ME-427 INTRODUCTION TO NUCLEAR ENGINEERING
PROBLEM SET 4

Due: 31 Dec 1984

Pr. 1. A fast rector core consists of a mixture of 50 % enriched uranium and liquid sodium. The uranium is present to 10 a/o (atomic percentage). The shape of the reactor is cylindrical (bare). The density of the homogeneous mixture is 2 g/cm^3. The radius is 1.5 m.

(a) Determine the minimum height of the core so that the system is critical.

(b) Find the maximum neutron flux.

Use one-group neutron diffusion equation and the following data:

Isotope	Cross Sections in barns				γ	η
	σ_s	σ_f	σ_a	σ_{tr}		
Na	0.0008	0	0.0008	3.3	-	-
U-235	0.25	1.4	1.65	6.8	2.6	2.2
U-238	0.6	0.095	0.255	6.9	2.6	0.97

Solution

For criticality $B_{geo} = B_{mat}$

$$B^2 = \frac{k_\infty - 1}{L^2} = \frac{\eta f - 1}{D/\Sigma_a}$$

$$\eta = \nu \frac{\Sigma_{f,F}}{\Sigma_{a,F}}$$

$$A_U = \frac{1}{\frac{0.5}{235.0439} + \frac{0.5}{238.0508}} = 236.76 \text{ g}$$

$$A_{tot} = (0.1)(236.76) + (0.9)(22.9898) = 44.367 \text{ g}$$

$$N_{U\text{-}235} = \frac{(2)(0.1)(236.76)}{44.367}(0.5)\frac{6.02\ 10^{23}}{235.044} = 1.367\ 10^{21} \text{ atom U-235/cm}^3$$

$$N_{U\text{-}233} = 1.35\ 10^{21} \text{ atom U-238/cm}^3$$

$$\Sigma_{f,F} = (1.4\ 10^{-24})(1.367\ 10^{21}) + (0.095\ 10^{-24})(1.35\ 10^{21}) = 0.00203 \text{ cm}^{-1}$$

$$\Sigma_{a,F} = (1.65\ 10^{-24})(1.367\ 10^{21}) + (0.255\ 10^{-24})(1.35\ 10^{21}) = 0.0026 \text{ cm}^{-1}$$

$$\eta = \nu \frac{\Sigma_{f,F}}{\Sigma_{a,F}} = 2.6\ \frac{0.00203}{0.0026} = 2.02$$

$$f = \frac{\Sigma_{a,F}}{\Sigma_a} \quad \text{where} \quad \Sigma_a = \sigma_{a_{U\text{-}235}} N_{U\text{-}235} + \sigma_{a_{U\text{-}238}} N_{U\text{-}238} + \sigma_{a_{Na}} N_{Na}$$

$$N_{Na} = (2)\ \frac{6.02\ 10^{23}}{44.367}\ (0.90) = 2.44\ 10^{22}\ \text{atoms Na/cm}^3$$

$$f = \frac{\Sigma_{a,F}}{\Sigma_a} = 0.99$$

$$D = \frac{1}{3\ \Sigma_{tr}} = \frac{1}{3\left[(6.8)\ (1.367\ 10^{21}) + (6.9)\ (1.35\ 10^{21}) + (3.3)\ (2.44\ 10^{22})\right]\ 10^{-24}}$$

$$= 3.36 \text{ cm}$$

$$L^2 = \frac{3.36}{0.0026 + 2.44\ 10^{22}\ (0.0068\ 10^{-24})} = 1282.7 \text{ cm}^2$$

$$B^2 = \frac{\eta\ f - 1}{D/\Sigma_a} = \frac{(0.99)\ (2.02) - 1}{1282.7} = 7.64 \text{ m}^{-2}$$

$$7.64 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{1.5}\right)^2 \quad \Rightarrow \quad H = 1.40 \text{ m}$$

(b)

$$\phi_{max} = A = \frac{3.64}{E_R}\ \frac{P}{\Sigma_f}\ \frac{1}{Vol} = \frac{3.64}{E_R}\ \frac{P}{\Sigma_f}\ \frac{1}{\pi\ R^2\ H}$$

$$= 1.81\ \frac{P}{\Sigma_f} \text{ neutrons/m}^2\text{.s}$$

Pr. 2. For a bare, hoogeneous, critical, finite, cyclindrical reactor, the flux function is:

$$\phi = A\ J_0\left(\frac{2.405}{R}\ r\right)\ \cos\left(\frac{\pi\ z}{H}\right)$$

Show that the constant A, in terms of the power P, is

$$A = \frac{3.63\ P}{E_R\ \Sigma_{g,F^*}\ (Vol)}$$

Solution

$$\phi = A\ J_0\left(\frac{2.45}{R}\ r\right)\ \cos\left(\frac{\pi\ z}{H}\right)$$

$$P = E_R\ \Sigma_{f,F} \int_0^R \int_{-H/2}^{H/2} A\ J_0\left(\frac{2.405}{R}\ r\right) \cos\left(\frac{\pi\ z}{H}\right) (2\ \pi\ r)\ dr\ dz$$

$$= E_R\ \Sigma_{f,F}\ A \left(\frac{H}{\pi} \sin\left(\frac{\pi\ z}{H}\right)\Bigg|_{-H/2}^{H/2}\right) 2\ \pi \int_0^R r\ J_0\left(\frac{2.405}{R}\ r\right) dr$$

Using $\int_0^R r\ J_0\left(\frac{2.405}{R}\ r\right) dr = \frac{R^2}{2.405}\ J_1(2.405)$

$$A = \frac{3.63\ P}{E_R\ \Sigma_{f,F}\ \pi\ R^2\ H}$$

since $J_1(2.405) \approx 0.5184$ (Table V.1, Lamarch)

$$A = \frac{3.63\ P}{E_R\ \Sigma_{f,F}\ (Vol)}$$

Pr. 3. A bare spherical reactor 50 cm in radius is composed of a homogeneous mixture U-235 and Beryllium. The reactor operates at a powr level of 50 thermal kilowatts. Using modified one-group theory, compute,

(a) The critical mass of U-235
(b) The thermal flux throughout the reator
(c) The leakage of neutrons from the reactor
(d) The rate of consumption of U-235

Solution

(a)

$$m_F = Z\ \frac{\sigma_{a,M}(E_0)\ M_F}{g_{a,F}\ \sigma_{a,F}(E_0)\ M_M}\ m_M$$

$$Z = \frac{1 + B^2\ \left(L^2_{TM} + \tau_{TM}\right)}{\eta_T - 1 - B^2\ \tau_{TM}}$$

where $B^2 = \left(\frac{\pi}{R}\right)^2 = \left(\frac{\pi}{50}\right)^2\ cm^{-2}$

$L^2_{TM} = 480\ cm^2 \quad \tau_{TM} = 102\ cm^2 \quad \eta_T = 2.065$

Substituting: $Z = 4.98$

$m_M = \rho_M\ \frac{4}{3}\ \pi\ R^3 = 986.6$ kg where $\rho_M = 1.35\ g/cm^3$

$g_a = 0.978$ for 20 °C

$\sigma_{aF}(E_0) = 681$ barns p.211 Lamarsh

$\sigma_{aM}(E_0) = 0.0092$ barns

$$m_F = \frac{(4.98)\,(235.0439)\,(0.0092)\,(9686)}{(0.978)\,(9.0121)\,(681)} = 1.74 \text{ kg}$$

(b)

$$\phi(r) = \frac{P}{4\,R^2\,E_R\,\Sigma_f}\;\frac{\sin\left(\frac{\pi\,r}{R}\right)}{r} \qquad \text{where} \quad \Sigma_f = N_f\;\sigma_f$$

$$\phi(r) = 3.57\;10^{13}\;\frac{\sin\left(\frac{\pi\,r}{50}\right)}{r}$$

(c)

Leakage = 4 π R^2 J(R)

$$\vec{J}(R) = -\,\bar{D}_M\left.\frac{d\phi}{dr}\right|_{r=R} = -\,\bar{D}_M\;(3.57\;10^{13})\left[\frac{R\,\frac{\pi}{R}\cos\left(\frac{\pi\,R}{R}\right) - \sin\left(\frac{\pi\,R}{R}\right)}{R^2}\right]$$

$$= \bar{D}_M\;(3.57\;10^{13})\left[\frac{\pi}{R^2}\right]$$

$$\text{Leakage} = 4\,\pi\,R^2\,\bar{D}_M\,\frac{\pi}{R^2} \qquad \text{where} \quad \bar{D}_M = 0.50 \text{ cm} \quad \text{Table 5.2, Lamarsh}$$

Leakage = 7.0 10^{14} neutrons/s

(d)

Rate of consumption of U-235:

$$= (50\;10^{13})\;\frac{1}{3.2\;10^{-11}}\;\frac{235.0439}{6.023\;10^{23}} = 6.1\;10^{-7}\text{ g/s}$$

Pr. 4. Consider a critical bare slab reactor 200 cm thick consisting of a homogeneous mixture of U-235 and graphite. The maximum thermal flux is 5 10^{12} neutrons/cm^2.s. Using modified on-group theory, calculate:

(a) The buckling of the reactor
(b) The critical atomic concentration of uranium
(c) The thermal diffusion area
(d) The value of k_∞
(e) The thermal flux and current throughout the slab
(f) The thermal power produced per cm_2 of this slab

Solution

(a)

Buckling $B^2 = \left(\frac{\pi}{200}\right)^2 = 0.247\ 10^{-3}\ \text{cm}^{-2}$

(b)

$\eta_T = 2.065$ $\bar{\sigma}_{aM} = 0.0034$ b

$L^2_{TM} = 3500\ \text{cm}^2$ $\bar{\sigma}_{aF} = 681$ b

$\tau_{TM} = 368\ \text{cm}^2$ $g_{aF} = 0.978$

$$z = \frac{1 + (0.247\ 10^{-3})\,(3500 + 368)}{2.065 - 1 - (0.247\ 10^{-3})\,(368)} = 2.006$$

$$m_F = \frac{(2.006)\,(0.0034)\,(235.05)}{(0.978)\,(631)\,(12)}\,(200)\,(1.6) = 0.064\ \text{g/(unit depth).(unit height)}$$

$$N_F = \frac{m_F\ N_A}{M_F\ V} = \frac{(0.064)\,(6.023\ 10^{23})}{(235.05)\,(200)} = 8.20\ 10^{17}\ \text{atoms U-235/cm}^3$$

(c)

$$L^2_T = (1 - f)\,L^2_{TM} \qquad f = \frac{z}{z+1} = \frac{2.006}{3.006} = 0.667$$

$$L^2_T = (1 - f)\,L^2_{TM} = (1 - 0.667)\,(3500) = 1166\ \text{cm}^2$$

(d)

$$k_\infty = \eta_T\ f = (2.065)\,(0.667) = 1.378$$

(e)

$$\phi(x) = \phi_{max}\cos\left(\frac{\pi\ x}{200}\right) = (5.10^{12})\cos\left(\frac{\pi\ x}{200}\right)$$

$$\vec{J} = -D_M\frac{d\phi}{dx} = -(5\ 10^{12})\left(-\frac{\pi}{200}\sin\left(\frac{\pi\ x}{200}\right)\right) = 6.60\ 10^{10}\ \sin\left(\frac{\pi\ x}{200}\right)$$

(f)

$$\phi_{max} = (5.10^{12}) = \frac{1.57\ P}{200\ E_R\ \Sigma_f}$$

$$\Sigma_f = N_F\ \sigma_F = (8.20\ 10^{17})\,(582\ 10^{-24}) = 4.77\ 10^{-4}\ \text{cm}^{-1}$$

$$P = \frac{(5\ 10^{12})\ (200)\ (3.2\ 10^{-11})\ (4.80\ 10^{-4})}{1.57} = 9.7\ \text{W}$$