ME-427 INTRODUCTION TO NUCLEAR ENGINEERING

PROBLEM SET 3

Due: 10 Dec 1984

Pr. 1. Starting from the Maxwellian distribution function, show that the most probable energy is

$$E_p = \frac{1}{2} k T$$

Solution

The Maxwellian distribution function: $N(E) = \frac{2 \pi N_{tot} E^{1/2}}{(\pi k T)^{3/2}} e^{-E/kT}$

Most probable energy E_p occurs at $\frac{dN}{dE} = 0$

$$\frac{dN}{dt} = 0 = \frac{2 \pi N_{tot}}{(\pi k T)^{3/2}} \left[\frac{1}{2} E_p^{-1/2} e^{-E_p/kT} - E_p^{1/2} \frac{1}{k T} e^{-E_p/kT}\right]$$

$$E_p = \frac{1}{2} k T$$

Pr. 2. Show that Th-232 isotope is not fissile. At what neutron energy would you expect fission to occur?

Solution

$^{232}Th + {}^{1}_{0}n \rightarrow {}^{233}Th$

Using Table 3.3 "Intro. to Nucl. Eng." by Lamarsh

B.E. of the last neutron in Th-232 = 5.1 MeV

Critical Energy for Fission of Th-232 = 6.5 MeV

Since B.E. of the last netron < E_{crit}, the neutrons having zero kinetic energy do not induce fission. Therefore, Th-232 is not fissile.

B.E. + $KE_n \geq E_{crit}$ => $KE_n \geq 1.4$ MeV

Pr. 3. Find the diffusion coefficient and diffusion length of (a) Beryllium, (b) graphite, for monoenergetic 0.0253 eV neutrons.

Solution

$$D = \frac{1}{3\left(\Sigma_t - \Sigma_s\, \mu_0\right)} \quad , \quad \mu_0 = \frac{2}{3\,A} \quad , \quad L = \sqrt{\frac{D}{\Sigma_a}}$$

(a) Beryllium

$\rho = 1.82\ g/cm^3$, $\sigma_a = 0.01$ b , $\sigma_s = 7$ b , A = 9.01219 g/mole

$$N_{Be} = \rho\, \frac{N_0}{A} = (1.82)\, \frac{6.023\ 10^{23}}{9.01219} = 1.2165\ 10^{23}\ \text{atoms/cm}^3$$

$$\mu_0 = \frac{2}{3\,A} = \frac{2}{3\,(9.01219)} = 0.074$$

$$D = \frac{1}{3\left(\Sigma_t - \Sigma_s\, \mu_0\right)} = \frac{1}{(3)\,(1.2165)\,\left(7.01 - (7)\,(0.074)\right)\, 10^{-1}} = 0.422\ \text{cm}$$

$$L = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{0.422}{(0.01)\,(1.2165)\,10^{-1}}} = 18.63\ \text{cm}$$

(b) graphite

$\rho = 2.22\ g/cm^3$, $\sigma_a = 0.0034$ b , $\sigma_s = 4.8$ b , A = 12.011 g/mole

$$N_C = \rho\, \frac{N_0}{A} = (2.22)\, \frac{6.023\ 10^{23}}{12.011} = 1.113\ 10^{23}\ \text{atoms/cm}^3$$

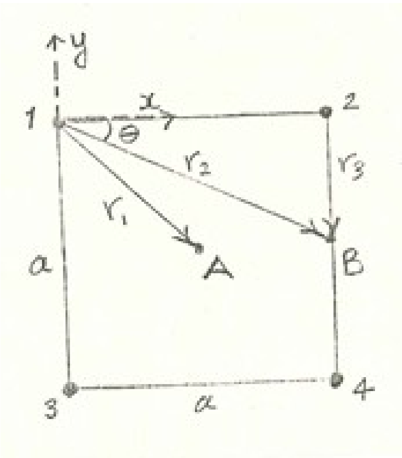
$$\mu_0 = \frac{2}{3\,A} = \frac{2}{3\,(12.011)} = 0.056$$

$$D = \frac{1}{3\left(\Sigma_t - \Sigma_s\, \mu_0\right)} = \frac{1}{(3)\,(1.113)\,\left(4.8 + 0.0034 - (4.8)\,(0.056)\right)\, 10^{-1}} = 0.66\ \text{cm}$$

$$L = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{0.66}{(0.0034)\,(1.113)\,10^{-1}}} = 41.76\ \text{cm}$$

Pr. 4. Isotropic point sources each emitting S neutrons per second are placed in an infinite moderator at the four corners of a square of side a. Compute the flux and current at the midpoint of any side of the square and its center.

Solution



$$\phi(r) = \frac{S}{4\,\pi\,D}\,\frac{e^{-r/L}}{r} \quad , \quad r > 0$$

$$r_1 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

$$r_2 = \sqrt{\left(\frac{a}{2}\right)^2 + a^2} = \frac{\sqrt{5}}{2}\,a$$

$$r_3 = \frac{a}{2}$$

Flux at point A:
$$\phi_A = 4\left[\frac{S}{4\,\pi\,D}\,\frac{e^{-\frac{a}{\sqrt{2}\,L}}}{\frac{a}{\sqrt{2}}}\right] = \frac{\sqrt{2}\,S}{\pi\,D\,a}\,e^{-\frac{a}{\sqrt{2}\,L}}$$

Flux at point B:
$$\phi_B = 2\,\frac{S}{4\,\pi\,D}\,\frac{e^{-\frac{a}{2\,L}}}{\frac{a}{2}} + 2\,\frac{S}{4\,\pi\,D}\,\frac{e^{-\frac{\sqrt{5}\,a}{2\,L}}}{\frac{\sqrt{5}\,a}{2}} = \frac{S}{\pi\,D\,a}\left[e^{-\frac{a}{2\,L}} + \frac{1}{\sqrt{5}}\,e^{-\frac{\sqrt{5}\,a}{2\,L}}\right]$$

Current at point A: $\vec{J}_A = -\,D\,\nabla\phi(r) = 0$ due to symmetry

Current at point B:
$$\vec{J}_B = -\,D\,\nabla\left[\frac{S}{2\,\pi\,D}\,\frac{e^{-\frac{r}{L}}}{r}\,\cos(\theta)\,\vec{i}\right]_{r\,=\,r_2}$$

Since currents due to sources 2 and 4 and the j-components of currents from sources 1 and 3 cance each other.

$$\vec{J} = -\,D\,\frac{S}{2\,\pi\,D}\,\frac{-\frac{1}{L}\,e^{-r_2/L}\,r - e^{-r_2/L}}{r_2^2}\,\cos(\theta)\,\vec{i} \quad , \quad r_2 = \frac{\sqrt{5}}{2}\,a \quad , \quad \cos(\theta) = \frac{a}{\frac{\sqrt{5}}{2}\,a} = \frac{2}{\sqrt{5}}$$

Substitute:
$$\vec{J} = \frac{2\,S}{5\,\pi}\,e^{-\frac{\sqrt{5}\,a}{2\,L}}\left[\frac{2}{\sqrt{5}\,a\,L} + \frac{4}{5\,a^2}\right]\frac{2}{\sqrt{5}}\,\vec{i}$$

$$\vec{J} = \frac{2\,S}{5\,\pi}\,e^{-\frac{\sqrt{5}\,a}{2\,L}}\left[\frac{1}{a\,L} + \frac{2}{\sqrt{5}\,a^2}\right]\vec{i}$$

Pr. 5. A point source emits S neutrons per second isotropically in an infinite vacuum. Show that the neutron flux is given by the expression $\phi = \frac{S}{4\pi r^2}$ where r is the distance from the source. What is the neutron current density vector at the same point?

Solution

$$\text{Flux} = \frac{\textit{Neutrons}}{\text{(unit area) (unit time)}}$$

(a)

Since the medium is vacuum, there is no loss of neotrons due to absorption, scattering, etc.

Hence, $\phi(r) = \frac{\text{S neutrons/second}}{\text{Area}} = \frac{S}{4\pi r^2}$

(b)

Since there is no scattering collisions in the medium, only the neutrons from the source will contribue to the value of neutron current density vector, J.

Hence, $\vec{J} = \frac{S}{4\pi r^2}\ \vec{i}$

Pr. 6. The neutron flux in a spherical medium of radius 50 cm is given by

$$\phi = 5\ 10^{13}\ \frac{\sin(0.0620\ r)}{r}\ \text{neutrons/cm}^2\text{.s}$$

where r is measured from the center of the sphere. The diffusion coefficient is 0.8 cm.

(a) What is the maximum value of the flux?

(b) Calculate the neutron current density as a function of r.

(c) How many neutrons escape from the medium per second?

Solution

$$\phi(r) = 5\ 10^{13}\ \frac{\sin(0.0620\ r)}{r}\ \text{neutrons/cm}^2\text{.s}$$

(a)

$\phi = \phi_{max}$ at r =0

$$\phi_{max} = \lim_{r\to 0}\left[5\ 10^{13}\ \frac{\sin(0.0620\ r)}{r}\right] = 5\ 10^{13}\ \lim_{r\to 0}\left[\frac{\sin(0.0620\ r)}{r}\right] = \left(5\ 10^{13}\right)\ (0.0628)$$

Hence $= 3.14\ 10^{12}$ neutrons/cm^2.s

(b)

$$\vec{J}(r) = -D\,\frac{d\phi}{dr} = -0.8\left[5\ 10^{13}\left(\frac{0.028\ r\cos(0.0628\ r) - \sin(0.0628\ r)}{r_2}\right)\right]\vec{i}$$

$$\vec{J}(r) = -4\ 10^{13}\left[\left(\frac{0.028\ r\cos(0.0628\ r) - \sin(0.0628\ r)}{r_2}\right)\right]\vec{i}$$

(c)

No of neutrons escaping:

$$J(R)\,(4\,\pi\,R^2) = -16\,\pi\,10^{13}\,R^2\left[\frac{(0.0628)\,(50)\cos\left((0.0628)\,(50)\right) - \sin\left((0.0628)\,(50)\right)}{R^2}\right]$$

$$= 1.58\ 10^{15}\ \text{neutrons/s}$$

Pr. 7. A sphere of moderator of radius R contains uniformly distributed sources emitting S neutrons/cm^2.s.

(a) Show that the flux in the sphere is given by

$$\phi(r) = \frac{S}{\Sigma_a}\left[1 - \left(\frac{R+d}{r}\right)\frac{\sinh\left(\frac{r}{L}\right)}{\sinh\left(\frac{R+d}{L}\right)}\right]$$

(b) How many neutrons leak from the sphere per second?

d is the extrapolation distance = 0.71 λ_{tr}.

Solution

Uniformy distributed sources S neutrons/cm^2.s

Diffusion equation: $D\,\nabla^2\phi - \Sigma_a\,\phi + S = 0$

BC's: 1) at r = 0 ϕ is finite or $\frac{d\phi}{dr} = 0$

2) at r = R + d ϕ = 0

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{1}{L^2}\,\phi = -\frac{S}{D} \quad \Rightarrow \quad \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{1}{L^2}\,\phi = -\frac{S}{D}$$

Replace $\phi(r) = \frac{y(r)}{r}$ => $\frac{d^2y}{dr^2} - \frac{1}{L^2}\,y = -\frac{S}{D}\,r$

Solution: $y(r) = C_1\sinh\left(\frac{r}{L}\right) + C_2\cosh\left(\frac{r}{L}\right) + \frac{S\,L^2}{D}\,r$

Or

$$y(r) = C_1 \frac{\sinh(r/L)}{r} + C_2 \frac{\cosh(r/L)}{r} + \frac{S L^2}{D}$$

BC 1 => $C_2 = 0$

BC 2 =>

$$0 = C_1 \frac{\sinh\left(\frac{R + d}{L}\right)}{R + d} + \frac{S L^2}{D}$$

$$C_1 = - \frac{S L^2}{D} \frac{R + d}{\sinh\left(\frac{R + d}{L}\right)}$$

$$\frac{S L^2}{D} = \frac{S}{\Sigma_a} \quad \Rightarrow \quad \phi(r) = \frac{S}{\Sigma_a}\left[1 - \frac{R + d}{r} \frac{\sinh(r/L)}{\sinh((R + d)/L)}\right]$$

(b)

No of neutrons leakşng per second = $J(R)\,(4\pi R^2)$

$$= - D\,(4\pi R^2)\left\{- \frac{S}{\Sigma_a} \frac{R + d}{\sinh((R + d)/L)}\left[\frac{\cosh(r/L)\,\frac{r}{L} - \sinh(r/L)}{r^2}\right]_{r = R}\right\}$$

$$= \frac{4\pi D S}{\Sigma_a} \frac{R + d}{\sinh((R + d)/L)}\left[\frac{R \cosh(R/L)}{L} - \sinh(R/L)\right]$$

Pr. 8. A radioactive sample with half life $T_{1/2}$ is placed at a point where the thermal neutron flux is ϕ_T. Show that the sample disappears as the result of its own decay and by neutron absorption with an effective half life given by

$$\left(\frac{1}{T_{1/2}}\right)_{eff} = \frac{1}{T_{1/2}} + \frac{\sigma_a \phi_T}{\ln(2)}$$

where σ is the average thermal absorption coefficient of the sample.

Solution

$$\left(\frac{dN}{dt}\right)_{eff} = \text{(Rate of its own decay)} + \text{(Rate of neutron absorption)}$$

$$= - \lambda N - \phi_t \Sigma_a = - N\left(\lambda + \phi_t \sigma_a\right)$$

$$N(t) = N_0\, e^{-(\lambda + \phi_t \sigma_a) t}$$

$$\frac{N_0}{2} = N_0\, e^{-(\lambda + \phi_t \sigma_a)(T_{1/2})_{eff}} \quad \Rightarrow \quad - \ln(2) = -\left(\lambda + \phi_t \sigma_a\right) (T_{1/2})_{eff}$$

$$\frac{1}{(T_{1/2})_{eff}} = \frac{\lambda}{\ln(2)} + \frac{\phi_t \ \sigma_a}{\ln(2)}$$

Since $\lambda = \dfrac{\ln(2)}{T_{1/2}}$ $\Rightarrow$ $\dfrac{1}{(T_{1/2})_{eff}} = \dfrac{1}{T_{1/2}} + \dfrac{\phi_t \ \sigma_a}{\ln(2)}$