

ME-427 INTRODUCTION TO NUCLEAR ENGINEERING
PROBLEM SET 1

Due: 22.10.1984

Pr. 1. A beaker contains 50 g of ordinary, i.e., naturally occurring, water.

- (a) How many moles of water are present?
- (b) How many hydrogen atoms?
- (c) How many deuterium atoms?

Solution

$$\text{Moles of H}_2\text{O} = \frac{50 \text{ g H}_2\text{O}}{((2)(1.00797) + 15.9994)} \frac{\text{g H}_2\text{O}}{\text{mole H}_2\text{O}} = 2.775 \text{ moles of H}_2\text{O}$$

(a)

$$\text{No of Hydrogen atoms} = (2.775 \text{ mole H}_2\text{O}) \left(\frac{2 \text{ moles H}}{1 \text{ mole H}_2\text{O}} \right) \left(\frac{6.023 \cdot 10^{23} \text{ atoms H}}{1 \text{ mole H}} \right)$$

(b) $= 3.34 \cdot 10^{24} \text{ atoms H}$

$$\text{No of Deuterium atoms} = \left(\frac{0.000151 \text{ Deuterium atoms}}{1 \text{ Hydrogen atom}} \right) (3.34 \cdot 10^{24} \text{ atoms H})$$

(c) $= 5.05 \cdot 10^{20} \text{ Deuterium atoms}$

Pr. 2. Compute the atomic weight of natural uranium which is composed of three isotopes, U-234, U-235, and U-238.

Solution

U-234: 0.0057 %

U-235: 0.714 %

U-238: 99.28 %

$$\begin{aligned} A_{\text{Uranium}} &= (0.000057)(234.0409) + (0.00714)(235.0439) + (0.9928)(238.0508) \\ &= 238.028 \text{ amu} \end{aligned}$$

Pr. 3. Carbon tetrachloride labeled with C-14 is sold commercially with an activity of 10 milli Curies per millimole (10 mCi/Mm). What fraction of the carbon atoms is C-14?

Solution

$$\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{5510 \text{ y}} = 3.946 \cdot 10^{-12} \text{ s}^{-1}$$

Activity = λN where

$$\lambda N = 3.946 \cdot 10^{-12} N = 10 \frac{\text{mCi}}{\text{m M}} = 3.7 \cdot 10^{11} \text{ disintegrations/s.Mole}$$

$$N = 9.38 \cdot 10^{22} \text{ atoms C-14/Mole}$$

$$\text{Fractions of C-14 atoms} = \frac{9.38 \cdot 10^{22} \text{ atoms C-14 /M CCl}_4}{6.023 \cdot 10^{23} \text{ atoms C /M CCl}_4} = 0.156$$

Pr. 4. Compute the average binding energy per nucleon of the following nuclei

(a) H-2 (b) C-12 (c) U-235

Solution

$$(a) {}_1^2\text{H}$$

$$\begin{aligned} \text{Mass defect} &= \Delta M = M_{{}_1^1\text{H}} + M_n - M_{{}_1^2\text{H}} \\ &= 1.007825 + 1.008665 - 2.01410 = 2.3905 \cdot 10^{-3} \text{ amu} \end{aligned}$$

$$\text{Binding energy} = BE = (2.3905 \cdot 10^{-3} \text{ amu}) (931.431 \text{ MeV/amu}) = 2.226 \text{ MeV}$$

$$\frac{BE}{\text{Nucleon}} = \frac{2.226}{2} = 1.113 \text{ MeV}$$

$$(b) {}_6^{12}\text{C}$$

$$\begin{aligned} \text{Mass defect} &= \Delta M = Z M_p + N M_n - M_A \\ &= 6 (1.007277) + 6 (1.008665) - (12.000 - 6 (0.000549)) = 0.09894 \text{ amu} \end{aligned}$$

$$\text{Binding energy} = BE = (0.09894 \text{ amu}) (931.431 \text{ MeV/amu}) = 92.164 \text{ MeV}$$

$$\frac{BE}{\text{Nucleon}} = \frac{92.164}{12} = 7.68 \text{ MeV}$$

$$(c) {}_{92}^{235}\text{U}$$

$$\begin{aligned} \text{Mass defect} &= \Delta M = Z M_p + N M_n - M_A \\ &= 92 (1.007277) + (235 - 92) (1.008665) - (235.0439 - 92 (0.000549)) \\ &= 1.915187 \text{ amu} \end{aligned}$$

$$\text{Binding energy} = BE = (1.915187 \text{ amu}) (931.431 \text{ MeV/amu}) = 1783.96 \text{ MeV}$$

$$\frac{BE}{\text{Nucleon}} = \frac{1783.96}{235} = 7.59 \text{ MeV}$$

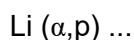
Pr. 5. What is the atom density of U-235 in 2.5 % enriched uranium if the density of uranium is 19 g/cm³.

Solution

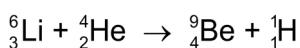
$$\text{Atom density of Uranium} = \rho \frac{N_A}{M}$$

$$\text{Atom density of U-235} = (0.025) (19) \frac{6.023 \cdot 10^{23}}{238.02891} = 1.20192 \cdot 10^{21} \text{ atoms of U-235 / cm}^3$$

Pr. 6. Complete the following reaction and find the Q value.



Solution



$$\begin{aligned} Q &= [(M_{\text{Li}} + M_{\text{He}}) - (M_{\text{Be}} + M_{\text{H}})] (931.481) \\ &= [(6.01513 + 4.00260) - (9.01219 + 1.007825)] (931.481) \\ &= -2.128 \text{ MeV} \end{aligned}$$

Pr. 7. The average kinetic energy of a fission neutron is 2 MeV. What is the velocity? Must it be considered relativistic?

Solution

$$KE = \frac{1}{2} m_n V^2 = 2 \text{ MeV} = (2 \cdot 10^6) (1.602 \cdot 10^{-19}) \text{ Joules}$$

$$m_n = (1.008665 \text{ amu}) (1.66053 \cdot 10^{-27} \text{ kg/amu})$$

$$V = 1.956 \cdot 10^7 \text{ m/s}$$

Since V < c it cannot be considered relativistic.

Pr. 8. How long will it take the activity of a 5 μCi Co-60 source to decrease to 1 μCi ?

Solution

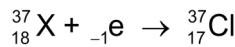
$$A(t) = A_0 e^{-\lambda t} \quad \text{where} \quad \lambda = \frac{\ln(2)}{5.2 \text{ y}} = 0.13228 \text{ y}^{-1}$$

$$1 \mu\text{Ci} = (5 \mu\text{Ci}) e^{-0.013228 t}$$

$$-0.13228 t = \ln\left(\frac{1}{5}\right) \Rightarrow t = 12.167 \text{ y}$$

Pr. 9. An unstable nuclide decays by electron capture to stable Cl-37. The Q-value of the reaction is 0.814 MeV. What is the unstable parent nucleus? What is the velocity of the recoiling Cl-37 nucleus?

Solution



The unstable nucleus is ${}^{37}_{18}\text{Ar}$

Given $Q = 0.814 \text{ MeV}$:

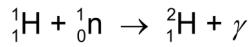
$$\frac{1}{2} M_{\text{Cl}} V_{\text{Cl}}^2 = (0.184 \cdot 10^6 \text{ eV}) (1.602 \cdot 10^{-19} \text{ J/eV})$$

$$\frac{1}{2} (36.96590 - (17)(0.00054)) (1.66053 \cdot 10^{-27}) V_{\text{Cl}}^2 = 1.304 \cdot 10^{-13}$$

$$V_{\text{Cl}}^2 = 2.062 \cdot 10^6 \text{ m/s}$$

Pr. 10. A light hydrogen atom absorbs a thermal neutron in a radioactive capture process. Compute the energy of the γ entity. Calculate the recoil velocity and kinetic energy of the deuteron.

Solution



$$Q = \left(m_{^1\text{H}} + m_{^1\text{H}} - m_{^2\text{H}} \right) (931.481) = 2.226 \text{ MeV}$$

$$(KE)_{\text{neutron}} + Q\text{-value} = (KE)_{\text{deuterium}} + E_{\gamma}$$

$$\frac{1}{2} (1.008665) (1.66053 \cdot 10^{-27}) V_n^2 = (0.0253) (1.602 \cdot 10^{-19}) \text{ J}$$

$$V_n = 2199.934 \text{ m/s}$$

Conservations of momentum: $m_n V_n = m_d V_d$

$$V_d = \frac{m_n}{m_d} V_n = \frac{1.006665}{2.01410} (2199.934) = 1101.73 \text{ m/s}$$

$$\begin{aligned} (KE)_{\text{deuterium}} &= \frac{1}{2} m_d V_d^2 \\ &= \frac{1}{2} (2.01410) (1.66053 \cdot 10^{-27}) (1101.73)^2 (6.242 \cdot 10^{18}) \\ &= 0.0126 \text{ eV} \end{aligned}$$

$$\text{Substitute: } 0.0253 + 2.226 \cdot 10^6 = 0.0126 + E_{\gamma}$$

$$E_{\gamma} \cong 2.226 \text{ MeV}$$