ME-427 INTRODUCTION TO NUCLEAR ENGINEERING

PROBLEM SET 1

Due: 22.10.1984

Pr. 1. A beaker contains 50 g of ordinary, i.e., naturally occuring, water.

(a) How many moles of water are present?

(b) How many hydrogen atoms?

(c) How many deuterium atoms?

Solution

(a)

$$\text{Moles of } H_2O = \frac{50 \text{ g } H_2O}{\left((2)\,(1.00797) + 15.9994\right)\ \frac{\text{g } H_2O}{\text{mole } H_2O}} = 2.775 \text{ moles of } H_2O$$

(b)

$$\text{No of Hydrogen atoms} = (2.775 \text{ mole } H_2O)\left(\frac{2 \text{ moles H}}{1 \text{ mole } H_2O}\right)\left(\frac{6.023\ 10^{23} \text{ atoms H}}{1 \text{ mole H}}\right)$$

$$= 3.34\ 10^{24} \text{ atoms H}$$

(c)

$$\text{No of Deuterium atoms} = \left(\frac{0.000151 \text{ Deuterium atoms}}{1 \text{ Hydrogen atom}}\right)\left(3.34\ 10^{24} \text{ atoms H}\right)$$

$$= 5.05\ 10^{20} \text{ Deuterium atoms}$$

Pr. 2. Compute the atomic weight of natural uranium which is composed of three isotopes, U-234, U-235, and U-238.

Solution

U-234: 0.0057 %

U-235: 0.714 %

U-238: 99.28 %

$$A_{Uranium} = (0.000057)\,(234.0409) + (0.00714)\,(235.0439) + (0.9928)\,(238.0508)$$

$$= 238.028 \text{ amu}$$

Pr. 3. Carbon tetrachloride labeled with C-14 is sold commercially with an activity of 10 milli Curies per millimole (10 mCi/Mm). What fraction of the carbon atoms is C-14?

Solution

Activity = λ N where $\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{5510\ y} = 3.946\ 10^{-12}\ s^{-1}$

λ N = 3.946 10^{-12} N = 10 $\frac{mCi}{m\ M}$ = 3.7 10^{11} distegrations/s.Mole

N = 9.38 10^{22} atoms C-14/Mole

Fractions of C-14 atoms = $\frac{9.38\ 10^{22}\ \text{atoms C-14 /M } CCl_4}{6.023\ 10^{23}\ \text{atoms C /M } CCl_4} = 0.156$

Pr. 4. Compute the average binding energy per nucleon of the following nuclei

(a) H-2 (b) C-12 (c) U-235

Solution

(a) $^{2}_{1}H$

Mass defect = $\Delta M = M_{^{1}_{1}H} + M_n - M_{^{2}_{1}H}$

= 1.007825 + 1.008665 - 2.01410 = 2.3905 10^{-3} amu

Binding energy = BE = $\left(2.3905\ 10^{-3}\ \text{amu}\right)\ \left(931.431\ \text{MeV/amu}\right)$ = 2.226 MeV

$\frac{BE}{Nucleon} = \frac{2.226}{2} = 1.113$ MeV

(b) $^{12}_{6}C$

Mass defect = $\Delta M = Z\ M_p + N\ M_n - M_A$

= 6 (1.007277) + 6 (1.008665) - $\left(12.000 - 6\ (0.000549)\right)$ = 0.09894 amu

Binding energy = BE = $\left(0.09894\ \text{amu}\right)\ \left(931.431\ \text{MeV/amu}\right)$ = 92.164 MeV

$\frac{BE}{Nucleon} = \frac{92.164}{12} = 7.68$ MeV

(c) $^{235}_{92}U$

Mass defect = $\Delta M = Z\ M_p + N\ M_n - M_A$

= 92 (1.007277) + (235 - 92) (1.008665) - $\left(235.0439 - 92\ (0.000549)\right)$

= 1.915187 amu

Binding energy = BE = $\left(1.915187\ \text{amu}\right)\ \left(931.431\ \text{MeV/amu}\right)$ = 1783.96 MeV

$$\frac{BE}{Nucleon} = \frac{1783.96}{235} = 7.59 \text{ MeV}$$

Pr. 5. What is the atom density of U-235 in 2.5 % enriched uranium if the desity of uranium is 19 g/cm^3.

Solution

Atom density of Uranium = $\rho \frac{N_A}{M}$

Atom density of U-235 = (0.025) (19) $\frac{6.023\ 10^{23}}{238.02891}$ = 1.20192 10^{21} atoms of U-235 / cm^3

Pr. 6. Complete the following reaction and find the Q value.

Li (α,p) ...

Solution

$^{6}_{3}Li + ^{4}_{2}He \rightarrow ^{9}_{4}Be + ^{1}_{1}H$

$$Q = \left[\left(M_{Li} + M_{He}\right) - \left(M_{Be} + M_{H}\right)\right] (931.481)$$

$$= \left[\left(6.01513 + 4.00260\right) - \left(9.01219 + 1.007825\right)\right] (931.481)$$

$$= - 2.128 \text{ MeV}$$

Pr. 7. The average kinetic energy of a fission neutron is 2 MeV. What is the velocity? Must it be considered relativistic?

Solution

$KE = \frac{1}{2} m_n V^2 = 2 \text{ MeV} = (2\ 10^6)\ (1.602\ 10^{-19})$ Joules

$m_n = (1.008665 \text{ amu})\ (1.66053\ 10^{-27} \text{ kg/amu})$

$V = 1.956\ 10^7$ m/s

Since V < c it cannot be considered relativistic.

Pr. 8. How long will it take the activity of a 5 μCi Co-60 source to decrease to 1 μCi?

Solution

$A(t) = A_0\ e^{-\lambda t}$ where $\lambda = \dfrac{\ln(2)}{5.2\ y} = 0.13228\ y^{-1}$

$1\ \mu Ci = (5\ \mu Ci)\ e^{-0.013228\ t}$

$-0.13228\ t = \ln\left(\dfrac{1}{5}\right) \quad \Rightarrow \quad t = 12.167\ y$

Pr. 9. An unstable nuclide decays by electron capture to stable Cl-37. The Q-value of the reaction is 0.814 MeV. What is the unstable parent nucleus? What is the velocity of the recoiling Cl-37 nucleus?

Solution

$^{37}_{18}X + {}_{-1}e \rightarrow {}^{37}_{17}Cl$

The unstable nucleus is $^{37}_{18}Ar$

Given Q = 0.814 MeV:

$\dfrac{1}{2}\ M_{Cl}\ V_{Cl}^2 = \left(0.184\ 10^6\ eV\right)\ \left(1.602\ 10^{-19}\ J/eV\right)$

$\dfrac{1}{2}\ \left(36.96590 - (17)\ (0.00054)\right)\ (1.66053\ 10^{-27})\ V_{Cl}^2 = 1.304\ 10^{-13}$

$V_{Cl}^2 = 2.062\ 10^6\ m/s$

Pr. 10. A light hydrogen atom absorbs a thermal neutron in a radiaoctive capture process. Compute the energy of the γ entity. Calculate the recoil velocity and kinetic energy of the deuteron.

Solution

$${}_1^1H + {}_0^1n \rightarrow {}_1^2H + \gamma$$

$$Q = \left(m_{{}_1^1H} + m_{{}_0^1n} - m_{{}_1^2H}\right)(931.481) = 2.226 \text{ MeV}$$

$$(KE)_{neutron} + \text{Q-value} = (KE)_{deuterium} + E_\gamma$$

$$\frac{1}{2}(1.008665)(1.66053\ 10^{-27})\ V_n^2 = (0.0253)(1.602\ 10^{-19})\ J$$

$$V_n = 2199.934 \text{ m/s}$$

Conservations of momentum: $m_n V_n = m_d V_d$

$$V_d = \frac{m_n}{m_d} V_n = \frac{1.006665}{2.01410}(2199.934) = 1101.73 \text{ m/s}$$

$$(KE)_{deuterium} = \frac{1}{2} m_d V_d^2$$

$$= \frac{1}{2}(2.01410)(1.66053\ 10^{-27})(1101.73)^2\ (6.242\ 10^{18})$$

$$= 0.0126 \text{ eV}$$

Substitute: $0.0253 + 2.226\ 10^6 = 0.0126 + E_\lambda$

$$E_\lambda \cong 2.226 \text{ MeV}$$