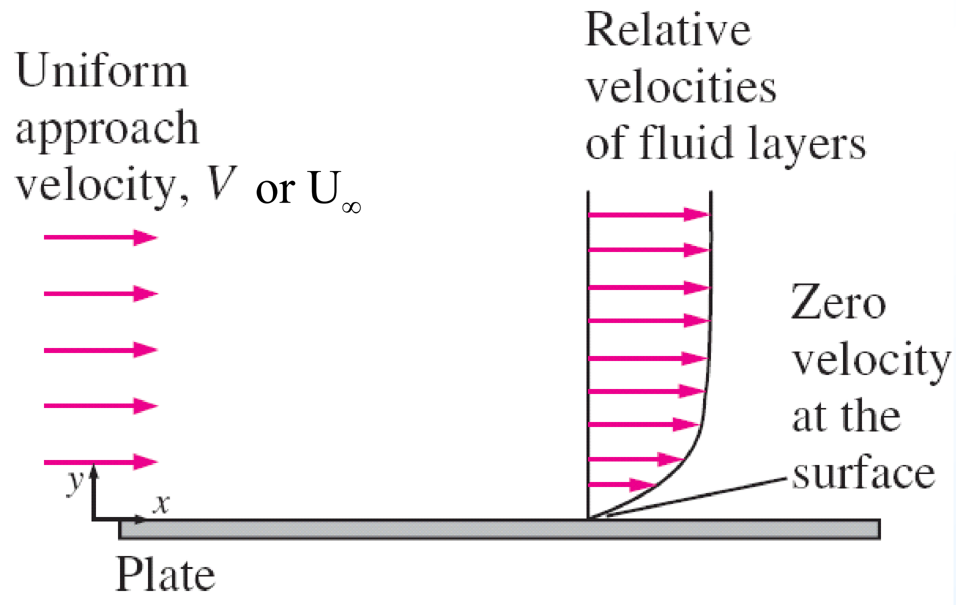
T_f or T_∞: Mean temperature of the fluid, in ˚C

h: Convective Heat Transfer Coefficient, in $W/m^2.˚C$

The convection heat transfer coefficient, ***h***, depends on fluid properties such as dynamic viscosity μ, thermal conductivity $\boldsymbol{k_f}$, density $\boldsymbol{\rho_f}$, specific heat $\boldsymbol{c_p}$, and fluid velocity ***V***, as well as geometry and surface roughness, and thus is very difficult to determine.

Type of Flow	h , W/m^2.°C (or K)
Free convection of gases	2 – 25
Free convection of liquids	10 – 1000
Turbulent forced convection of air (gases) inside pipes	25 – 250
Turbulent forced convection of water inside pipes	50 – 20 000
Boiling of water	2 500 – 60 000
Condensation of steam	5 000 – 100 000

All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface (**no-slip condition**).



The **no-slip** condition is responsible for the development of the **velocity profile**.

The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**.

An implication of the **no-slip condition** is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by ***pure conduction***, and can be expressed as

$$\dot{q}_{conv} = \dot{q}_{cond} = - k_{fluid} \left. \frac{\partial T_{fluid}}{\partial y} \right|_{y=0} \quad \text{in W/m}^2$$

Equate this to the Newton's law of cooling: $\dot{q}_{conv} = h \left(T_s - T_\infty \right)$

$$h = \frac{- k_{fluid} \left. \dfrac{\partial T_{fluid}}{\partial y} \right|_{y=0}}{T_s - T_\infty} \quad \text{in W/m}^2\text{.K}$$

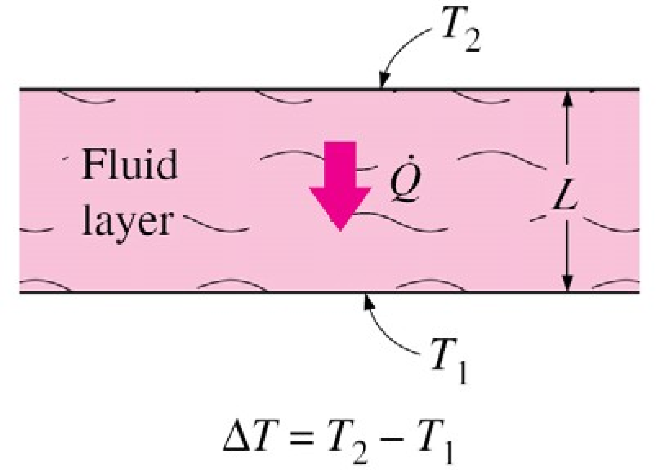
The convection **heat transfer coefficient**, in general, **varies along the flow direction**.

The Nusselt Number

It is common practice to non-dimensionalize the convective heat transfer coefficient, *h,* with the Nusselt number:

$$Nu = \frac{h\,L_c}{k_{fluid}}$$

Characteristic length



Heat flux through the fluid layer by *convection* and by *conduction* can be expressed as, respectively:

$$\dot{q}_{conv} = h\,\Delta T \qquad \dot{q}_{cond} = k_f\,\frac{\Delta T}{L_c}$$

Take the ratio: $$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\,\Delta T}{k_f\,(\Delta T/L_c)} = \frac{h\,L_c}{k_f} = Nu$$

Ernst Kraft Wilhelm Nußelt

German Engineer

1882 - 1957

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\ \Delta T}{k_f\ (\Delta T/L_c)} = \frac{h\ L_c}{k_f} = Nu$$

The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

$Nu = 1$ implies pure conduction

Remember definitions:

h is the convective heat transfer coefficient in $W/m^2.K$

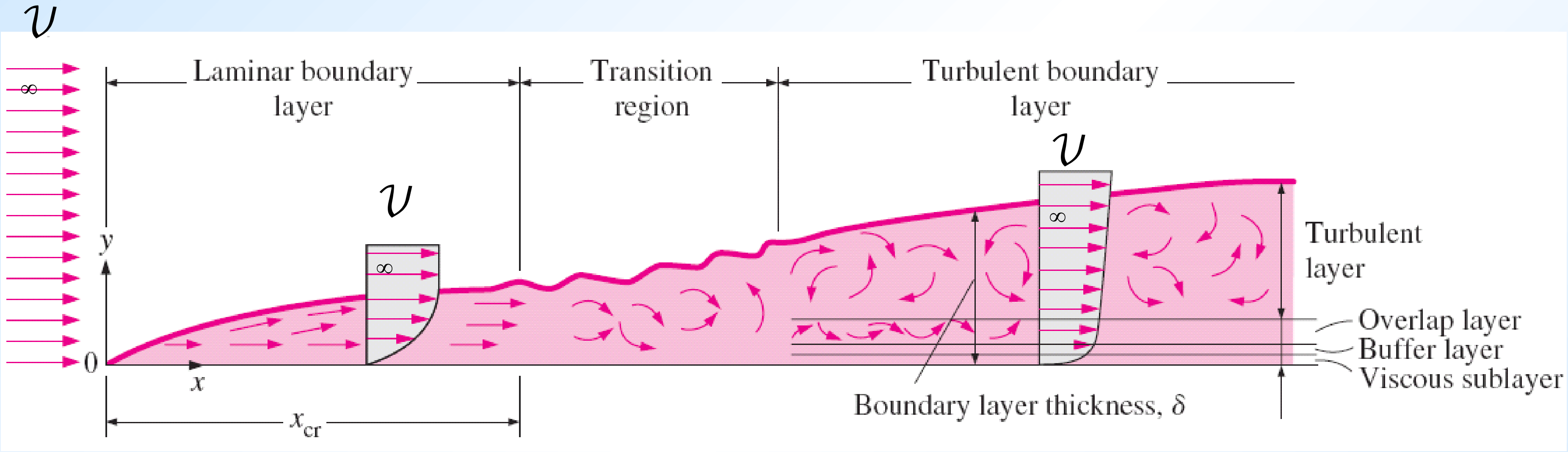
L_c is the characteristic length, adjacent to the solid surface, in which heat flow occurs (boundary layer thickness? What boundary layer?)

k_f is the thermal conductivity of the fluid

Remember Classification of Fluid Flows

- Viscous versus inviscid regions of flow (viscosity)
- Internal versus external flow (type of flow area)
- Compressible versus incompressible flow (density)
- Laminar versus turbulent flow (mixing)
- Natural (or unforced) versus forced flow (gravity)
- Steady versus unsteady flow (time)
- One-, two-, and three-dimensional flows

Remember development of velocity boundary layer (external flow)

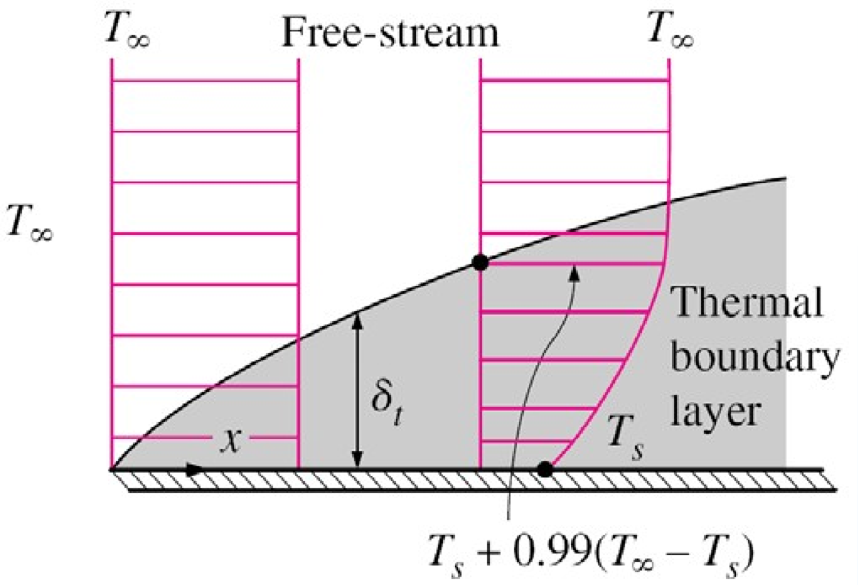


The velocity profile in turbulent flow is much fuller than that in laminar flow (a sharp drop near the surface).

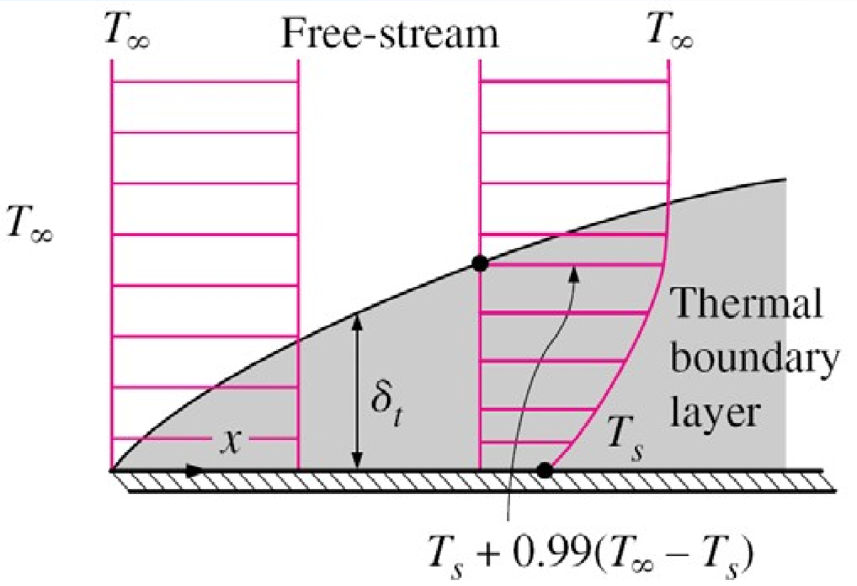
The *intense mixing in turbulent flow enhances heat and momentum* transfer, which increases the friction force on the surface and the convection heat transfer rate.

Thermal boundary layer

Like the velocity, a *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature.



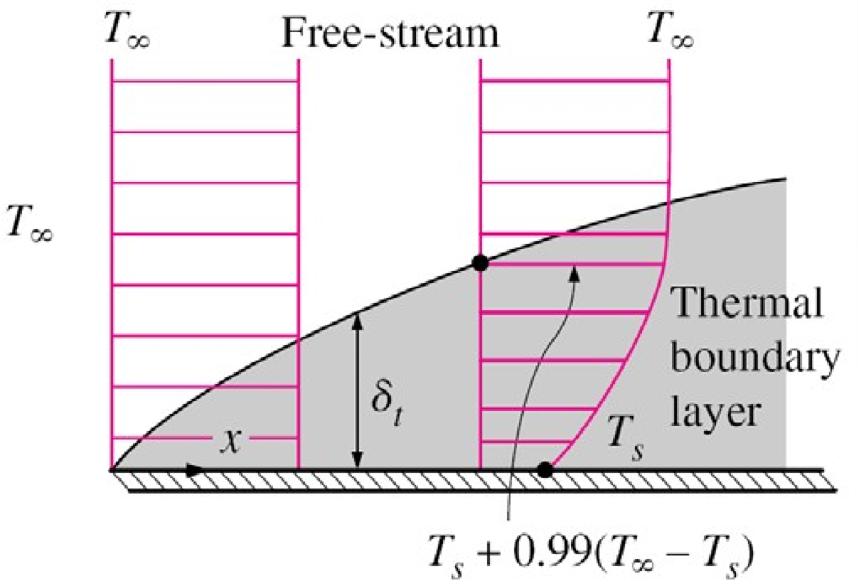
Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s.



The fluid particles in the layer adjacent to the boundary assume the surface temperature T_s.

A temperature profile develops that ranges from T_s at the surface to T_∞ sufficiently far from the surface.

The **thermal boundary layer** is the flow region over the surface in which the temperature variation in the direction normal to the surface is significant.



The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference*

$$T(y = \delta_t) - T_s = 0.99\ (T_\infty - T_s).$$

The thickness of the thermal boundary layer increases in the flow direction.

The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location.

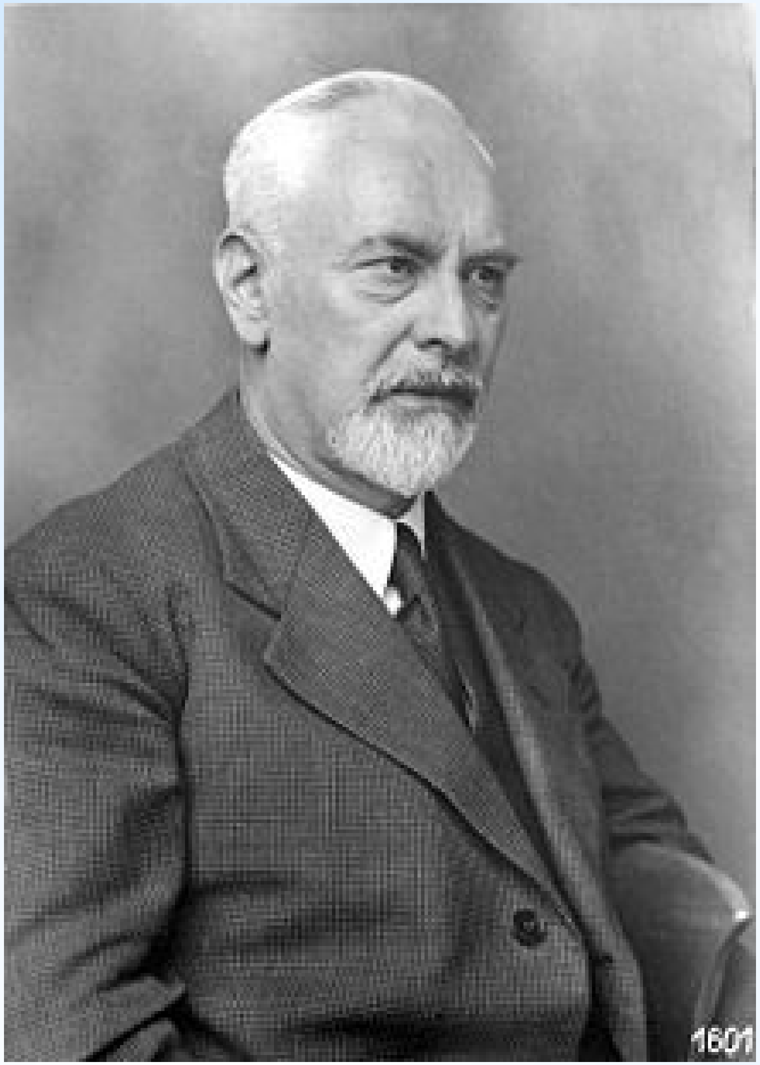
The Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number**, defined as

$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu\ c_p}{k}$$

Heat diffuses very quickly in liquid metals (Pr « 1) and very slowly in oils (Pr » 1) relative to momentum.

Consequently **the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer**.

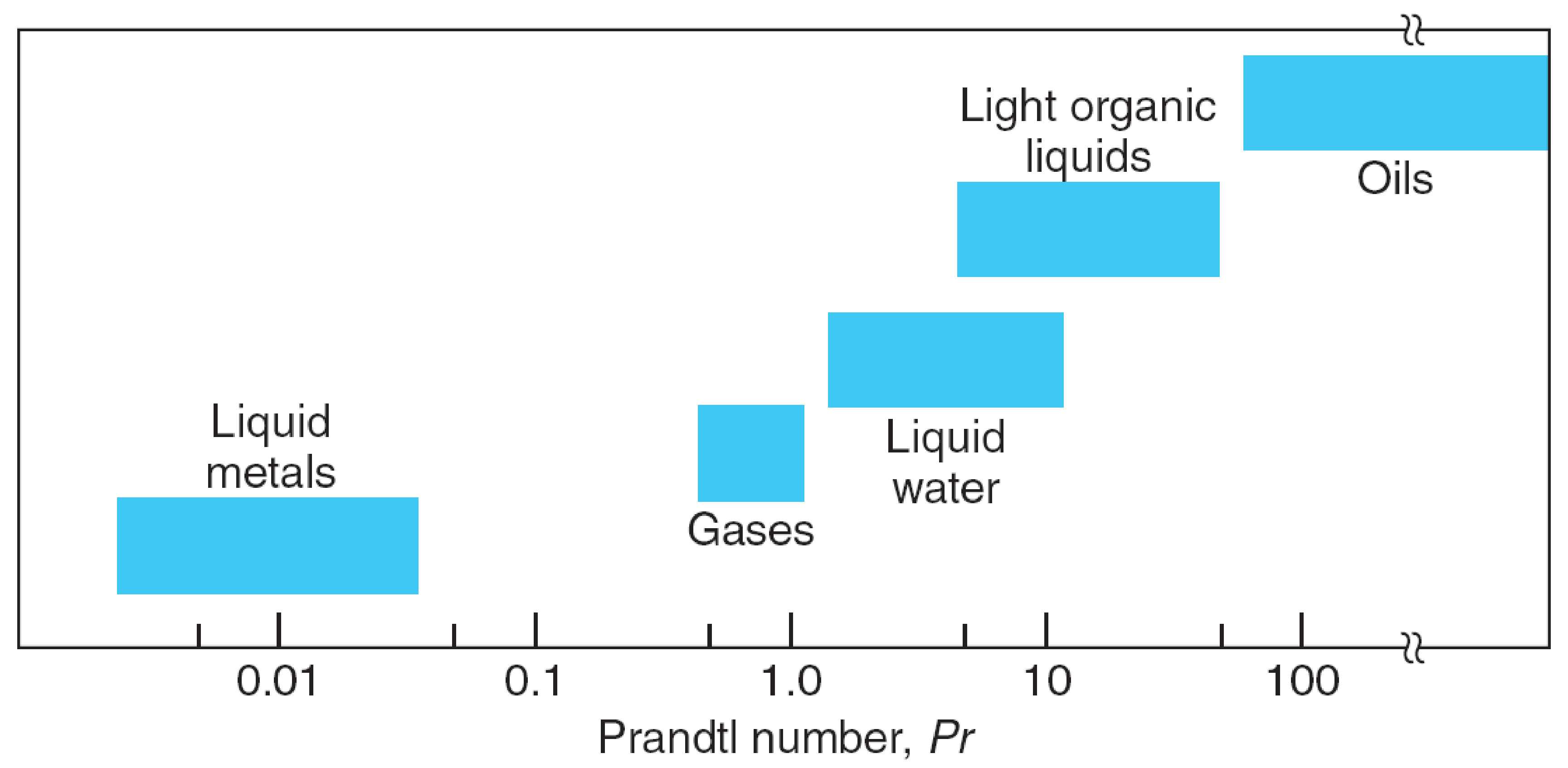


Ludwig Prandtl

German Scientist

1875 - 1953

Typical Prandtl Numbers for common fluids:



Remember the Reynolds Number

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid.*

The flow regime depends mainly on the ratio of the *inertia forces to viscous forces in the fluid.*

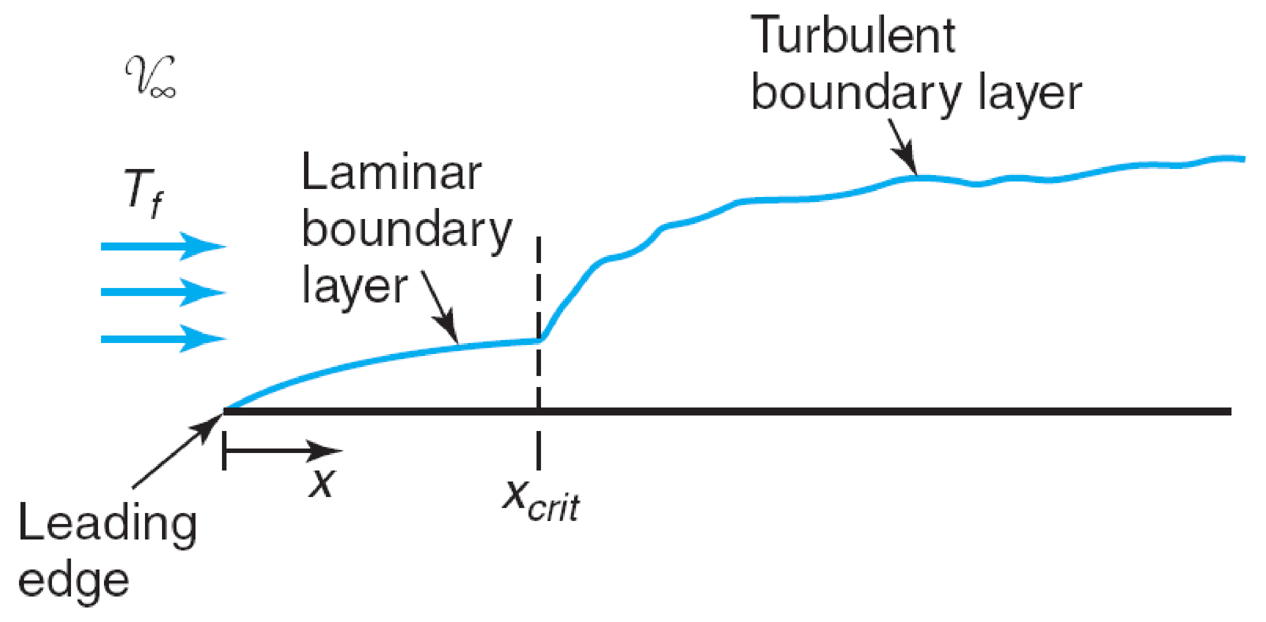
- $$Re = \frac{Inertia\ forces}{Viscous\ forces} = \frac{\rho\, \mathcal{V}\, L_c}{\mu} = \frac{\mathcal{V}\, L_c}{\nu}$$

ρ: Density of the fluid, kg/m^3

$\mathcal{V}$: Velocity of the fluid, m/s

μ: Dynamic viscosity of the fluid: N.s/m^2

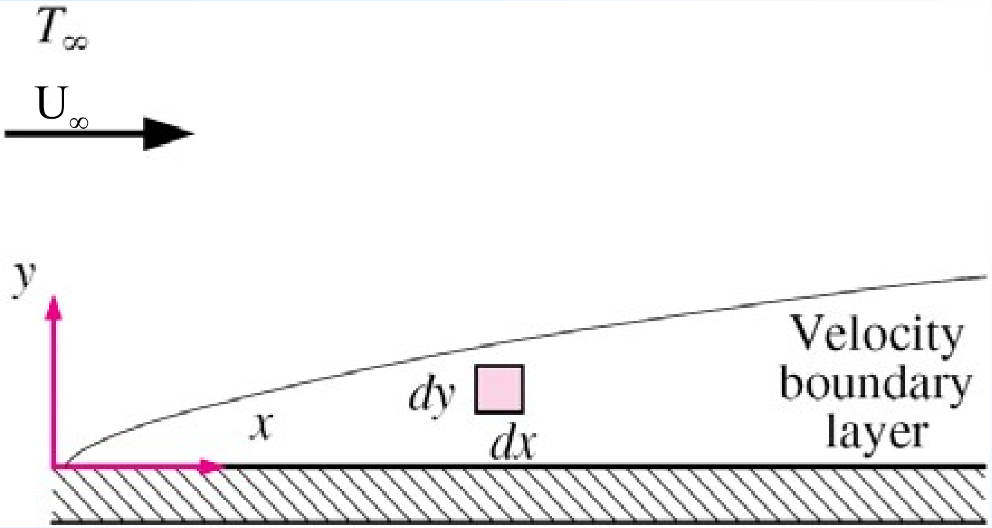
L_c: Characteristic length (of the conduit), m



- $$Re_{\text{crit}} = \frac{\rho\, \mathcal{V}\, x_{\text{crit}}}{\mu} = 5\ 10^5$$

Differential Equations of Convective Heat Transfer

Consider the parallel flow of a fluid over a surface.



Assumptions:

- laminar flow;
- steady two-dimensional flow;
- Newtonian fluid; and
- constant properties.

The fluid flows over the surface with a uniform free-stream velocity U_∞, *but the velocity within boundary* layer is two-dimensional [$U=U(x,y)$, $V=V(x,y)$].

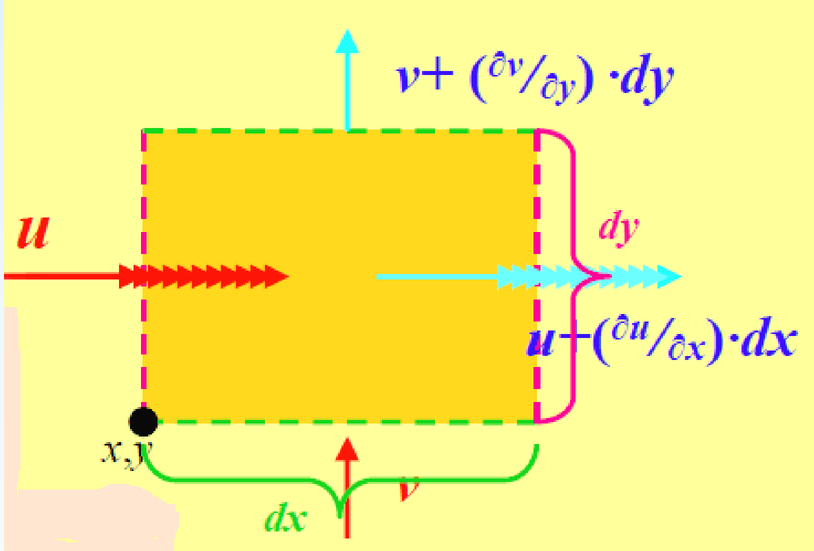
THREE FUNDAMENTAL LAWS:

- Conservation of mass => Continuity Equation
- Conservation of momentum => Momentum Equation
- Conservation of energy => Energy Equation

Conservation of mass: $\begin{pmatrix}\text{Rate of mass flow}\\ \text{into control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow}\\ \text{out of control volume}\end{pmatrix}$

$$\rho\, u\,(dy\;.\;1) + \rho\, v\,(dx\;.\;1) \;=$$

$$\rho\left(u + \frac{\partial u}{\partial x}\,dx\right)(dy\;.\;1) + \rho\left(v + \frac{\partial v}{\partial y}\,dy\right)(dx\;.\;1)$$



Simplify and divide by dy dx

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Continuity equation

Conservation of momentum:

The differential forms of the equations of motion in the velocity boundary layer are obtained by applying Newton's second law of motion to a differential control volume element in the boundary layer.

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a given direction}\end{pmatrix} = \begin{pmatrix}\text{Net force acting}\\ \text{in that direction}\end{pmatrix}$$

There are two types of forces: Body Forces and Surface Forces

$$\delta m\, a_x = F_{surface,x} + F_{body,x}$$

mass in the control vol.

due to pressure and viscosity

due to gravity magnetism, etc

$$\delta m \, a_x = F_{surface,x} + F_{body,x}$$

Mass in the control volume: $\delta m = \rho \left(dx \, . \, dy \, . \, 1 \right)$

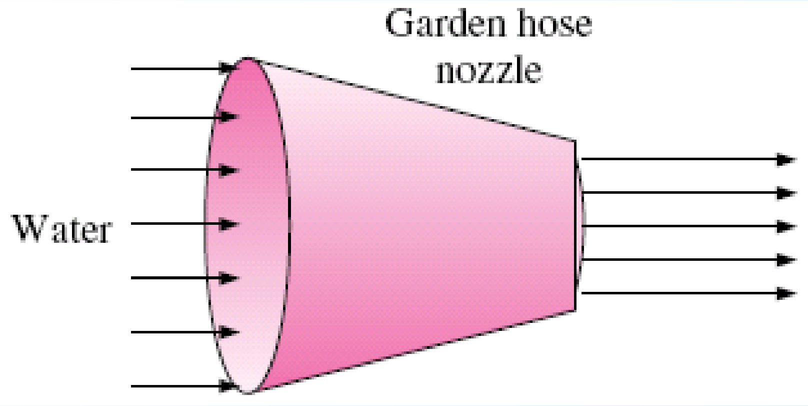
Acceleration of the mass in the control volume:

$$u(x,y)$$

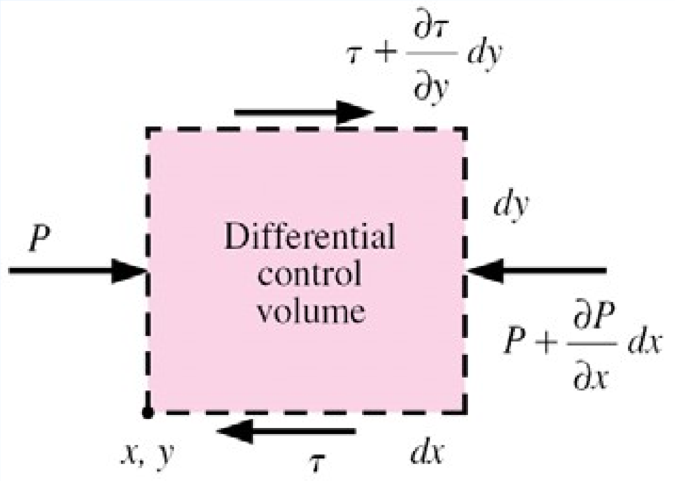
$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$



Forces acting on a surface due to pressure and viscous effects :



$$F_{surface,x} = \left(\frac{\partial \tau}{\partial y} dy\right)(dx \,.\, 1) + \left(\frac{\partial \tau}{\partial x} dx\right)(dy \,.\, 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \,.\, 1)$$

$$= \left(\frac{\partial \tau}{\partial y} + \frac{\partial \tau}{\partial x} - \frac{\partial P}{\partial x}\right)(dx \,.\, dy.\, 1)$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 u}{\partial x^2} - \frac{\partial P}{\partial x}\right)(dx \,.\, dy.\, 1)$$

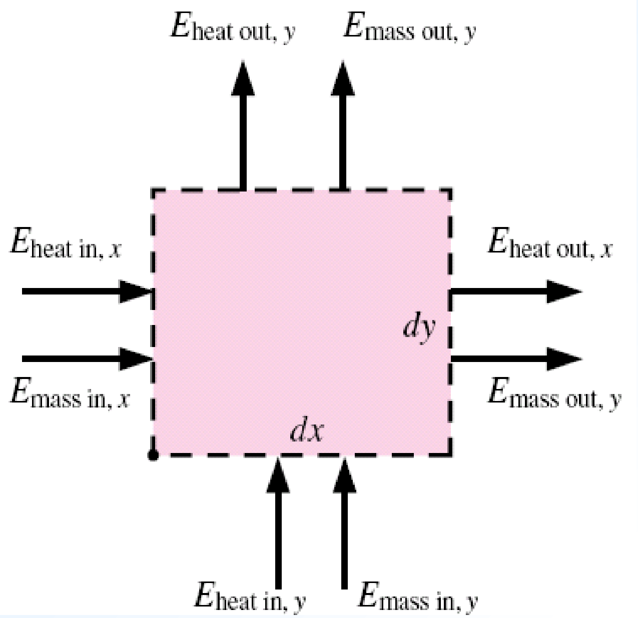
Momentum equation in the x-direction:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=F_{body,x}-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}\right)$$

Momentum equation in the y-direction:

$$\underbrace{\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)}_{\text{Inertia term}}=\underbrace{F_{body,y}}_{\text{Body force}}-\underbrace{\frac{\partial P}{\partial y}}_{\text{Pressure force}}+\underbrace{\mu\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}\right)}_{\text{Viscous force}}$$

Conservation of energy



$$\rho\, c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu\, \phi$$

Energy flow
due to mass transfer

Energy flow
due to heat transfer

Viscous dissipation term

$$\phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2$$

Summary: Two-dimensional, steady, incompressible flow with constant properties

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = F_{body,x} - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = F_{body,y} - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right)$$

Energy:

$$\rho\, c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu\, \phi$$

Define total derivative:

$$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\left(+\cancel{w\frac{\partial}{\partial z}} + \cancel{\frac{\partial}{\partial t}}\right)$$

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum:

$$\rho\frac{Du}{Dt} = F_{body,x} - \frac{\partial P}{\partial x} + \mu\nabla^2 u$$

$$\rho\frac{Dv}{Dt} = F_{body,y} - \frac{\partial P}{\partial y} + \mu\nabla^2 v$$

Energy:

$$\rho c_p\frac{DT}{Dt} = k\nabla^2 T + \mu\phi$$

Non-dimensional parameters

Define:

$x^+ = \frac{x}{L}$ and $y^+ = \frac{y}{L}$ where L is some characteristic length

$u^+ = \frac{u}{U_\infty}$ and $v^+ = \frac{v}{U_\infty}$ where U_∞ is some characteristic velocity

$$P^+ = \frac{P}{\rho U_\infty^2}$$

$T^+ = \frac{T - T_\infty}{\Delta T}$ where T_∞ is some characteristic temperature and

ΔT is a characteristic (reference) temperature difference

The differential equations become (neglecting body forces):

Continuity:

$$\frac{\partial u^+}{\partial x^+} + \frac{\partial v^+}{\partial y^+} = 0$$

Momentum:

$$\frac{Du^+}{Dt^+} = -\frac{\partial P^+}{\partial x^+} + \overbrace{\boxed{\frac{\mu}{\rho\, U_\infty\, L}}}^{\boxed{\frac{1}{Re}}} \nabla^2 u^+$$

$$\frac{Dv^+}{Dt^+} = -\frac{\partial P^+}{\partial y^+} + \overbrace{\boxed{\frac{\mu}{\rho\, U_\infty\, L}}}^{\boxed{\frac{1}{Re}}} \nabla^2 v^+$$

Energy:

$$\frac{DT^+}{Dt^+} = \underbrace{\boxed{\frac{\nu}{L\, U_\infty}\ \frac{\alpha}{\nu}}}_{\boxed{\frac{1}{Pe} = \frac{1}{Re\ Pr}}} \nabla^2 T + \overbrace{\boxed{\frac{\mu}{\rho\, c_p}\ \frac{U_\infty}{L\, \Delta T}}}^{\boxed{\frac{E}{Re} = \frac{U_\infty^2/(c_p\, \Delta T)}{(L\, U_\infty)/\nu}}} \phi^+$$

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous force}} = \frac{U_\infty \, L_c}{\nu} = \frac{\rho \, U_\infty \, L_c}{\mu}$$

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu \, c_p}{k}$$

Peclet number: $$\mathrm{Pe} = \mathrm{Re} \; \mathrm{Pr} = \frac{\text{Thermal energy transported by convection}}{\text{Thermal energy transported by conduction}}$$

Eckert number: $$E = \frac{U_\infty^2 \, / \, c_p}{\Delta T} = \frac{\text{Dynamic temperature due to fluid motion}}{\text{Characteristic temperature difference}}$$

Nusselt number: $$\mathrm{Nu} = \frac{h \, L_c}{k} = \left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=0} = f(\mathrm{Re}, \mathrm{Pr}, E)$$

Physical significance

If Re is very large => Viscous forces are negligible

If Re is very small => Viscous forces dominate

If Pe is very large => Conduction is negligible besides convection

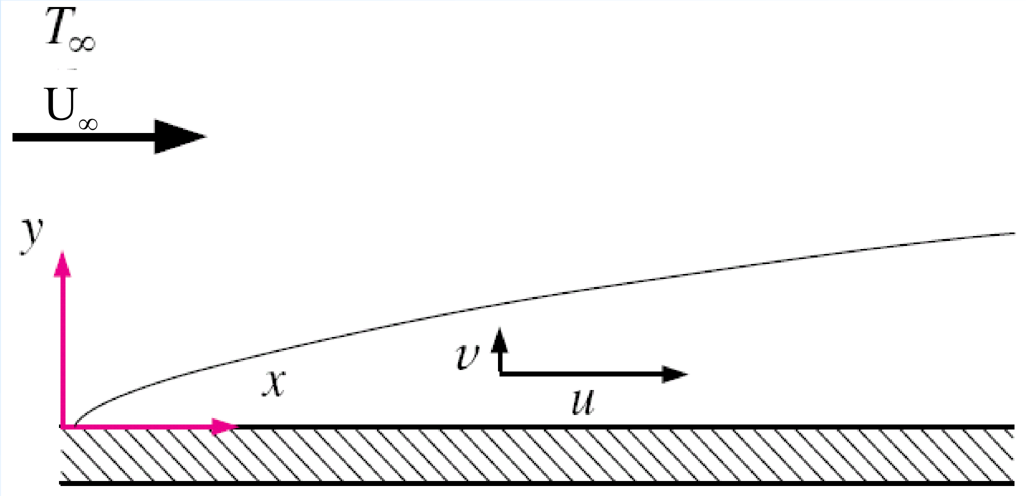
If Pe is very small => Conduction dominates

$Pr >> 1$ for oils => Diffusivity of momentum dominates

$Pr \approx 1$ for gases => Heat and momentum diffusivities are comparable

$Pr << 1$ for liquid metals => Diffusivity of heat dominates

Boundary layer simplifications (approximation): in the boundary layer



$$u >> v$$

$$\frac{\partial v}{\partial x} \approx 0 \qquad \frac{\partial v}{\partial y} \approx 0$$

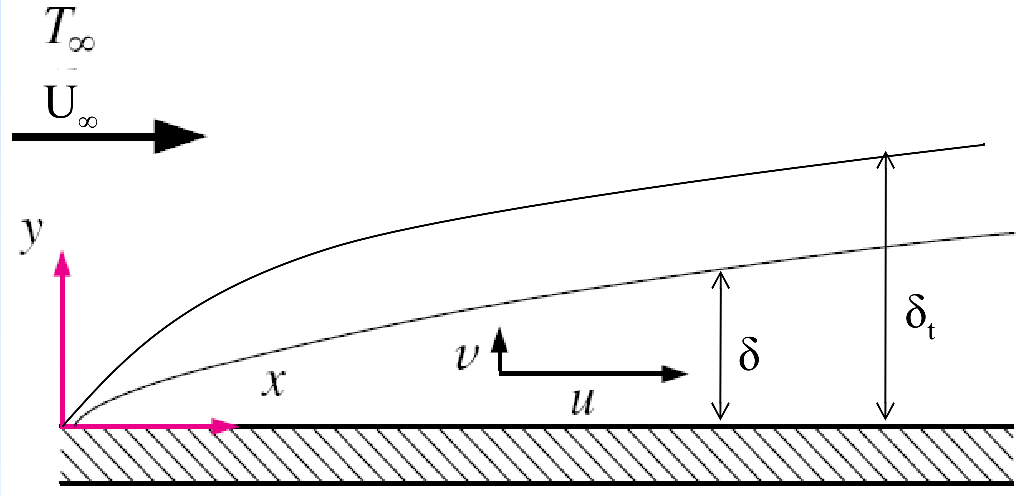
$$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}$$

$$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$$

When gravity and other body forces are neglected

Momentum equation in the y-direction becomes:

$$\frac{\partial P}{\partial y} = 0$$



δ(x) : Thickness of velocity boundary layer

δ_t(x): Thickness of thermal boundary layer

δ and δ_t are of the same order of magnitude but not necessarily the same

If Pr > 1 $\delta(x) > \delta_t(x)$

If Pr = 1 $\delta(x) = \delta_t(x)$

If Pr < 1 $\delta(x) < \delta_t(x)$

$$\frac{\delta_t}{\delta} \propto \frac{1}{\sqrt{\Pr}} \quad \text{and} \quad \frac{\delta}{L} \propto \frac{1}{\sqrt{\mathrm{Re}}}$$

The differential equations with boundary layer simplifications become (neglecting body forces):

Continuity:

$$\frac{\partial u^+}{\partial x^+} + \frac{\partial v^+}{\partial y^+} = 0$$

Momentum:

$$\frac{Du^+}{Dt^+} = - \frac{\partial P^+}{\partial x^+} + \frac{1}{Re} \nabla^2 u^+$$

Energy:

$$\frac{DT^+}{Dt^+} = \frac{1}{Re\ Pr} \nabla^2 T^+ + \frac{E}{Re} \phi^+$$

Analogy between heat transfer and momentum transfer

Under certain conditions, there is a relation between coefficient of friction C_f and Nusselt number, Nu. One may be used to obtain the other.

x - Momentum:

$$\frac{Du^+}{Dt^+} = - \frac{\partial P^+}{\partial x^+} + \frac{1}{Re} \nabla^2 u^+$$

Energy:

$$\frac{DT^+}{Dt^+} = \frac{1}{Re\ Pr} \nabla^2 T^+ + \frac{E}{Re} \phi^+$$

If $dP^+/dx^+ = 0$, $Pr = 1$, and $E = 0$, these two equations look alike (have the same form) and the boundary conditions are the same, indicating that the functions for u^+ and T^+ must be identical. Hence:

$$\left.\frac{\partial u^+}{\partial y^+}\right|_{y=0} = \left.\frac{\partial T^+}{\partial y^+}\right|_{y=0}$$

Analogy between heat transfer and momentum transfer

$$\left.\frac{\partial u^+}{\partial y^+}\right|_{y=0} = \left.\frac{\partial T^+}{\partial y^+}\right|_{y=0} = \mathrm{Nu}$$

Shear stress at the surface: a $\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu\, U_\infty}{L} \left.\frac{\partial u^+}{\partial y^+}\right|_{y=0}$

Coefficient of friction: $C_f = \frac{\tau_s}{\rho \frac{U_\infty^2}{2}} = \frac{2\,\mu}{\rho\, L\, U_\infty} \left.\frac{\partial u^+}{\partial y^+}\right|_{y=0} = \frac{2}{\mathrm{Re}} \left.\frac{\partial u^+}{\partial y^+}\right|_{y=0}$

Therefore: $C_f = \frac{2}{\mathrm{Re}}\, \mathrm{Nu}$ **Reynolds Analogy**

Analogy between heat transfer and momentum transfer

Reynolds Analogy $C_f = \frac{2}{\text{Re}} \text{Nu}$

Chilton-Colburn Analogy $C_f = \frac{2}{\text{Re}} \text{Nu Pr}^{1/3}$

Define Stanton number: $\text{St} = \frac{\text{Nu}}{\text{Re Pr}}$

Chilton-Colburn Analogy $C_f = \text{St Pr}^{2/3}$